# Full energy transfer between breathers in soft potential lattices

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**Abstract:** In this paper we present the phenomenon of full interchange of energy between two discrete breathers. It consists of two breathers with close but different energies separated a certain distance *d*. Then, there is an interchange of energy while the breathers oscillate and, in a very different time scale, an energy transfer [1,2] between the two breathers occurs, achieving a full-energy interchange which repeats periodically. The system is a Klein-Gordon system with an on-site Morse potential and harmonic coupling. Discrete breathers are periodic localized vibrations in nonlinear systems. Two breathers with close frequencies below the optical phonon band are constructed from the anticontinuous limit. Then, they are located in a larger lattice separated a distance *d,* defined as the number of particles between the three central particles of each breather. They can be located in phase or with a phase difference of π (out of phase), but the full interchange only occurs for out of phase breathers. The breathers at the initial time are represented in Fig. 1, with a distance *d*=4. When they are left to interact their energies can be seen in Fig. 2,

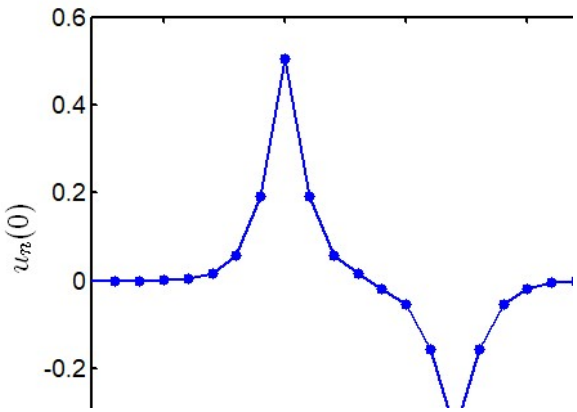


Fig. 1. Profile of the two breathers

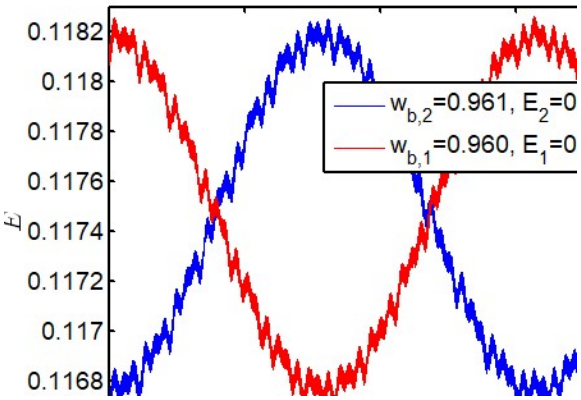


Fig. 2. Full energy transfer

The dependence of the full-interchange characteristics and periods on the distance and energies are analyzed. The phenomenon is not susceptible to enhancement with an external frequency, showing the intrinsic character of the phenomenon, an explanation of which is intended. Similar phenomenon was observed for FPU systems and hard quartic potentials in Refs. [3,4], an important difference is that with soft potential the interchange appear only in out-of-phase breathers.

**Keywords:** breather, energy transfer, nonlinearity, intrinsic localization, emergent phenomenon

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