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Lifetime of breathers in hard and soft potentials

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Abstract—We study the lifetime of breathers when they are produced in a thermalized background and the evolution to thermalization is observed. We deduce a theoretical value of a magnitude called the participation number P that indicates the arrival to thermal equilibrium. It is found that the lifetime has a great variability, but their average have an exponential dependence of the breather energy. This property holds for several systems although the exponential coefficients differ from system to system.

1. Procedure and results

We consider both symmetric hard and soft Klein-Gordon potentials and harmonic coupling. The Hamiltonian of the system in scaled variables is given by:

$$H = \sum_n \frac{p_n^2}{2} + \omega_0^2\Big(\frac{u_n^2}{2} + s\frac{u_n^4}{4}\Big) + \varepsilon\frac{1}{2}(u_{n+1} - u_n)^2 ,$$

where ω_0 is the frequency of the isolated oscillator at the linear limit and it is taken as $\omega_0 = 1$, and $\varepsilon = 0.05$ is the coupling parameter. The nonlinearity parameter $s = \pm 1$ selects a hard or soft potential. Other values of the parameters can be scaled to the previous equation, except for the ratio ε/ω_0^2.

We first obtain breathers [1] from the anticontinuous limit [2] to have an indication of the energies involved. We can deduce the frequencies and their stability, being $\omega_b \in (1.1, 2)$ for hard potentials and $\omega_b \in (0.54, 1)$ for soft potentials. The energies are of a few tenths depending of the frequencies but smaller for soft potentials.

A measure of localization is the participation number $P = E^2/\sum e_n^2$, where e_n is the local energy and $E = \sum e_n$. P takes values between 1, when all the energy is only at one particle and the number of particles N if all of them have the same energies. We deduce that at thermal equilibrium

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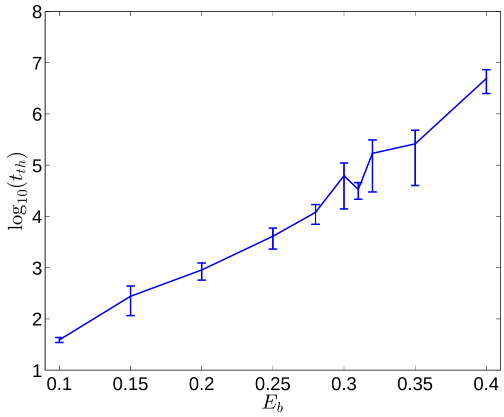

$P = N/2$. Then we thermalize the system with a given average local energy $\langle e_n \rangle = 0.02$ and add a kinetic energy E_b at a single site, thereafter observing the time until P becomes $N/2$. These times vary greatly but their averages follow an exponential law as can be seen in the figure. This confirms that K-G systems have similar relaxation to equilibrium as the DNLS in spite of not having a conserved norm [3].

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