Motion of discrete solitons assisted by nonlinearity management

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Introduction

- Array of nearly isolate droplets of BEC in a deep optical lattice
- Application of an AC Feshbach Resonance Management
- DNLS model with time-dependent nonlinear coefficient

$$i\dot{u}_n + u_{n+1} + u_{n-1} - 2u_n + g(t)|u_n|^2 u_n = 0$$

$$g(t) = g_{\mathrm{dc}} + g_{\mathrm{ac}} \sin(\omega t)$$

- Variational approach predicts vanishing of Peierls-Nabarro barrier
- Objective: Numerical study of the effect of Feshbach Resonance Management on moving solitons

Variational approximation results

- Peierls-Nabarro barrier:

$$U_{\mathrm{PN}}(\xi,\dot{\xi}) = \frac{1}{2}\sqrt{\frac{\pi}{a}}A^2 \exp\left(-\frac{\pi^2}{4a}\right)\left\{4\sqrt{2}\exp\left(-\frac{\pi^2}{4a}\right)\left[1+e^{-a/2}\cos\left(\dot{\xi}/2\right)\right] - gA^2\sqrt{\frac{\pi}{a}}\right\}\cos\left(2\pi\xi\right)$$

- Consequences:
  - Dependence on $\cos(2\pi\xi) \longrightarrow$ Soliton amplitude oscillates during movement
  - Dependence on soliton velocity ($\dot{\xi}$)
  - Vanishing when the average soliton velocity fulfills:

$$\dot{\xi}_0 = (c_{\mathrm{res}})_N^{(M)} \equiv M\omega/2\pi N, \qquad N, M \in \mathbb{Z}$$

Numerical results

- Dynamical equations:

$$i\dot{u}_n + u_{n+1} + u_{n-1} - 2u_n + g(t)|u_n|^2 u_n = 0$$

$$g(t) = g_{\rm dc} + g_{\rm ac}\sin(\omega t)$$

- Initial configuration (static soliton) $\rightarrow u_n(t) = v_n \exp(it)$ with $g_{\rm ac} = 0$:

$$v_n = v_{n+1} + v_{n-1} - 2v_n + g_{\rm dc} v_n^3$$

- Initial condition (static soliton + thrust):

