Existence and stability of Floquet breathers in a nonlinear system with parametric modulation

Juan F.R. Archilla[†], Masayuki Kimura[‡], Yusuke Doi[⋆], Víctor J. Sánchez-Morcillo[§]

† Group of Nonlinear Physics, Department of Applied Physics I
Universidad de Sevilla, ETSII, Avda Reina Mercedes s/n, 41012-Sevilla, Spain
‡ Department of Electrical and Electronic Engineering, Faculty of Science and Engineering
Setsunan University, 17-8 Ikeda-Nakamachi, Neyagawa, Osaka 572-8508, Japan
⋆ Laboratory of Microdynamics, Department of Mechanical Engineering, Graduate School of Engineering
Osaka University 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan
§ IGIC, Universitat Politècnica de València, Paranimf 1, 46730 Grao de Gandia, Spain
Email: archilla@us.es, masayuki.kimura@setsunan.ac.jp, doi@mech.eng.osaka-u.ac.jp, victorsm@fis.upv.es

Abstract—Parametric time-modulation bring about important changes in the phonon dispersion relation, the conditions of existence and the stability analysis of breathers. These changes are analyzed and explained.

Model and results

The system considered is an array of coupled cantilevers with a restoring force increased by an electromagnet fed with both DC and AC currents. In this way, a parameter, the restoring constant is modulated in time and it can be also be modulated in space by implementing a phase difference in the AC current through simple electronic means. The dynamical equation are deduced modifying a simpler system [1]. The system differential equations after re-scaling become [2]:

$$\begin{aligned} \ddot{u}_n &= -\omega_0^2 u_n \\ &- \big(-\delta_1 u_n + (1+\delta_2\cos(hn-\Omega t))\frac{u_n}{(u_n^2+d_0^2)^{3/2}}\big) \\ &+ \kappa(u_{n+1}+u_{n-1}-2u_n)\,, \end{aligned} \qquad (1)$$

with $\omega_0 = 1$, δ_1, δ_2, and d_0 constants.

Using Bloch theorem, it is deduced that the dispersion relation becomes:

$$\omega = -m\Omega + \sqrt{\omega_0^2 + 2\kappa(1-\cos(q+mh))}\,, \qquad (2)$$

with m an integer.

We call the breathers in this type of system Floquet breathers, similarly to Floquet solitons in optical systems [3]. Modifying the breather theory as described in [4], a sufficient condition for breather existence is obtained: the moving-frame frequency of the breather and the modulating wave are commensurate. For stationary breathers the condition is applied to the breather frequency. We obtain stationary breathers experiencing period-doubling, period-halving, and many other combination of frequencies. Some breathers are in phase and other in quadrature with the modulating frequency.

It is observed and deduced that the system can be embedded into an extended system which is both symplectic and autonomous, therefore with the Floquet multipliers at the unit circle when a breather is stable. For the original non-autonomous system, the Floquet multipliers are also at the unit circle but without the phase and growth mode multipliers at +1. The existence of the breather appears as a pair of localized Floquet multipliers at the unit circle corresponding to the internal *breathing* mode of the breather. We analyze the evolution of breather stability with the frequency for a large number of breather frequencies.

ORCID iDs JFR Archilla: 0000-0001-6583-6114,
M Kimura: 0000-0002-1445-6266,
Y Doi: 0000-0003-3749-5353,
VJ Sánchez-Morcillo: 0000-0003-4766-1263

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References

[1] M. Kimura, T. Hikihara, Phys. Lett. A **373**, 1257 (2009)

[2] M. Kimura, J.F.R. Archilla, Y. Doi, V.J. Sánchez-Morcillo, Chaos **36**, 033151 (2026)

[3] M. Johansson, et al., Chaos **35**, 123107 (2025)

[4] J.F.R. Archilla, Y. Doi, M. Kimura, Phys. Rev. E **100**, 022206 (2019)