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Optimal control of investments for quality of supply improvement in electrical energy distribution networks☆

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Abstract

This paper considers the problem of deciding multi-period investments for maintenance and upgrade of electrical energy distribution networks. After describing the network as a constrained hybrid dynamical system, optimal control theory is applied to optimize profit under a complex incentive/penalty mechanism imposed by public authorities. The dynamics of the system and the cost function are translated into a mixed integer optimization model, whose solution gives the optimal investment policy over the multi-period horizon. While for a reduced-size test problem the pure mixed-integer approach provides the best optimal control policy, for real-life large-scale scenarios a heuristic solution is also introduced. Finally, the uncertainty associated with the dynamical model of the network is taken care of by adopting ideas from stochastic programming.

Keywords: Investment planning; Optimal control; Hybrid systems; Mixed-integer programming; Stochastic systems

1. Introduction

National regulations were recently applied in several countries for encouraging electrical energy distribution companies to improve the continuity of energy supply. Such regulations include incentives/penalties to energy distribution companies that depend on a few quality indicators. The introduced incentives/penalty mechanisms usually reflect customers' preferences and requirements, and their willingness to pay for quality.

Quality management has become a strategic issue for electricity suppliers. The aforementioned regulations not only impact considerably the economic activities of the supplier, but also provide guidelines to the company management for deciding the multi-annual investment plans for renovating energy distribution lines, in order to maximize the quality of energy supply perceived by customers while satisfying financial and operational constraints. By taking into account national regulations, the state of the network, previous actions, and other historical data, the managers of the company currently decide the multi-annual investment plan for maintaining and upgrading the energy distribution lines according to a manual trial-and-error procedure. This activity is time-consuming, does not always bring to optimal choices that exploit the available resources to maximize the resulting quality of energy supply, and in any case always makes the management wonder if better plans could have been made. The above disadvantages are amplified by the fact that such decisions should be re-iterated during the multi-period horizon in order to take into account poorly forecasted or unexpected events.

In this paper we consider the Italian regulation system introduced in 2004 (Authority for Electrical Energy & Gas). We propose an automatic method for taking decisions about multi-annual investments that is based on optimal control ideas. After modeling the network as a simple (yet large-scale) hybrid dynamical system with integrating dynamics and a piecewise

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affine output function, and after expressing the nonlinear incentive/penalty function by means of mixed-integer variables, an optimal control problem is solved to optimize the profit of investments. In order to cope with the actual large-scale scenario of a regional network, a heuristic solution is introduced. Finally, by taking into account the uncertainty associated with the nonlinear function relating investments to benefits, we propose a model to optimize the profit of maintenance and upgrade of a electrical distribution network under uncertainty.

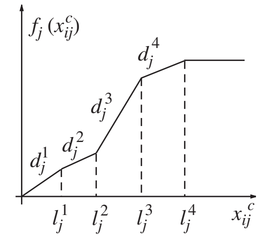

Fig. 1. Quality of electricity supply improvement function.

2. Optimal control problem

The aim of this section is to set up an optimal control problem to determine the optimal allocation of investments for maintenance and upgrade of electrical energy distribution networks on a multi-period (four years) time basis. In our context, a control action is considered optimal if the profit of the electrical distribution company is maximized (indirectly, this also implies a high quality of energy supply perceived by customers).

We treat the electrical distribution network as a (large-scale) discrete-time dynamical system whose sampling time is one year (decisions are taken on a yearly basis), whose states define the quality of energy supply in each individual district, and whose input is the amount of money invested in that district at a given year. The optimal investment decision depends on the prediction over a certain number N of future years of the evolution of the quality of the network. The prediction is repeated every year, according to the so-called "receding-horizon" (or "rolling-horizon") principle, over a multi-annual horizon that has been shifted forward by one year.

2.1. Dynamical system

An electrical distribution company must decide the amount of money that must be invested in each district $j \in \mathscr{D}$, where $\mathscr{D} = \{1, 2, \dots, D\}$ is a finite set of districts, in order to maintain a certain quality of energy supply, which is measured by the amount of minutes of power outage per customer per year (customers' minutes lost, CML), an indicator of the continuity of supply service.

