Constrained Predictive Control of an Irrigation Canal

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ABSTRACT

This paper presents the application of a Distributed Model Predictive Controller (DMPC) to the control of an accurate model of an actual irrigation canal in Spain. The canal is modelled using the Saint-Venant equations and implemented using the well known modelling software for irrigation canals SIC. The DMPC algorithm has been implemented in Matlab and interfaced to SIC. In the distributed control algorithms, the local controllers exchange information so that their control policies are optimal in the sense of getting the best values of a performance index. The results show that the proposed distributed control algorithm obtains better control performance than a more conventional decentralized control scheme without information exchange. This better performance translates directly into money and resource savings.

Keywords: Model Predictive Control, irrigation canal, distributed control, control algorithms

1 INTRODUCTION

Water is a limited resource. In addition, nowadays there are some regions in Europe and all over the world with long seasons of drought. As a consequence, the development of innovative control techniques that optimize water management is a relevant issue.

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The main objective of irrigation canals is to supply water to farmers according to a specific schedule. An irrigation canal is composed by several reaches, connected by gates, and usually following a tree structure. In a typical irrigation canal the length can be hundred of kilometers, there are tens of gates and hundreds of off-take points, used by farmers to take water from the canal.

Irrigation canals management involves operating gates, pumps and valves in order to satisfy user demands and minimize costs and water loses. In addition, a set of constraints imposed by the physical system and management policies has to be considered, for example, maximum and minimum water level and flow.

Automatic control techniques are widely used in irrigation canals, most of them based on a local control of gates using classic approaches as PI (Proportional-Integral) controllers (See (Malaterre et al. 1998) for a detailed classification of these algorithms). These decentralized approaches provide reasonable behavior in many cases, but as the coupling effect among the different local controllers (agents) is not taken into account, sometimes they produce important loss in the control performance.

Another approach based on PI is discussed in (Ooi and Weyer 2008), where the controller is a PI controller augmented with a first order low pass filter in order not to amplify waves present in the channel. The developed routine for controller design is based on frequency response design, and configurations with and without feedforward from downstream gate are considered.

The use of a single global controller for the control of the whole system (centralized control) is an alternative to deal with this problem. Model Predictive Control (MPC) (See (Camacho and Bordons 2004)) approaches have been widely and successfully applied in water systems. However, MPC is a technique with strong computational requirements that hinder its application to large-scale systems such as water networks in a centralized way. Moreover, the communication difficulties in a system extended in a geographical area of hundreds of kilometers make not sensible the use of a centralized real-time control system based on long distance communications. Another problem to use centralized approaches is the fact that sometimes different sections of the canal can be managed by different control centers and even by different organizations.

Distributed Model Predictive Control techniques to optimize the management of water in irrigation canals provide a reasonable trade-off between complexity and performance. Basically, the idea is to provide communication among local controllers, in such a way that agents can exchange information or even negotiate and reach agreements. In this paper, a Distributed MPC algorithm is presented where the communication requirements are adapted to the complexity of the control of the different subsystems, from a simple information exchange to a negotiation in problematic reaches.

There are several works that address the canal control with Predictive control techniques with decentralized and centralized approaches.

In (Rodellar et al. 1993), a model predictive algorithm is presented to control the downstream discharge of a canal reach. (Gómez et al. 2002) presents a decentralized predictive control for an irrigation canal composed by a series of pools. In order to decouple the system, the controller used an estimation of the future discharges and the hypothesis of being linearly approaching the reference, to finally reach it, at the end of the prediction horizon. Because the control law solution was given in terms of reach's inflow discharge, they used a local controller to adjust the gate opening to the required discharge.

In (Sawadogo et al. 1998), and later in (Sawadogo et al. 2000), a similar decentralized adaptive predictive control is presented , but that used the reach's head gate opening as controllable variable and the reach's tail gate opening and the irrigation off-take discharge as known disturbances.

Several centralized MPC approaches also have been proposed. (Malaterre and Rodellar 1997) performed a multivariable predictive control of a two reaches canal using a state space model. They observed that the increase of the prediction horizon produced a change in the controller behavior, varying the control perspective from a local to a global problem.

(Wahlin 2004) tested a Multivariable Constrained Predictive controller using a state space model based on Schuurmans first-order Integrator Delay model (Schuurmans et al. 1999). They performed tests where the controller either knew or did not know the canal parameters and with and without the minimum gate movement restriction.

