Path to equilibrium of breathers in nonlinear lattices

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Abstract: A simple parameter that characterizes the degree of localization in a lattice is the participation number *P*, defined as the ratio between the square of the total energy and the sum of the squares of the local energies. It takes values between 1 when all the energy is in one particle and N, when all the particles have the same energy. An approximate deduction proves that at thermal equilibrium $<P>=N/2$, where *N* is the number of particles in the system. Numerical simulations result in a slightly larger value. However, we can take it as a parameter indicating that the system is very close to thermal equilibrium as the variance or *P* is anyway relatively large.
We produce a thermalized state at thermal equilibrium with a given average energy per particle $h=E/N$, where *E* is the total energy of the system. Then, we add a localized energy E_b to a single site, mimicking the interaction of an incident particle as a neutron. We call the localized excitation a *discrete breather* [1,2], as it tends to perform localized oscillations during some time in the thermalized environment. We let the system evolve until $P=N/2$ after some time that we call the *thermalization* time t_{th}. We perform many simulations which lead to many different values of t_{th}, as it should be expected because the initial conditions are very different, including but not limited to the value of *P*.

We observe that t_{th} decreases exponentially with the energy delivered E_b both for hard and soft quartic potentials, a result previously observed in [3] for the DNLS. This suggests that the conservation of the norm in the DNLS is not necessary for the exponential behavior of the thermalization time. The similitude of the exponents with both hard and soft potentials is striking, and we are repeating the study for some different systems as the Frenkel-Kontorova lattice, and models for Josephson-junction arrays, with the objective of determining the generality of the phenomenon.

Keywords: Nonlinear lattices. Breathers. Thermal equilibrium. Lifetime.

References

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