In order to improve the CML in their districts of competence, distribution companies invest money in maintenance and upgrade of the energy distribution network. Districts are usually heterogeneous among them, have different sizes and levels of quality, so that each district must have its own investment project. Each project has a cost and provides an expected improvement of quality (i.e., a lower CML). The relation between quality improvement and invested money is nonlinear, and is modeled here as a piecewise affine function (see Fig. 1).

In this paper we consider two different kinds of investment projects: local improvement interventions on a district, and big network upgrade projects (such as the construction of a distribution substation) which may affect more than one district and are defined by a fixed cost and a fixed gain. The latter are yes/no interventions, and, if carried out, have a fixed cost and produce a fixed improvement of quality.

To evaluate an investment policy along a multi-period time basis, we use a hybrid dynamical model (Bemporad & Morari, 1999). Let each district be indexed by $j \in \mathscr{D}$, each time period by $i \in \mathscr{Y}$, where $\mathscr{Y} = \{0, 1, \dots, N\}$, $N \in \mathbb{N}$, and let x_{ij}^{c} be the money invested in the local improvement of district $j \in \mathscr{D}$ from the initial time period up to year $i \in \mathscr{Y}$. For the investment problem tackled in this paper, the time period goes from 2003 ($i = 0$) to 2007 ($i = N = 4$). This is the period of definition of the Italian normative described in (Authority for Electrical Energy & Gas).

As mentioned earlier, the investment problem also allows yes/no upgrade investments affecting more than one district. The lth of such investments, $l \in \mathscr{I} = \{1, \dots, I\}$, is defined by the cost of the investment and the quality increase in each affected district. For each possible upgrade $l \in \mathscr{I}$, the binary state x_{il}^{b} indicates if the upgrade has been done by time period $i \in \mathscr{Y}$. The cost of investment l is denoted by C_l^{up} and the quality increase in district j by ΔI_{lj}. Matrix $\Delta I \in \mathbb{R}^{I \times D}$ contains the quality increase factors for all upgrades and districts ($\Delta I_{lj} = 0$ if upgrade investment l does not affect district j).

At each time period $i \in \mathscr{Y}$, one must decide the *continuous* inputs u_{ij}^{c}, which is the money invested in each district $j \in \mathscr{D}$ for maintenance, and the *binary* input u_{il}^{b}, which decides whether network upgrade l must be realized. Hence, the equations that determine the dynamics of the system are

$$x_{i+1,j}^{\mathrm{c}} = x_{ij}^{\mathrm{c}} + u_{ij}^{\mathrm{c}}, \quad \forall j \in \mathscr{D}, \tag{1a}$$

$$x_{i+1,l}^{\mathrm{b}} = x_{il}^{\mathrm{b}} \vee u_{il}^{\mathrm{b}}, \quad \forall l \in \mathscr{I}, \tag{1b}$$

where "$\vee$" denotes the logical or. The CML of district j at time period i is denoted by $\mathscr{C}_{ij}$ and can be described as

$$\mathscr{C}_{ij}(x_{ij}^{\mathrm{c}}) = \mathscr{C}_{0j} - f(x_{ij}^{\mathrm{c}}) - \sum_{l \in \mathscr{I}} x_{il}^{\mathrm{b}} \Delta I_{lj}, \tag{2}$$

where ΔI_{lj} is the quality increase in district j if upgrade l is realized and $f_j : \mathbb{R} \mapsto \mathbb{R}$ is the piecewise affine function

$$f_j(x_{ij}^{\mathrm{c}}) = \begin{cases} d_j^1 x_{ij}^{\mathrm{c}} & \text{if } x_{ij}^{\mathrm{c}} \leqslant l_j^1, \\ f_j(l_j^1) + d_j^2(x_{ij}^{\mathrm{c}} - l_j^1) & \text{if } l_j^1 \leqslant x_{ij}^{\mathrm{c}} \leqslant l_j^2, \\ f_j(l_j^2) + d_j^3(x_{ij}^{\mathrm{c}} - l_j^2) & \text{if } l_j^2 \leqslant x_{ij}^{\mathrm{c}} \leqslant l_j^3, \\ f_j(l_j^3) + d_j^4(x_{ij}^{\mathrm{c}} - l_j^3) & \text{if } l_j^3 \leqslant x_{ij}^{\mathrm{c}} \leqslant l_j^4, \\ f_j(l_j^4) & \text{if } x_{ij}^{\mathrm{c}} \geqslant l_j^4 \end{cases} \tag{3}$$

that relates quality increase to money invested in each district $j \in \mathscr{D}$,[1] see Fig. 1.