In (Silva et al. 2007), a predictive controller, based on a linearization of the Saint-Venant equations, has been also implemented on an experimental water canal. (Begovich et al. 2004; Begovich 2007) proposed a multivariable predictive controller with constraints which was implemented in real-time to regulate the downstream levels of a four-pool irrigation canal prototype. In (Lemos et al. 2009), several control structures are applied to a pilot canal, ranging from decentralized MPC, multivariate control using only neighbour reaches, to centralized multivariable control. Also, an adaptive MPC based on multiple models is evaluated. A complete state-of the art of MPC applications can be found in (van Overloop 2006) and (Sepulveda 2007).

Distributed control has been also a focus of research during the last few years. (Tricaud and Chen 2007; Li and Cantoni 2008; Li and De Schutter 2010) presented different distributed approaches based on control techniques different to MPC. (Negenborn et al. 2009) presented a distributed MPC based on Lagrange multipliers. At every sample interval the controllers perform several iterations of local optimization problem and communication with their neighbour based on a serial communication scheme. Finally, in (Zafra-Cabeza et al. 2011) a distributed MPC method based on game theory for multiple agents is applied to irrigation canals. The controller were tested by a simulation in which the canals were modelled using the integrator delay model. Also, the controlled variables were water flows at each gate, assuming an underlying low level control structure that managed to get the flows set by the distributed MPC controller. The distributed MPC algorithm used, presented in (Maestre et al. 2011), provides a reasonable trade-off between performance and low communication requirements needed to reach a cooperative solution.

In this paper we present the modelling of a section of a real canal in the South-East of

Spain and its control using predictive controllers based on the distributed MPC algorithm presented in (Maestre et al. 2011) and also on feedforward techniques. The controller uses an iterative game theory algorithm for the two most coupled subsystems and for the remaining a non iterative distributed scheme in which the information exchanged for each controller is used to compensate the interactions in a feedforward manner.

The model of the canal used to test the control structure is a very realistic one developed using the well known SIC software (Simulation of Irrigation Canals), which is based on a mathematical model that can simulate the hydraulic behaviour of most of the irrigation canals or rivers, under steady and unsteady flow conditions. The SIC hydraulic model solves the complete Saint Venant equations using the classical implicit Preissmann scheme.

Moreover, different control scenarios are illustrated in the paper, and in each of them different control structures are tested. The performance of each control structure is illustrated by means of a performance index and an estimation of the economical costs incurred by each controller. These merit figures shows that distributed decentralized predictive controllers obtain the best results compared with decentralized local controllers.

The rest of the paper is organized as follows: Sections 2 and 3 present the irrigation canal benchmark and some issues regarding the control of canals. The proposed control strategy is presented in section 4 and experimental results for several simulations are shown in section 5. Finally the conclusions are presented in section 6.

2 CONTROL OF IRRIGATION CANALS

The control of irrigation canals presents some specific details that should be considered before choosing any control strategy. First, it is important to start with the variables involved in the control scheme. In the case of irrigation canals, there are two types of variables that one could wish to control (i.e., controlled variables or system outputs): water levels and flows (measured at each or some of the gates). On the other hand, to achieve the control goals, two variables can be manipulated (i.e., control inputs or manipulated variables): the degree of gate aperture and also flows (usually only at the head of the canal). Note that in the case that the manipulated variable is chosen to be the flow at some specific gate (instead of the head of the canal), then a lower level controller has to be used to attain that flow using the gate aperture.

An important issue in the control of the canal system is the location of controlled variable relative to the control structure (i.e, gates). Mainly, two alternatives are considered. In downstream control strategy, control structure adjustments are based upon information measured by a sensor located downstream. Downstream control transfers the downstream canal-side off-takes demands to the upstream water supply source (or canal head works). On the other hand, in upstream control, control structure adjustments are based upon information from upstream. Upstream control transfers the upstream water supply (or inflow) downstream to points of diversion or to the end of the canal. Upstream control has to be used when the flow at the head of the canal is fixed, normally by an external organization. In any other circumstances, downstream control has demonstrated to be more efficient.

Measurable disturbances play an important role in the control of irrigation canals. This is because the coupled nature of irrigation channels, which extend for hundreds of kilometers and have multiple controllers that disturb their neighbours with each change in their manipulated variables. Specifically, downstream control actions mean disturbances that could be considered when computing a control action somewhere in the canal. When calculating the opening/closing of any gate at any sample period, the opening/closing of the following downstream gate could be considered as a measurable disturbance and its effect could be taken into account in the optimal control sequence calculation.