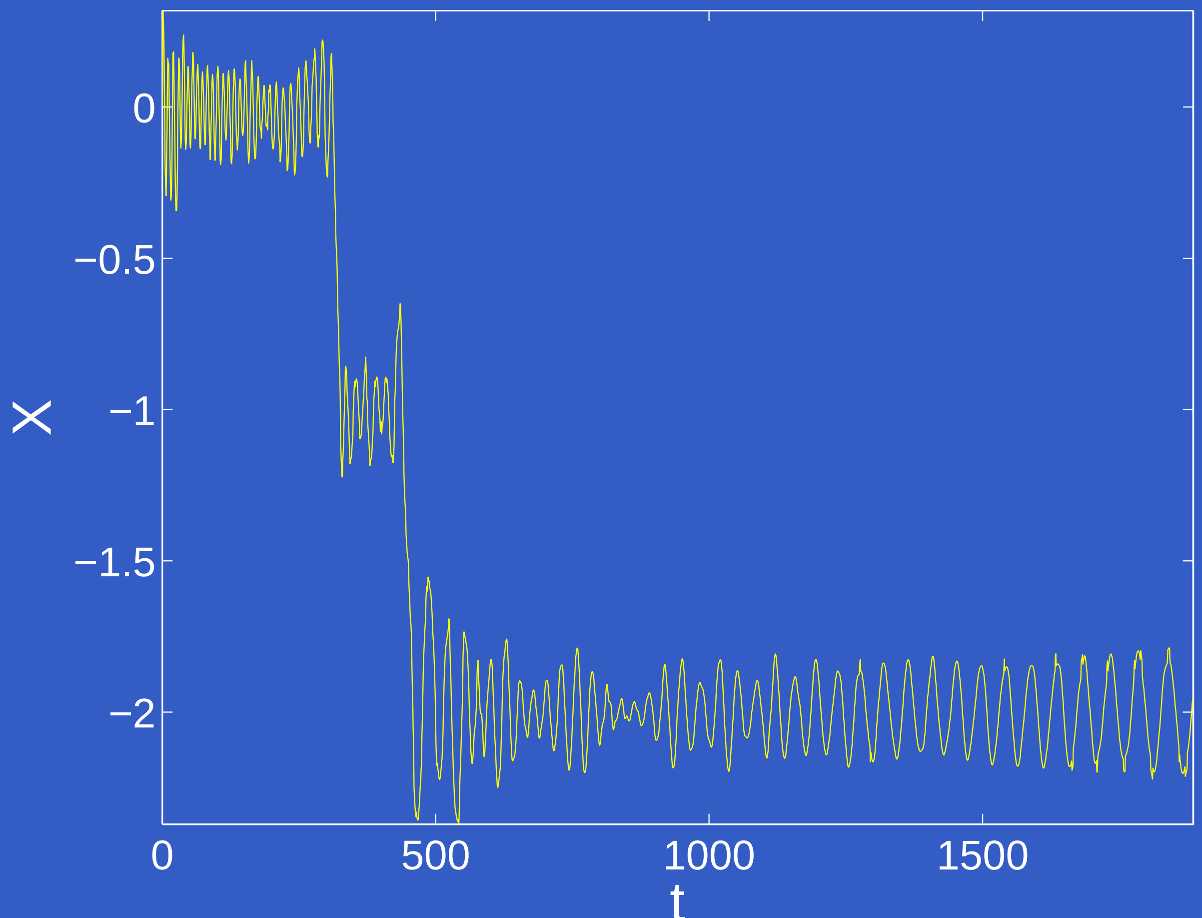
$$u_n(0) = v_n \exp(inq/2)$$

- $q = 0.5$, $g_{\rm dc} = 1 \rightarrow$ No motion for $g_{\rm ac} = 0$.

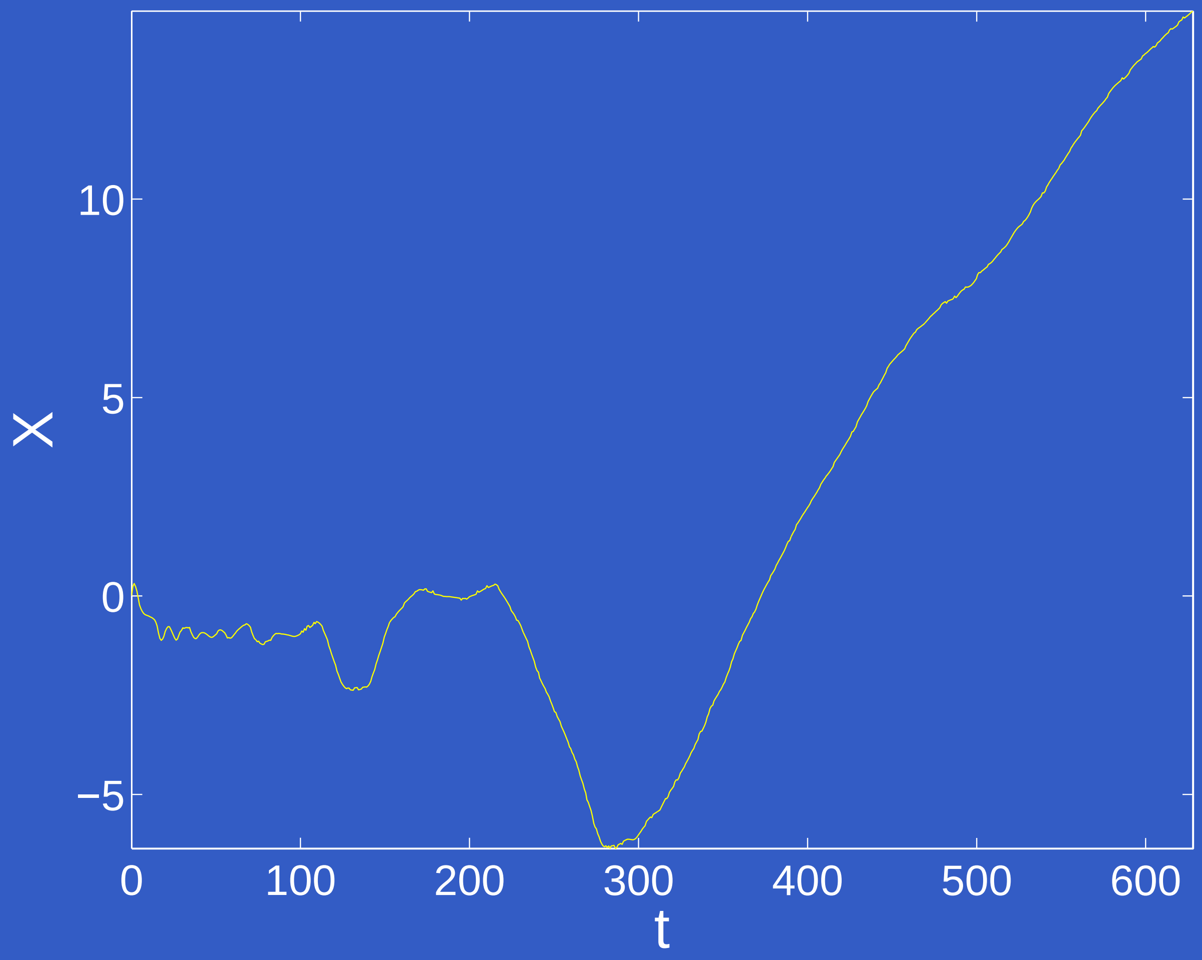
Regimes

- Pinning
- Irregular motion
- Regular forward/backward motion:
  - Stable motion (almost dispersionless)
  - Unstable motion
- Asymmetric splitting