The dynamics of each district j are only coupled by binary upgrade decisions. A further coupling is due to constraints on the capital that can be invested at each time period i, namely

$$\sum_{j\in\mathscr{D}} u^{\rm c}_{ij} + \sum_{l\in\mathscr{I}} u^{\rm b}_{il} C^{\rm up}_l \leqslant \mathrm{U}_{\max}, \quad \forall i \in \mathscr{Y}, \tag{4}$$

where $\mathrm{U}_{\max}$ is the maximum amount of money that can be invested each year i.

2.2. Cost function

Distribution agencies must select the allocation of investments depending on complex incentive–penalty mechanisms imposed by national authorities for energy. In this paper we consider the Italian normative described in (Authority for Electrical Energy & Gas), which we summarize here below.

Each year and for each district the company is given an incentive if the two-year moving average CML of that district is under a given basic standard, or must pay a penalty if it is higher. The objective is to maximize the overall profit, i.e., the difference between the incentive obtained and the invested money or paid penalties during the time period 2004–2007.

The incentive–penalty mechanism is defined by the following set of rules (see Bemporad, Muñoz de la Peña, & Piazzesi, 2005 for more details):

- *Incentive–penalty evaluation*: The incentive value for a given district j and period i is a piecewise affine function of (i) the two-year moving average of the CML of the district, and of (ii) the basic standard.
- *Dead band*: For each district, a dead band is defined around the basic standard level where neither incentives nor penalties are due.
- *Maximum and minimum incentive–penalty*: Incentives and penalties are saturated.
- *Discount rate*: We suppose a discount rate $r = 7\%$ to evaluate the weighted average cost of capital (WACC) with respect to year 2004.
- *Penalty cancelation for 2004–2005*: The law provides a special treatment for the penalties of years 2004 and 2005. If the distribution company incurs a penalty in either year 2004 or 2005, the debt is not paid immediately. It is instead subdivided into three installments to be paid in the following three years. In addition, if at any time the CML goes under the basic standard, then the remaining installments are canceled.

The above regulations define a complex nonlinear function relating investments to profits. Such a function is composed by a piecewise affine function and by a set of logical conditions, which can be both handled by a suitable mixed-integer optimization model.

[1] The coefficients in (3) are obtained by processing historical data.

3. Optimization model

In this section we propose a mixed integer optimization model which takes into account both the hybrid dynamics (1)–(2) and piecewise affine/logical relations describing the objective function described in the previous section.

For solving optimal control problems for discrete-time hybrid dynamical systems subject to linear and logical constraints, the mixed logical dynamical (MLD) formalism was introduced in Bemporad and Morari (1999). The key idea of the approach consists of embedding the logic part in the state equations by transforming Boolean variables into 0–1 integers, and by expressing the relations as mixed-integer linear inequalities (Bemporad & Morari, 1999; Torrisi & Bemporad, 2004; Williams, 1993).

The hybrid optimal control problem tackled in this paper can be treated using similar techniques. Here the objective is to maximize the total profit cumulated over a set of periods $i \in \mathscr{Y}$ for a given set of districts $j \in \mathscr{D}$. The decision variables are the sums invested each year for improvements and for upgrades: $u^{\rm c}_{ij}$ is the money invested during period i in district j, $u^{\rm b}_{il}$ is a binary variable that indicates if upgrade project l is realized during period i.