Off-takes and in-takes comprise another kind of disturbance. An off-take is a point where water is taken for a particular purpose (for example, irrigation).The flows are usually scheduled, so their value and moment of apparition can be predicted in advance. Nevertheless, the off-take gates are manipulated directly by farmers, so an uncertainty must be considered in this prediction. Sometimes there is only partial information about off-take flows, for example, an aggregate value of the flows of the off-take in a determined area. Also in-takes can be considered, for example, rainfall. The operation of off-takes and in-takes is considered as a measurable disturbance in the same manner as the gate movements, with exactly the same treatment.

Finally, the aforementioned coupled nature of irrigation canals together with the usual geographical dispersion found in the actual control hardware used leads to the consideration of distributed control schemes as a practical control solution. Thus, the overall performance of the canal control system will be greatly improved if distributed control strategies are used at least in those segments of the canal in which the coupling is so strong that a measurable disturbance management only is not enough.

2.1 The irrigation canal of La Pedrera (Murcia, Spain)

This work is focused on the control of a section of the ”postrasvase Tajo-Segura” in the South-East of Spain. The ”postrasvase Tajo-Segura” is a set of canals which distribute water coming from the Tajo River in the basin of the Segura River. This water is mainly used for irrigation (78%), although 22% of it is drinking water. The selected section is a Y-shape canal (see Figure 1), a main canal that splits into two canals with a gate placed at the input of each one of them:

- ”Canal de la Pedrera”, 6.680 kilometres long.
- ”Canal de Cartagena”, 17.444 kilometres long.

It is a gravity-fed canal without pumps in the considered section. The total length of the canals is approximately of 24 kilometers with a trapezoidal section. There are five main sluice-type overshot gates (in red in Figure 1) and 5 gravity off-take gates in the section selected (green arrows in Figure 1).

The objective of this paper is to control the distance downstream level at each one of the reaches in Canal de Cartagena (ref1, ref2, ref4 and ref5 in 1) and the flow at the head of Canal de la Pedrera (ref3). To reach this objective, the control system will manipulate the flow at the head of the main canal (f1) and the position of the main gates (g2 to g5).

Notice that typically the source of water at the head of the canal (a reservoir or a main river, as in this benchmark) is managed by a different organization than the canal operator. That includes also the control or supervision of the gate at the head of the canal (typically a set of undershot sluice gates), fixing at any time the head flow of the canal or, at least, the constraints on the flow. In this benchmark, the proposed control system provide flow set-points to the head gate but considering flow limits imposed by the external organization

The following constraints are also considered:

- Minimum level to guarantee that off-take points are submerged.
- Maximum level to prevent the canal from bursting its banks and causing floods.
- The flow at the head of the canal is limited.
- Maximum and minimum gates opening. Maximum gate opening is fixed by the water level, when the whole gate becomes above the water line and increasing its opening is pointless. Besides, gate opening has a physical limit which depends on the gate itself.

A combination of local and distributed MPC approaches is proposed for the control of this section of the canal.

3 CONSTRAINED PREDICTIVE CONTROL OF IRRIGATION CANALS

This section presents control algorithms and techniques used to control the irrigation canal described in the previous section. These techniques are briefly reviewed, and only the main ideas are presented. Thus, the reader interested in more technical details is encouraged to consult the Model Predictive Control works cited in this section. Section 3.1 presents the main constrained predictive control used to compute the control signal applied to each gate. How to take into account the effect of measurable disturbances in the control signal computation is shown in section 3.2. Finally, the coordination of a pair of local controllers using a Distributed Model Predictive Control scheme is presented in section 3.3.

3.1 Constrained Model Predictive Control

Model Predictive Control (MPC) is one of the most popular techniques in the field of automatic control. It is in fact one of the few advanced control techniques that are nowadays available in commercial industrial control solutions. The reasons of that success are mainly the ability to consider constraints in the computation of the control signal, the possibility of taking into account measurable disturbances and process dead-time and, also, that the extension to the multivariable case is relatively straightforward. All these features yield a control performance that can be much better than that obtained with conventional control methods (i.e., PID controllers). Furthermore, MPC can be applied to a wide range of problems and the tuning of MPC controllers involve the choice of a reasonable number of design parameters.