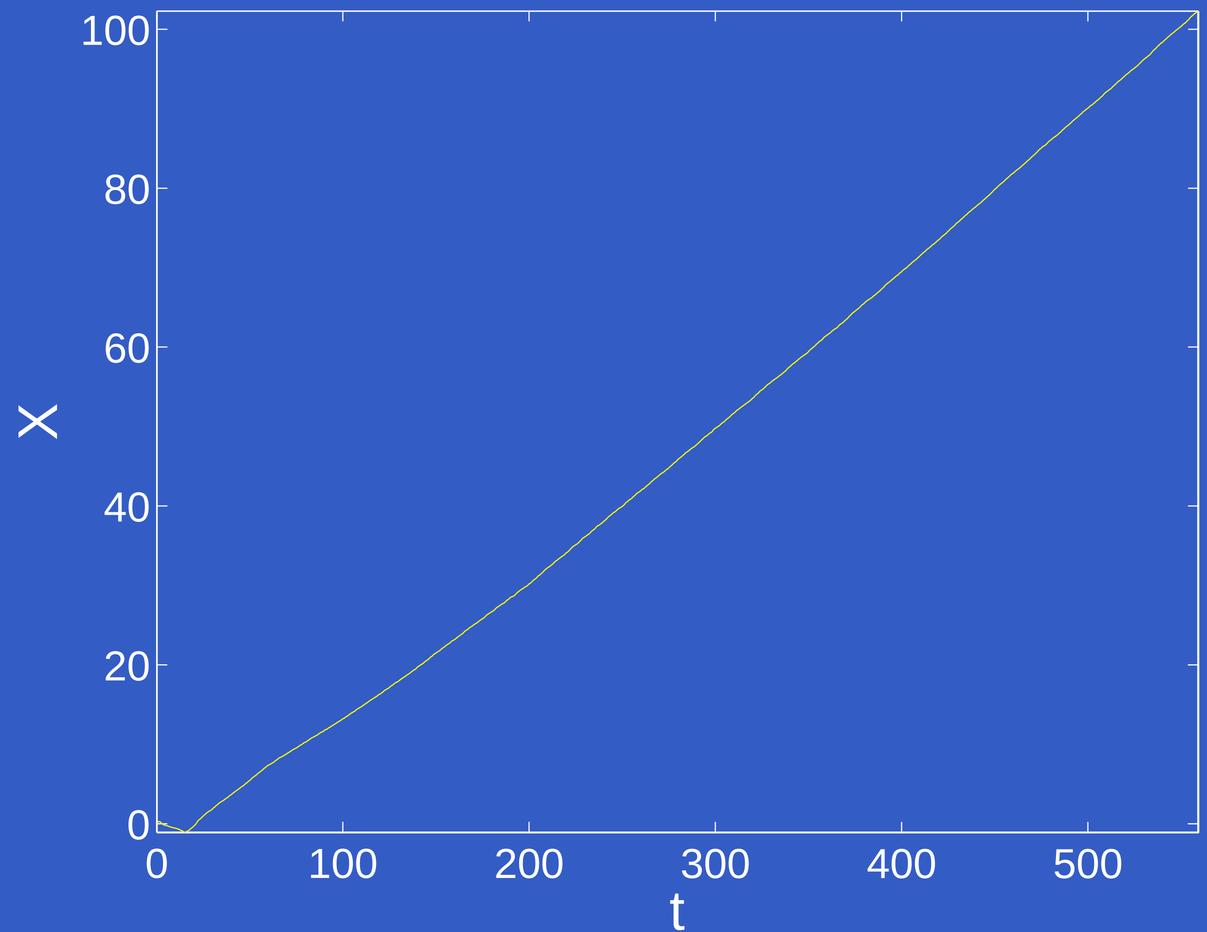
Pinning regime



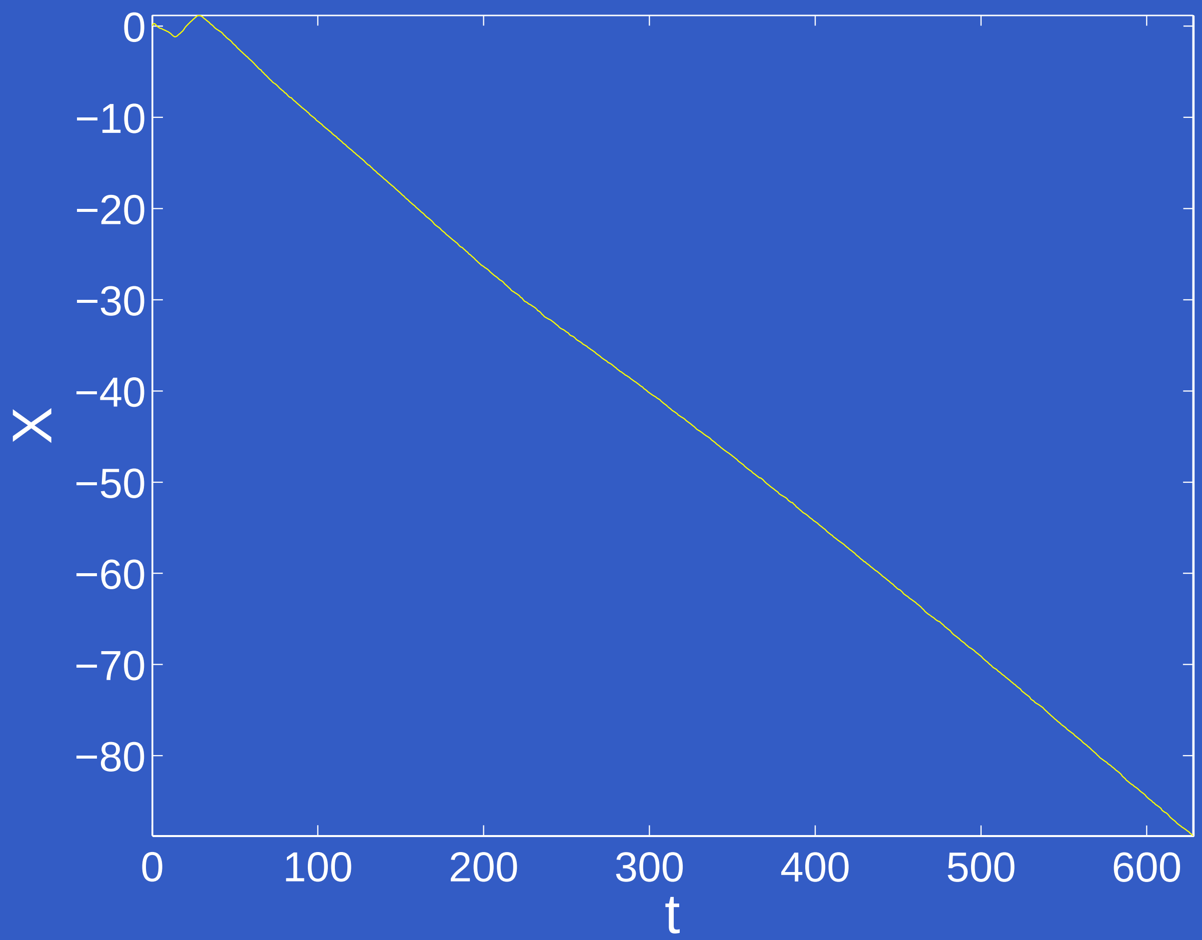
Irregular motion regime



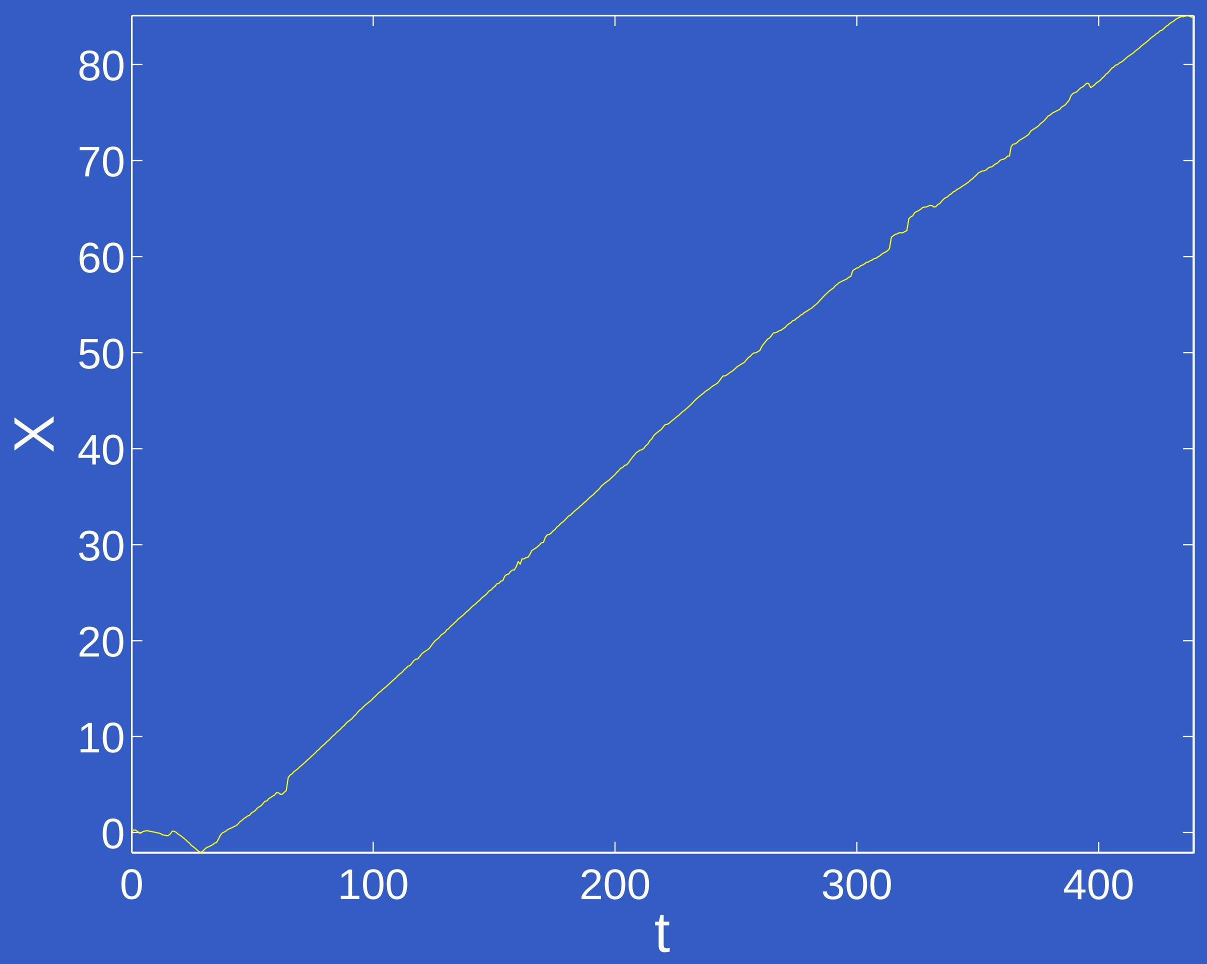