The optimum control problem providing the desired investment allocation can be recast as the following optimization problem (see Bemporad et al., 2005, Section 3 for the complete formulation):

$$\begin{aligned} \max_{\mathbf{u},\mathbf{z}} \quad & c^{\rm T}\mathbf{u} + f^{\rm T}\mathbf{z} \\ \text{s.t.} \quad & A\mathbf{u} \leqslant b, \\ & W\mathbf{z} \leqslant h + T\mathbf{u}, \\ & \mathbf{u} \in \mathbb{R}^{n_{\rm c}} \times \{0,1\}^{n_{\rm b}}, \quad \mathbf{z} \in \mathbb{R}^{m_{\rm c}} \times \{0,1\}^{m_{\rm b}}, \end{aligned} \tag{5}$$

where matrices c, f, A, b, W, h, T depend on the problem parameters. The optimization variables $\mathbf{u}, \mathbf{z}$ have been subdivided into two categories: "actual" decision variables $\mathbf{u} = \{u^{\rm c}_{ij}, u^{\rm b}_{il}\}$ that constitute the investment policy over the prediction horizon, and the auxiliary variables $\mathbf{z}$ that are introduced to evaluate the dynamics and the cost function. Note that both $\mathbf{u}$ and $\mathbf{z}$ vectors have binary and real components.

3.1. Pure MILP approach

Problem (5) can be solved by standard MILP solvers, such as GLPK (public domain), Cplex (ILOG, Inc., 2004) (commercial), or Xpress-MP (commercial), for which Matlab interfaces are available at ⟨http://www.dii.unisi.it/hybrid/tools.html⟩.

Although the aforementioned MILP solvers are very efficient, MILP is still an $\mathscr{N}P$-hard problem, and its complexity is in general exponential with the number of binary variables. The size of Problem (5) grows linearly with the number N of time periods and the number D of districts. In many cases of practical relevance, the problem may be too large to be solved to optimality. In order to handle such complex cases, the next section provides a heuristic approach to get good solutions to Problem (5) with limited computations.

3.2. Heuristic approach

One of the most widely used heuristics are greedy algorithms. The idea is to start from a simple solution, change relevant variables sequentially, each time selecting the variable that achieves the greatest immediate improvement in the objective function. Moreover, once the value of a variable is fixed it is not changed further.

In the case at hand, the algorithm starts from the initial feasible solution where no investment is done, i.e. $u^{\mathrm{c}}_{ij}(0)=0$ and $u^{\mathrm{b}}_{il}(0)=0$ for all periods $i\in\mathscr{Y}$, districts $j\in\mathscr{D}$ and upgrades $l\in\mathscr{I}$. The initial profit, $J^{\mathrm{profit}}(0)$, is the predicted profit of this policy. The algorithm keeps track of the money that has not been invested (remaining budget) through variables $\mathrm{U}^{\max}_i(k)$. At iteration k of the greedy algorithm, the value of $\mathrm{U}^{\max}_i(k)$ is the money left for period i. The initial money constraint is equal to the maximum budget, $\mathrm{U}^{\max}_i(0)=\mathrm{U}^{\max}$ for all $i\in\mathscr{Y}$.

At each iteration k of the greedy algorithm, an investment U is added to the current solution and the cost of U is subtracted from the remaining budget. The added investment U is chosen according to an optimality index, which is the ratio between the corresponding obtained profit and the cost of investment U. The investment with the highest optimality index is chosen among the local improvement projects of each district and the different network upgrades. For each district j for which an investment has not been fixed yet, the obtained profit and the corresponding optimal investment is obtained solving (5) subject to the following additional constraints:

$$
\begin{aligned}
&u^{\mathrm{c}}_{ih}=u^{\mathrm{c}}_{ih}(k-1),\quad \forall i\in\mathscr{Y},\ \forall h\neq j\in\mathscr{D},\\
&u^{\mathrm{b}}_{il}=u^{\mathrm{b}}_{il}(k-1),\quad \forall i\in\mathscr{Y},\ \forall l\in\mathscr{I},\\
&u^{\mathrm{c}}_{ij}\leqslant \mathrm{U}^{\max}_i(k-1),\quad \forall i\in\mathscr{Y}.
\end{aligned}\tag{6}
$$

This problem evaluates the profit $J^{\mathrm{profit}}_j(k)$ of the best investment that can be done in district j, with the remaining budget after $k-1$ iterations. The optimality index for a given investment option in district j is then given by

$$
\mathrm{I}_j(k)=\frac{J^{\mathrm{profit}}_j(k)-J^{\mathrm{profit}}(k-1)}{\sum_{i\in\mathscr{Y}}1/(1+r)^{i-1}u^{\mathrm{c}}_{ij}}.\tag{7}
$$

We suppose a discount rate $r=7\%$ to evaluate the WACC with respect to year 2004.