All MPC strategies are based on a process model that is used to predict the evolution of the system state or output[1] along an interval of time called the prediction horizon (see figure 2). The prediction of the system output are computed iteratively using the prediction model using present and past values of the system input and output as initial conditions. Predicted values for system output or input at time $k+j$ using the information available at time k are denoted by $y_{k+j|k}$ or $u_{k+j|k}$. On the other hand, the prediction horizon comprises all sampling times between $k+N_1$ and $k+N_2$. There is also a control horizon, comprising sampling times between k and $k+N_u-1$, after which the system input is considered to be constant (see figure 2).

Usually, the process model (or prediction model) considered is a discrete time model that can be nonlinear or linear. The theory of MPC using linear models is much more developed than that of nonlinear models, thus almost all commercial implementations are based on a linear prediction model. From a practitioner point of view, the most natural and easy choice is an input-output model based on the transfer function of the process to be controlled. Thus, we propose the use of a CARIMA (Controlled Auto-Regressive Integrated Moving

[1]In this work we use input-output models, thus we consider here predictions of the system output.

Average) model, which in time domain can be written as:

$$y_k = a_1 y_{k-1} + \cdots + a_{na} y_{k-na} + b_0 \Delta u_{k-d-1} + \cdots + b_{nb} \Delta u_{k-d-nb-1} + e_k \qquad (1)$$

where d is the model dead time measured in sampling times and y_k and Δu_k denote the value of the process output and process input increment at sampling time k. Furthermore, this model considers a noisy disturbance denoted as e_k which can be modelled as a white noise. Note also that, in practice, the a_i, b_i parameters would be usually obtained through identification from the real process to be controlled.

Model (1) can be used to obtain at time k predictions of the future values of the process output along a prediction horizon defined by the sampling times $k + j$, $j \in [N_1, N_2]$. Those predicted values of the process output computed at time k will be denoted as $y_{k+j|k}$. Note that these predictions will depend on some information that is readily available at time k (namely the present and past values of the process output and the past values of the input increments) and also they will depend on the present and future values of the input increments, which have to be computed by the predictive controller. These present and future values of the input increments will be considered along a control horizon defined by the sampling times $k + j$ with $j \in [0, N_u - 1]$[2].

As mentioned at the beginning of this section, one of the most remarkable features of MPC is that constraints can be taking into account in the computation of the control signal. Thus, here constraints on the values of the input signal, input increments and predicted outputs are considered:

$$\begin{array}{rcccll} \underline{u} & \leq & u_{k+i|k} & \leq & \overline{u}, & i = 0, \ldots, N_u - 1 \\ \underline{\Delta u} & \leq & \Delta u_{k+i|k} & \leq & \overline{\Delta u}, & i = 0, \ldots, N_u - 1 \\ \underline{y} & \leq & y_{k+i|k} & \leq & \overline{y}, & i = N_1, \ldots, N_2 \end{array} \qquad (2)$$

[2]Note that if the control horizon is smaller than the prediction horizon, i.e., $N_u < N_2$, then, the input increments after the control horizon are assumed to be zero, i.e, $\Delta u_k = 0$, $k \in [N_u, N_2]$.

Note that, being model (1) linear, all these constraints are linear on the input increments, so they can be rewritten as:

$$R\mathbf{u} \leq c \tag{3}$$

where R and c are a matrix and a vector of appropiate dimensions and $\mathbf{u}$ is the sequence of present and future input increments defined as $\mathbf{u} = \left[\Delta u_{k|k}, \Delta u_{k+1|k}, \ldots, \Delta u_{k+N_u-1|k}\right]$ (see (Camacho and Bordons 2004) for details on how to find R and c). Only those sequences $\mathbf{u}$ that satisfy (3) will be considered as admissible by the controller.

Once the admissible sequences $\mathbf{u}$ are characterized, the next step to the formulation of a MPC is to provide some means of getting a measure of how good is a sequence $\mathbf{u}$ in terms of control performance. This can be achieved by means of a quadratic cost function of the future set point tracking errors plus a term weighting the input increments is added:

$$J(\mathbf{u}) = \sum_{j=N_1}^{N_2} (y(k+j|k) - r(k+j))^2 + \lambda \sum_{j=0}^{N_u-1} \Delta u(k+j|k)^2 \tag{4}$$

where $\lambda > 0$ is the weighting factor for present and future input increments[3]. Note that this term is added to penalize the use of unnecessary arbitrarily large values of the input increments as these increments are usually related to economical costs. With this definition of $J(\mathbf{u})$ the best control sequence will be that which obtains the smallest tracking errors with the smallest control input increments. This sequence will be the one that minimizes the cost function $J(\mathbf{u})$.