Regular stable motion regime



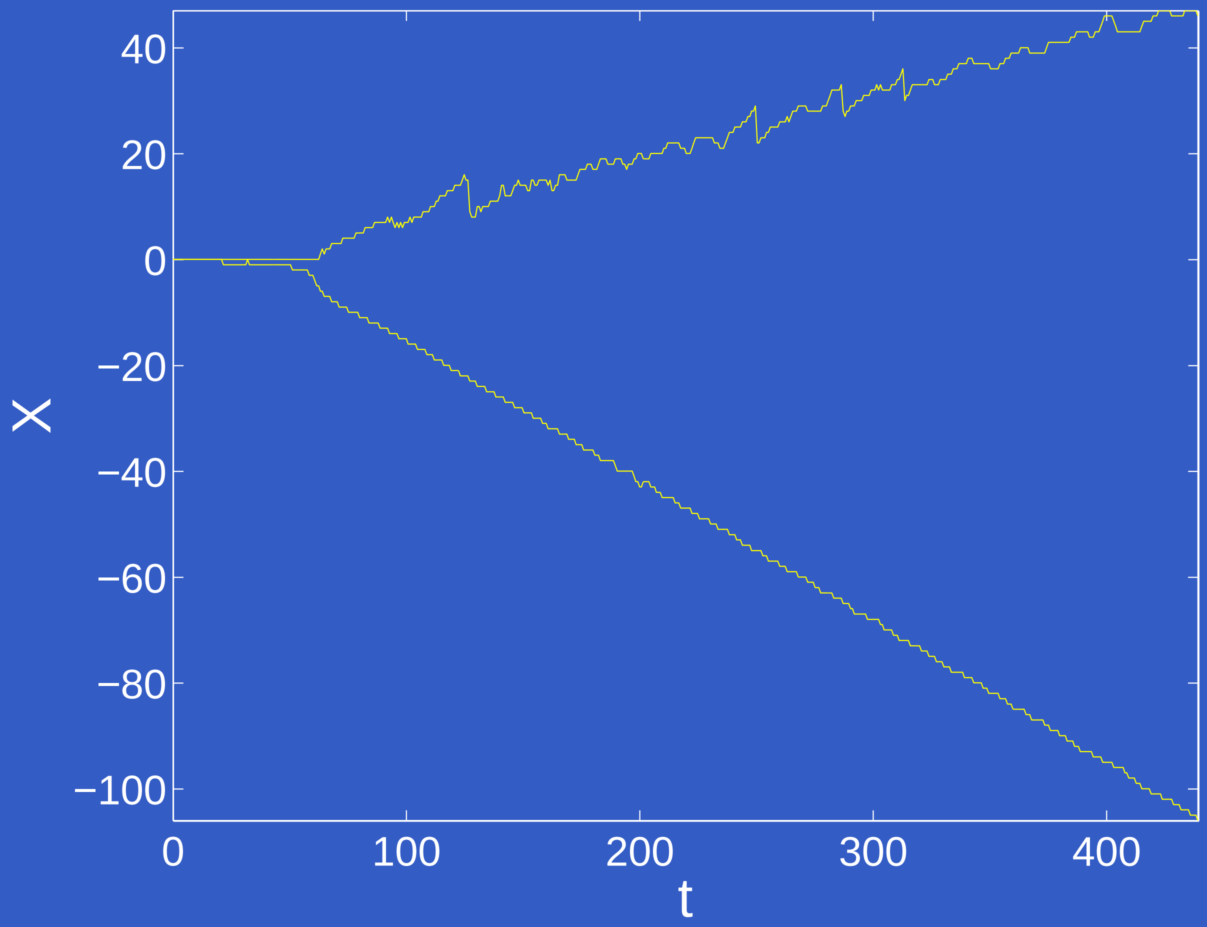
Regular stable motion regime



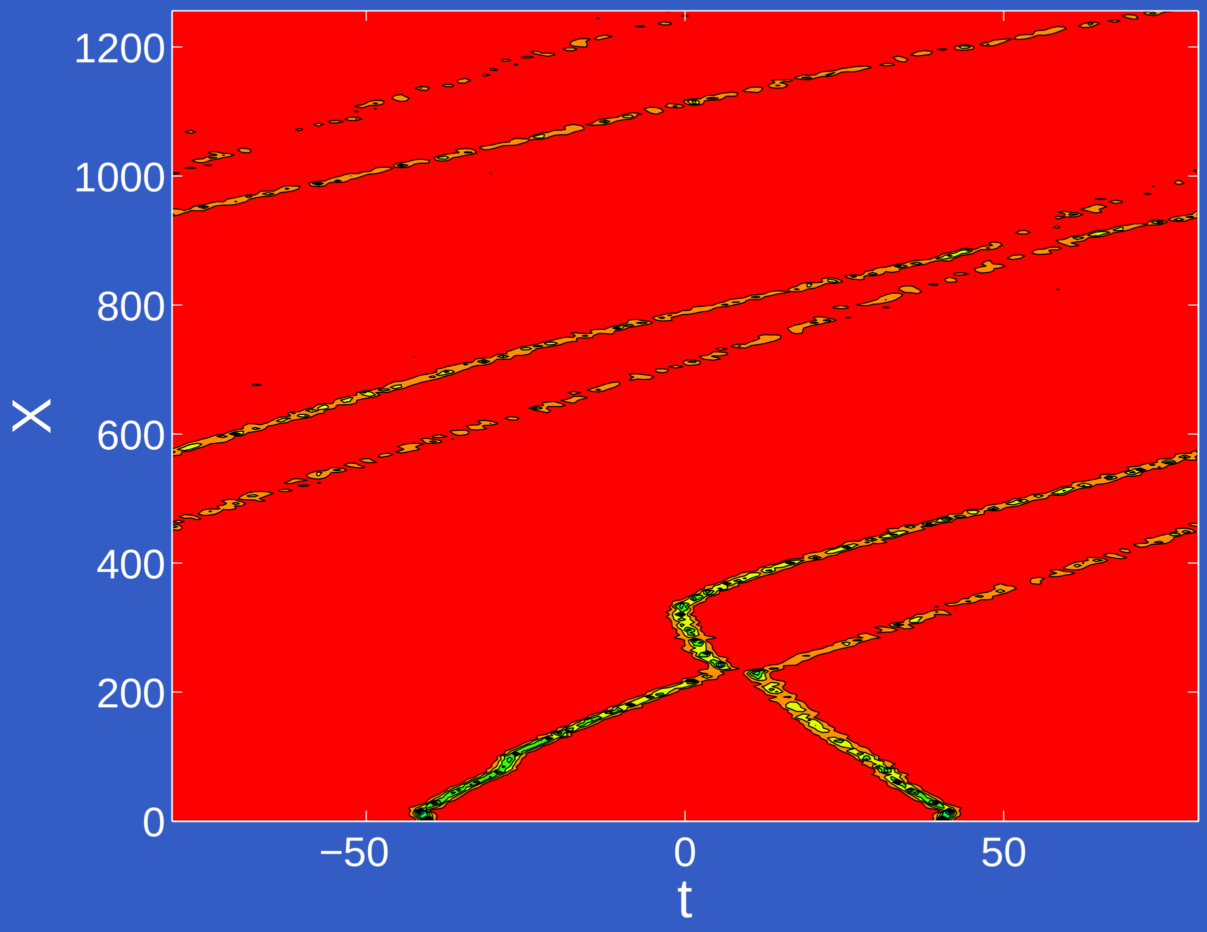
Regular unstable motion regime