Note that this optimization problem is easy to solve because most of the decision variables (i.e. the inputs to the system) are fixed. In particular, as the dynamics of each district are independent, only the variables and constraints of that district have to be taken into account to solve the corresponding problem. At each iteration the investment project with the maximum optimality index is chosen and the current solution and budget is updated as

$$
\mathrm{I}^*(k)=\max_{j\in\mathscr{D}}\mathrm{I}_j(k).
$$

If $\mathrm{I}_j(k)=\mathrm{I}^*(k)$ then

$$
\begin{aligned}
&J^{\mathrm{profit}}(k)=J^{\mathrm{profit}}_j(k),\\
&\mathrm{U}^{\max}_i(k)=\mathrm{U}^{\max}_i(k-1)-u^{\mathrm{c}}_{ij},\quad \forall i\in\mathscr{Y},\\
&u^{\mathrm{c}}_{ih}(k)=u^{\mathrm{c}}_{ih}(k-1),\quad \forall i\in\mathscr{Y},\ \forall h\neq j\in\mathscr{D},\\
&u^{\mathrm{c}}_{ij}(k)=u^{\mathrm{c}}_{ij},\quad \forall i\in\mathscr{Y},\\
&u^{\mathrm{b}}_{il}(k)=u^{\mathrm{b}}_{ih}(k-1),\quad \forall i\in\mathscr{Y},\ \forall l\in\mathscr{I}.
\end{aligned}
$$

The algorithm iterates until the solution does not improve any more, i.e., $J^{\mathrm{profit}}(k)=J^{\mathrm{profit}}(k-1)$. The feasible suboptimal solution is given by $\{u^{\mathrm{c}}_{ij}(k),u^{\mathrm{b}}_{lj}(k)\}$.

The heuristic algorithm presented above deals only with single optimization projects on each district. It can be easily modified to include big investments projects (the modification has not been included here for lack of space).

4. Stochastic optimal control

A large number of problems in production planning and scheduling, location, transportation, finance, and engineering design require that decisions have to be made in the presence of uncertainty. In the problem tackled in this paper uncertainty affects the function between quality of supply and money invested in improvement projects.

Stochastic programming (SP) is a special class of mathematical programming that involves optimization under uncertainty (see Birge & Louveaux, 1997; Kall & Wallace, 1994; Ross, 1983). The first applications of SP date back to the 1950s and nowadays it is becoming a mature theory that is successfully applied in several domains (Sahinidis, 2004). In this section, a two-stage stochastic integer programming formulation is proposed for the investment problem described in the previous sections.[2]

The increase of quality caused by an investment project is very difficult to predict. For this reason the CML of each district is affected by uncertainty, and therefore the hybrid dynamical model (1)–(2) is uncertain. As a consequence, all the variables (and so the constraints) defined to evaluate the profit, which depend on the quality level of each district and period, are affected by uncertainty.

Stochastic programming optimizes the mean value of the cost function taking into account *causality*. This means that the decision on how to invest the available money must be done "before" knowing the real effect of the investment projects, i.e., before knowing the value of the random variables and so the actual evolution of the CML of the districts.

Variable $\Xi=\{\xi_i,\xi_l\}$ collects all the random variables associated with uncertainty of the relation between improvement/upgrade investments and CML. We resort to sampling each of the independent continuous distributions $\xi_{j,l}$. The number q of possible values of each uncertain variable determines the complexity of the stochastic optimal control problem. In this way, as each variable in independent, the uncertain variable

[2] A multi-stage formulation would be more appropriate here, although not hardly solvable by standard solvers.

Ξ may take values $\Xi_1, \ldots, \Xi_Q$ with probabilities $p_1, \ldots, p_Q$, respectively, where $Q = q^{D+I}$.

