Space-time modulation of a nonlinear system

Juan F.R. Archilla†, Masayuki Kimura‡, Yusuke Doi⋆, Víctor J. Sánchez-Morcillo§

† Group of Nonlinear Physics, Department of Applied Physics I
Universidad de Sevilla, ETSII, Avda Reina Mercedes s/n, 41012-Sevilla, Spain
‡ Department of Electrical and Electronic Engineering, Faculty of Science and Engineering
Setsunan University, 17-8 Ikeda-Nakamachi, Neyagawa, Osaka 572-8508, Japan
⋆ Laboratory of Microdynamics, Department of Mechanical Engineering, Graduate School of Engineering
Osaka University 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan
§ IGIC, Universitat Politècnica de València, Paranimf 1, 46730 Grao de Gandia, Spain
Email: archilla@us.es, masayuki.kimura@setsunan.ac.jp, doi@mech.eng.osaka-u.ac.jp, victorsm@fis.upv.es

Abstract—We consider a model that can be modulated in time and space and can be reproduced physically as a cantilever array. We explore the linear spectrum and show that localized nonlinear excitations, as breathers, can be obtained in such a complex system.

Model and results

Dynamic metamaterials, are artificial structures that incorporate non only space modulation, but also time modulation, or space-time modulation. The modulation can be achieved with acoustic waves, voltage waves, or electromagnetic waves, that may change media parameters as permittivity, magnetic permeability, refractive index, or other properties [1].

One of the authors constructed a physical model for cantilever arrays with an on-site potential provided by electromagnets, which is therefore both nonlinear and tunable by changing the electric current [2] described by the dynamical equation:

$$\ddot{u}_n = -(\omega_0^2 - \delta\cos(hn - \Omega t))u_n + \kappa(u_{n+1} + u_{n-1} - 2u_n)\,, \quad (1)$$

The modulation in time and space produce a transformation of the the phonon bands in $\omega - q$ space where new bands and forbidden bands appear. We develop the spectral theory [3] for the modulated systems and were able to obtain the equations of the bands (see Fig. 1):

$$\omega = +m\Omega + \sqrt{\omega_0^2 + 2\kappa(1 - \cos(q + mh))} \quad (2)$$

A modification of the system introduces nonlinearity and we are able to find breathers, both site centered and bond centered. The properties of those breathers and analyzed and related with the band spectrum.

ORCID iDs JFR Archilla: 0000-0001-6583-6114,
M Kimura: 0000-0002-1445-6266,
Y Doi: 0000-0003-3749-5353,
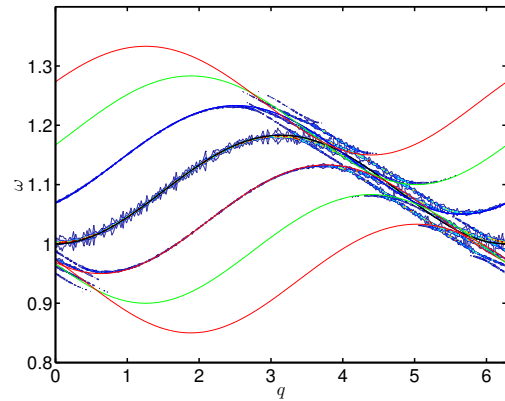VJ Sánchez-Morcillo: 0000-0003-4766-1263

Figure 1: Theoretical and numerical frequency-momenta representation

In conclusion, modulated systems offer an striking change in the phonon spectrum providing for new nonlinear excitations to appear. As they can be engineered, they allow for the design of breather properties.

Acknowledgments

References

[1] C. Caloz, Z.L. Deck-Léger, A. Bahrami, O.C. Vicente, Z. Li, IEEE Antenn. Propag. M. **65**, 50 (2023)

[2] M. Kimura, T. Hikihara, Phys. Lett. A **373**, 1257 (2009)

[3] J.F.R. Archilla, Y. Doi, M. Kimura, Phys. Rev. E **100**(2), 022206 (2019)