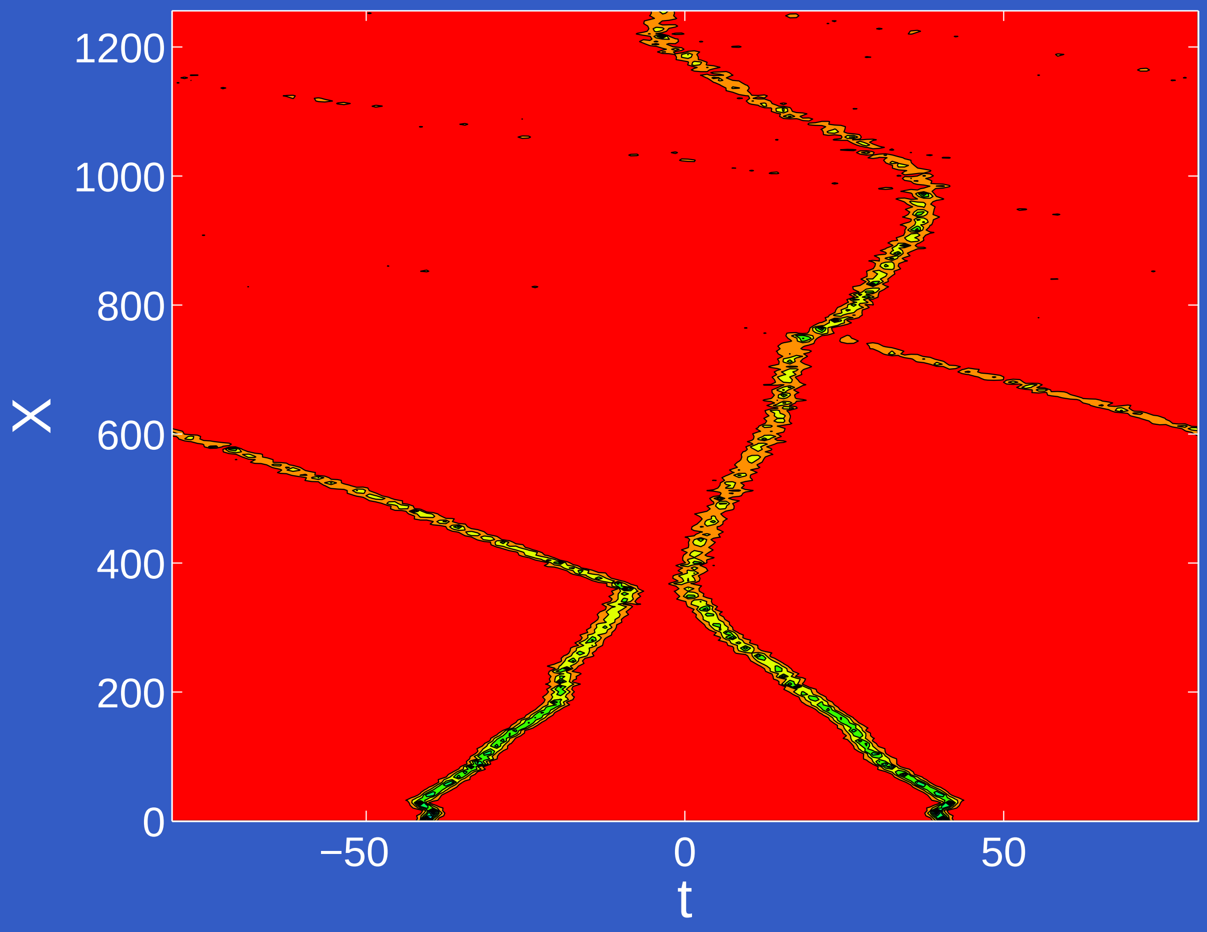
Splitting regime



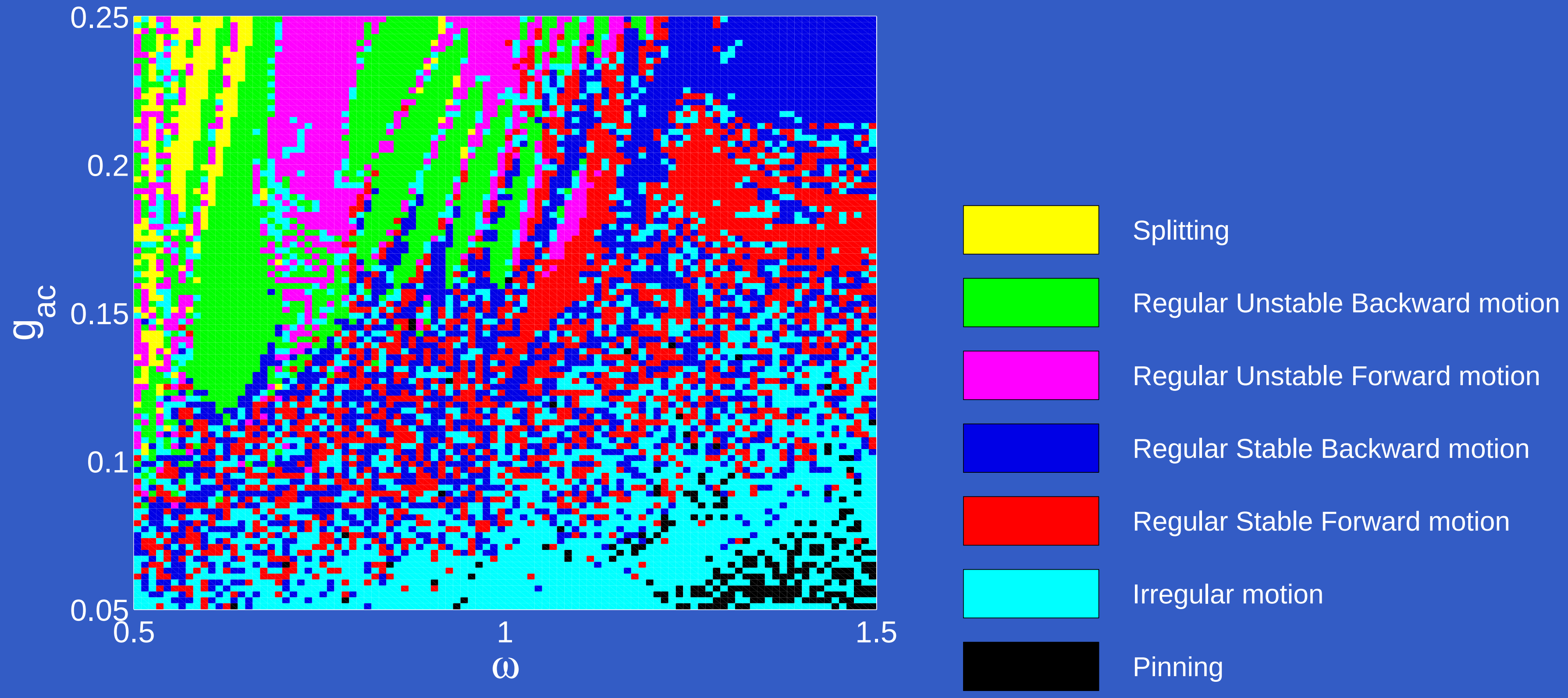
Collisions



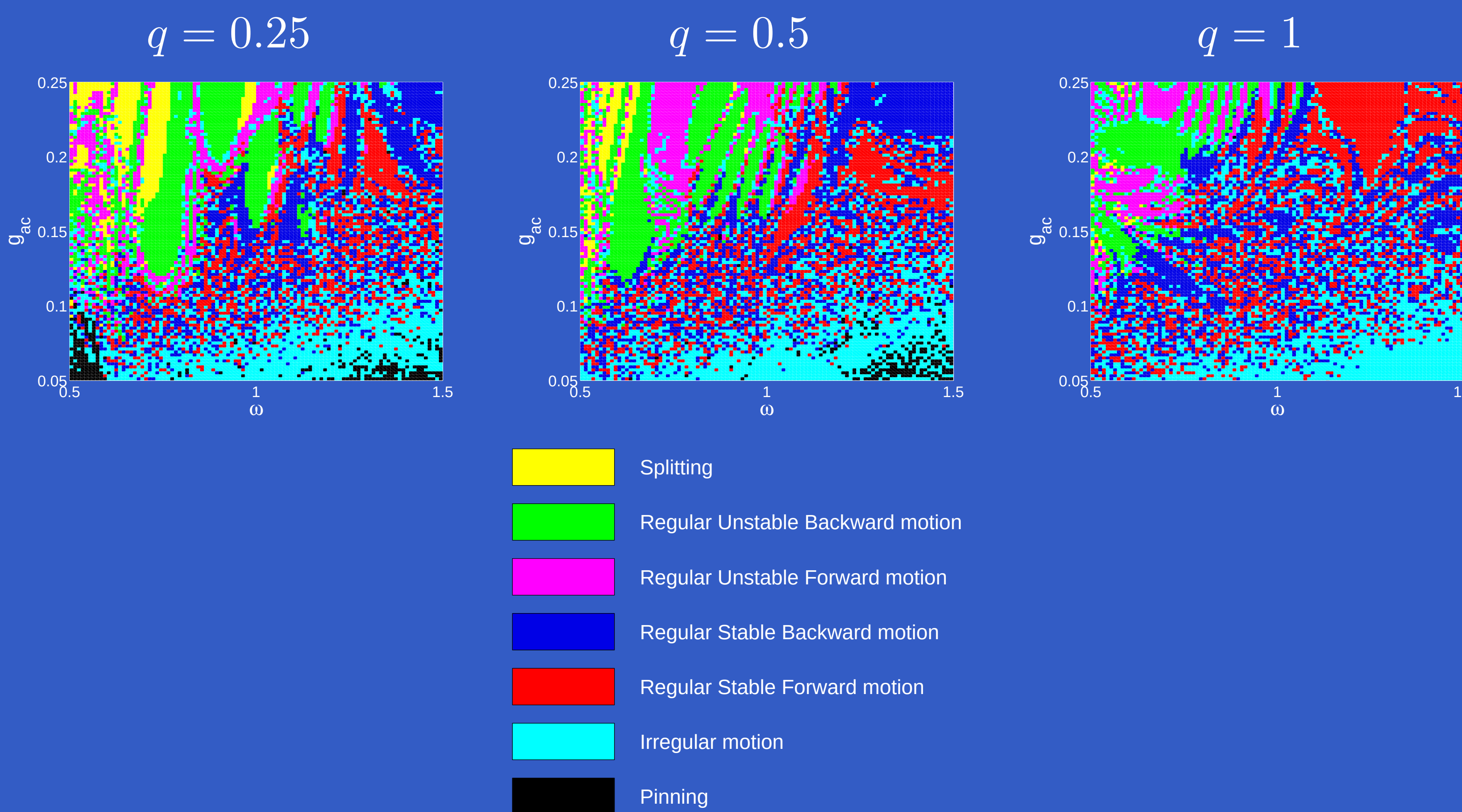
Collisions



Regimes (ω, g_{ac}) plane



Regimes (ω, g_{ac}) plane



Conclusions

- AC nonlinearity management facilitates creation of travelling solitons (vanishing of Peierls-Nabarro barrier).
- Several dynamical regimes are observed.
- Moving stable solitons survive for long times.
- BECs implementation:
  - Quasi-1-d, cigar-shaped BEC in strong optical lattice.
  - Nonlinearity modulation induced by the Feshbach resonance in the ac regime.
  - Setting similar to repulsive (^{87}Rb) condensate.
- More information in:
  - `arXiv:cond-mat/0411352`
  - `http://www.grupo.us.es/gfnl`