With all the previous elements, the optimal control sequence $\mathbf{u}^*$ produced by the MPC controller is defined as the solution of the following optimization problem:

$$\begin{aligned} \mathbf{u}^* = \arg \quad & \min_{\mathbf{u}} \quad J(\mathbf{u}) \\ & \text{s.t.} \quad R\mathbf{u} \leq c \end{aligned} \tag{5}$$

[3]Note that more complex weighting schemes exist (like using time variable weight factors or weighting both terms in (4)). We use here the scheme proposed in (Clarke et al. 1987) as practice shows that a similar performance can be achieved with the added benefit of a simpler tuning procedure (as only one weighting factor has to be tuned).

The solution to this optimization problem is applied using a receding horizon scheme, that means that every sampling time problem (5) is solved, and at each sampling time, only the first component of $\mathbf{u}^*$ is in fact applied to the system, whereas the remaining components are discarded. The reason to use such receding horizon scheme is to close the control loop, that otherwise would result in an open-loop control scheme. Note that, being the model and constraints linear and the cost function quadratic, the optimization problem (5) is a Quadratic Program that can be efficiently solved using the current computer hardware.

There are some different ways to implement MPC algorithms. All of them are discussed in depth in (Camacho and Bordons 2004). We have chosen Generalized Predictive Control (GPC), which can easily be extended for the distributed case.

3.2 Consideration of measurable disturbances in the computation of the control signal

Measurable disturbances can be easily included in an MPC scheme like the one presented so far in this section. The only modification that has to be done is in the prediction model, that now has two deterministic inputs: the manipulated input u (which it is used to control the system) and the disturbance v (which has to be measurable). Thus, model (1) will be rewritten as:

$$\begin{aligned} y_k &= a_1 y_{k-1} + \cdots + a_{na} y_{k-na} + b_0 \Delta u_{k-d_u-1} + \cdots + b_{nb} \Delta u_{k-d_u-nb-1} + \\ &\quad + d_0 \Delta v_{k-d_v-1} + \cdots + d_{nd} \Delta v_{k-d_v-nd-1} + e_k \end{aligned} \tag{6}$$

Note that the delay from each input to the output is not necessarily equal, and that the measurable disturbance behaves just like an extra input that it is not under our control. Besides this modification, the MPC controller formulation remains the same. This way of taken into account measurable disturbance is essentially the same as in the classic feedforward disturbance compensation techniques (Camacho and Bordons 2004).

3.3 Cooperative Distributed MPC

The MPC strategy discussed so far involves a number of controllers that operate independently without exchanging any information about the optimal sequences computed by each one. Thus, each controller operates independtly, having its own data, which are just a part of the whole information. However, it is possible to establish a communication link between two or more controllers in order to share information and to work in a collaborative manner. In this way, the controllers would have more information available, which would improve the overall control performance. This observation leads to the development of cooperative Distributed MPC (DMPC) strategies (see (Zafra-Cabeza et al. 2011)). The algorithm used here, which is discussed in detail in (Maestre et al. 2011), involves only a pair of controllers (although it can be extended to consider any number of controllers), and it is based on cooperative game theory. The goal is to control a pair of constrained coupled linear systems where a communication link is established between controllers. Each controller has only a part of the information related to the model and the state of the overall system, although they can exchange information about their optimal control sequences. Game theory is used to implement a coordination scheme in which both controllers have to cooperate to achieve their control goals, even in the case of conflicting goals. The coordination problem is reduced to a cooperative game where each agent have to make a choice among three possibilities. Only two communication cycles will be required for each choice.

The proposed distributed MPC algorithm,for a pair of controllers, is the following:

1. At sample time k Each controller $i \in [1,2]$ reads its controlled variables. Denote the optimal sequence computed in the previous sample time as $U_i^S(k)$.
2. Each controller $i \in [1,2]$ solves its local MPC problem minimizing its own cost function J_i and considering the effect of the control actions of the other controller as a measurable disturbance. It is assumed that the other controller will keep applying the optimal control sequence computed in the previous sample time (that is, $U_j^S(k)$)[4].