For each fixed value of the uncertainty Ξ_i, referred to as *scenario*, all problem parameters $f(\Xi_i)$, $T(\Xi_i)$, $W(\Xi_i)$, $h(\Xi_i)$ become fixed. By enumerating all possible Q scenarios, a large-scale MILP problem can be posed. To each scenario, a set of "recourse" auxiliary variables $\mathbf{z}_i$ are assigned, but the problem optimizes only a single set of decision variables $\mathbf{u}$. In this way, the causality of the decision process is maintained. The (large-scale) equivalent MILP problem is

$$\begin{aligned} \max_{\mathbf{u},\mathbf{z}_i} \quad & c^{\mathrm{T}}\mathbf{u} + \sum_{i=1}^{Q} p_i f(\Xi_i)^{\mathrm{T}}\mathbf{z}_i \\ \text{s.t.} \quad & A\mathbf{u} \leqslant b, \\ & W(\Xi_i)\mathbf{z}_i \leqslant h(\Xi_i) + T(\Xi_i)\mathbf{u}, \quad i = 1, \ldots, Q, \\ & \mathbf{u} \in \mathbb{R}^{n_c} \times \{0,1\}^{n_b}, \quad \mathbf{z}_i \in \mathbb{R}^{m_c} \times \{0,1\}^{m_b}. \end{aligned} \qquad (8)$$

4.1. Solution strategies

The complexity of the two-stage stochastic problem (8) heavily depends on the number Q of scenarios. In the optimization model proposed in this paper the uncertainty affects investment projects. If each project is supposed to be independent, and the uncertainty is supposed to take q different values, the number of scenarios of the optimization problem is $Q = q^{D+I}$. Although stochastic programming for linear and quadratic problems is nowadays a mature field, stochastic MILP problems are still in general hard to solve, we refer again to Sahinidis (2004) for a complete survey of the state of the art.

Given that even the deterministic MILP approach (5) is too complex to handle instances of the investment allocation problem of interest, we avoid solving (8) using stochastic MILP solution techniques, but rather propose again a heuristic algorithm. By decoupling each investment project using the heuristic approach of Section 3.2, one can deal with each random pair $\{\xi_i, \xi_l\}$ independently. Each local improvement project can be dealt without taking into account the realization of the random variables associated with other districts, as such variables only affect the corresponding CML, and hence evaluating the investment project of district j only requires the enumeration of q scenarios, corresponding to the possible realizations of ξ_j. Districts affected by upgrade projects must take into account also the uncertainty associated with upgrade projects, so the number of scenarios may be larger than q.

Having reduced the number of scenarios in each smaller subproblem associated with a single district, one can use an MILP formulation to solve the stochastic equivalent of the optimization problems described in Section 3.2. In this way, it is possible to obtain a feasible suboptimal investment policy that takes into account the uncertain nature of each investment project.

5. A case study

We apply the optimal control approach developed in the previous sections for solving a real-life investment allocation problem. We consider three different problems[3] : P9 (9 districts), P18 (18 districts), P36 (36 districts). As a reference, the sizes of the corresponding MILP problems is provided in Table 1. The relaxed LP bounds of this problem is several orders of magnitude greater than the optimal value. For P9, the relaxed LP bound is 2×10^7.

Table 1
Number of continuous variables, binary variables and constraints for P9, P18 and P36

	$n_c + m_c$	$n_b + m_b$	Constraints
P9	603	234	1278
P18	1206	468	2556
P36	2412	936	5112

Table 2
Optimization results for P9, P18, P36 and for different solution strategies

	J_{det}	$E[J_{\text{stc}}]$	Time (s)	Gap (%)
P9				
MILP	1829	1534	1.7	0
Greedy	1807	1363	2.3	–
SP	1649	1603	72.17	–
P18				
MILP	3445	2343	200	1.2
MILP	3983	2680	2000	0.2
Greedy	3795	2530	7.2	–
SP	3450	3332	125.3	–
P36				
MILP	13079	12185	1000	2
Greedy	13021	12285	18.6	–
SP	12861	12306	563.3	–

J_{det} is the profit obtained for the deterministic problem, $E[J_{\text{stc}}]$ is the expected profit for the stochastic formulation, "time" is the time for computing the solution, "gap" is the optimality gap of the MILP solution.