[4]Given $i \in [1,2]$ and $j \in [1,2]$, when i=1 then j=2 and vice versa. So, in general terms, we use the

Denote the optimal control sequence as $U_i^*(k)$.

3. Each controller $i \in [1, 2]$, assuming that it applies the optimal sequence previously obtained in step 2, computes the control sequence for neighbour j that gets the smallest value of its own cost function J_i. That is, each controller computes the neighbour input that it is more beneficial for its own performance. Denote this sequence as $U_j^w(k)$[5]. Note that each controller assumes that its neighbour behaves in an altruist way, thus it will "agree" to use $U_j^w(k)$ instead of $U_j^*(k)$.
4. Both controllers communicate the sequences computed in the previous steps. Controller 1 sends to controller 2 the sequences $U_1^*(k)$ and $U_2^w(k)$, whereas controller 2 sends to controller 1 $U_2^*(k)$ and $U_1^w(k)$. Thus, at the end of this step both controllers know all the sequences that have been computed so far.
5. Each controller i evaluates its own cost function for all the sequences it could choose. That is, controller 1 computes the set:

$$\mathbf{J_1} = \left\{ J_1(U_1^S(k)),\ J_1(U_1^*(k)),\ J_1(U_1^w(k)) \right\}$$

and controller 2 computes the set:

$$\mathbf{J_2} = \left\{ J_2(U_2^S(k)),\ J_2(U_2^*(k)),\ J_2(U_2^w(k)) \right\}.$$

6. Both controllers communicate the values obtained in the previous step. That is, controller 1 sends the set $\mathbf{J_1}$ to controller 2, whereas controller 2 sends the set $\mathbf{J_2}$ to controller 1.
7. Both controllers consider the 9 possible pairs (J_1, J_2) of optimal costs in $\mathbf{J} = \mathbf{J_1} \times \mathbf{J_2}$ and pick the one that gives the minimum sum $J = J_1 + J_2$. Note that this pair has a pair of associated optimal sequences, which will be denoted as $U_1^d(k)$ and $U_2^d(k)$ respectively.

subindex i when referring to the controller we are dealing with and j when referring to the other one

[5]Note that controller 1 computes $U_2^w(k)$ and controller 2 computes $U_1^w(k)$.

8. Each controller i apply to the controlled system the first component of $U_i^d(k)$ and the whole procedure is repeated again at the next sampling time.

To summarize the procedure, the goal is to construct a 3x3 matrix. Each row contains a possible optimal sequence which can be chosen by controller 1, and each column contains a possible optimal sequence which can be chosen by controller 2. Cells contain the sum of cost functions for each of the possible optimal sequence combinations. Thus, there are 9 options, and the combination that minimize the cost function sum will be chosen.

4 MODELING AND CONTROL STRUCTURE

Models that involve water movement are generally obtained making use of simplifications of the Navier-Stokes equations, because of the complexity in dealing directly with them. For irrigation canals, one of the most accepted and used model in simulations is the system given by the Saint-Venant Equations, because of its capacity to represent the dynamic characteristics of real interest. However, this system is a nonlinear partial differential equation system, which has analytical solution only in very special cases, forcing the employment of numerical methods to solve it properly. Since the early 60s researchers have devoted important efforts to developing efficient solutions methods for those equations. Most numerical methods can be included in the finite difference or finite element categories. As a model for computational simulation it is very accurate, but as model for control, it is clearly not appropriate because of its complexity. Linearizations or simplifications of the Saint-Venant equations are used for control purposes.

Making use of Saint Venant equations, a reach can be modelled by two partial differential equations representing a mass balance (continuity equation) and a momentum balance.

$$\begin{cases} \frac{\partial q(t,z)}{\partial z} + \frac{\partial s(t,z)}{\partial t} = 0 \\ \frac{1}{g}\frac{\partial}{\partial t}\left(\frac{q(t,z)}{s(t,z)}\right) + \frac{1}{2g}\frac{\partial}{\partial z}\left(\frac{q^2(t,z)}{s^2(t,z)}\right) + \frac{\partial h(t,z)}{\partial z} + I_{\mathrm{f}}(t,z) - I_0(z) = 0 \end{cases} \tag{7}$$

The variables represent the following quantities:

- z is the spatial variable which increases along the flow main direction;
- $q(t, z)$ is the river flow (or discharge) at time t and space coordinate z;
- $s(t, z)$ is the wetted surface;
- $h(t, z)$ is the water level w.r.t. the river bed;
- g is the gravitational acceleration;
- $I_f(t, z)$ is the friction slope;
- $I_0(z)$ is the river bed slope.