In the three cases deviations of up to 20% of estimated CML predictions in all districts are considered as explained in Section 4. We suppose a 20% error margin in the predictions: for each district j and upgrade project l we define a pair of independent continuous random variables $\xi_i, \xi_l \in [0.8, 1.2]$. These random variables model the possible error $\Delta\mathscr{C}_{ij} = \xi_j f(x_{ij}^c)$, $\Delta I_{lj}(\xi_l) = \xi_l \Delta I_{lj}$ in the forecasted quality increase of each project. The uncertain variables take five possible values ($\{0.8, 0.9, 1, 1.1, 1.2\}$) with equal probability ($p = 0.2$). It is also taken into account that there is a probability $p = 0.2$ that some of the investments fail during the first two years for unexpected reasons.

Table 2 shows the profit and computation time of the different solutions strategies developed in the previous sections for both the deterministic and the stochastic case.[4] For the deterministic problem, the optimal solution has been obtained only

[3] Numerical data were generated by perturbing actual confidential data provided by the Italian electrical company ENEL and are available on request from the authors.

[4] The results were obtained in MATLAB 6.3 using CPLEX 9.0 on a AMD Athlon$^{\text{TM}}$XP 2800+ with 512 MB of RAM.

for P9. In this case, the MILP solver of CPLEX obtains the optimal investment policy even faster than the greedy algorithm of Section 3.2. For P18 the computer runs out of memory after 2300 s while solving the MILP problem (5) to optimality. The solutions obtained by CPLEX after 200 and 2000 s are shown. For P36, the computer runs out of memory after 1100 s. The reported solution is the one obtained after 1000 s of CPU time. The heuristic algorithm is instead very fast and converges in less that 20 s for P36. Although suboptimal, the obtained solution is considered a valid decision by the company. The optimization problem associated with the stochastic formulation is in all cases too large to be solved by one large MILP. A suboptimal solution is obtained by using the greedy approach of Sections 3.2, 4.1.

6. Conclusions

In this paper we have proposed a novel application of optimal control of hybrid dynamical systems for solving a management problem in allocation of investments for maintenance of the electrical distribution network in a territory composed by several districts. Because of nontrivial national regulations and of complex relations between invested money and resulting quality of supply, allocating investments for both local improvements and major upgrades is a complex problem, usually solved heuristically after a series of tedious iterations and without any guarantee of having taken the best decision. We have shown that an optimal control setup and optimization tools provide a systematic way of making the best (or, at least, a good) investment allocation, even when model uncertainty is taken into account.

References

Bemporad, A., & Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica*, *35*(3), 407–427.

Bemporad, A., Muñoz de la Peña, D., & Piazzesi, P. (2005). Optimal control of investments for quality of supply improvement in electrical energy distribution networks. *Technical Report*, Dip. Ingegneria dell'Informazione, University of Siena ⟨http://www.dii.unisi.it/~bemporad/publications⟩.

Birge, J. R., & Louveaux, F. V. (1997). *Introduction to stochastic programming*. New York: Springer.

Authority for Electrical Energy and Gas. *Delibera n4/04 - Testo integrato delle disposizioni dell'Autorità per l'energia elettrica e il gas in materia di qualità dei servizi di distribuzione*, *misura e vendita dell'energia elettrica*, Periodo di regolazione 2004–2007 [in Italian].

ILOG, Inc. (2004). *CPLEX 9.0 User Manual*. France: Gentilly Cedex.

Kall, P., & Wallace, S. W. (1994). *Stochastic programming*. Chichester: Wiley.

Ross, S. (1983). *Introduction to stochastic dynamic programming*. New York: Academic Press.

Sahinidis, N. V. (2004). Optimization under uncertainty: State-of-the-art and opportunities. *Computers & Chemical Engineering*, *28*(6–7), 971–983.

Torrisi, F. D., & Bemporad, A. (2004). HYSDEL—A tool for generating computational hybrid models. *IEEE Transaction on Control Systems Technology*, *12*(2), 235–249.

Williams, H. P. (1993). *Model building in mathematical programming*. 3rd ed., New York: Wiley.

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