Different approaches have been used to model the friction slope such as the Gauckler-Manning-Strickler equation:

$$S_f(t, z) = \frac{q(t, z)^2 \left(p(t, z)\right)^{4/3}}{k_{\text{str}}^2 \left(s(t, z)\right)^{10/3}} \tag{8}$$

where $p(z)$ is the wet section perimeter and k_{str} is the Gauckler-Manning-Strickler coefficient. The Gauckler-Manning-Strickler coefficient changes accordingly to the kind of river bed surface.

In order to have a realistic simulation of the irrigation canal of La Pedrera, Saint-Venant equations, the well known SIC (Simulation of Irrigation Canals) software has been used. SIC provides a mathematical model which can simulate the hydraulic behaviour of most of the irrigation canals or rivers, under steady and unsteady flow conditions. Steady flow and unsteady flow computations can be performed on any type of hydraulic networks (linear, looped or branched). Any reach can be composed of a minor, a medium and a major bed. Storage pools can also be modelled. The SIC model is an efficient tool allowing canal managers, engineers and researchers to quickly simulate a large number of hydraulic conditions at the design or management level. Moreover, it can be interfaced (by means of its regulation module) to different mathematical software like Matlab and Scilab, a very convenient feature for research purposes.

The SIC model of the irrigation canal of La Pedrera comprises a set of data which are obtained from a topographic source or planes. From the software processing point of view, hydraulic canals are usually described on the basis of a set of cross-sections. Each section has some associated information such as the section shape (circular, square, trapezoidal), the coordinates of the significant points of the section (usually vertices), the position measured from the origin of the canal, the Manning coefficient or water leakage losses. In addition, points where either water injection or extraction exists are indicated by using nodes. As indicated above, these points are called off-takes. A reach is a canal portion situated between a pair of nodes. SIC has a editing tool (see figure 3)which allows to characterize the canal by introducing the data related to each cross-section(see figure 4).

The Saint-Venant model of the irrigation canal is a very realistic one and it will be used as a test bed for the control structure proposed in this paper. But for control purposes a less complex model is usually needed. Moreover, the control structure proposed here relies on model predictive controllers, which use a linear prediction model to compute the gate openings that are necessary to attain the target flows at each gate. Thus a Multi Input Multi Output (MIMO) model of five inputs and five outputs has been identified using the well known Least Squares method (this method and others used in identification processes are discussed in depth in (Cellier and Greifeneder 1991; Johansson 1993; Landau and Landau 1990)) around the operating point shown in table 1. In the model, the five inputs are the flow at the head of the main canal (u1), and the position of the main gates g2 to g5 (u2 to u5 in table 1). On the other hand, the five outputs that are to be controlled are water level at each one of the reaches in Canal de Cartagena (y1, y2, y4 and y5) and the flow at the head of Canal de la Pedrera (y3). The linear models for each input-output pair are first or second order models plus a transport delay (system modelling and concretely first and second order approaches are deeply discussed in (Ogata 2010)) caused by the distance between reaches. Note that being the model a MIMO one, there can be couplings between different pairs of input-outputs, thus a given output can be affected not only by its paired input but also by any other input in the model. These couplings or interactions can be weaker or more intense, and in this latter case they cannot be neglected when designing the control structure.

Once the hydraulic canal has been modelled as a MIMO plant, the following step is to design an optimal control structure. Firstly an appropriate input-output pairing must be chosen. During this research several pairings were tested. The chosen input-output pairing is detailed in table 2. In this table, an input-output pair is detailed in every row and information about the involved magnitudes and the measurement points is shown. Two data are necessary to locate these points: the branch where they are situated and the kilometric distance to the end of the branch. Figure 5 gives an idea of the location of both inputs and outputs and distances between them. Some control structures will be explained below.

Figure 6 shows a totally decentralized control structure based on predictive control. Five GPC controllers govern each input-output pairing aforementioned. GPC1 tracks a downstream water level reference by regulating the incoming water flow at the canal head gate. GPC3 monitors the downstream flow through its corresponding gate by manipulating its degree of aperture. Finally, GPC2, GPC4 and GPC5 track a water level reference using the degree of gate aperture as a manipulated variable.

An hydraulic canal is such a coupled system that every control command sent to the plant in order to obtain a desirable behaviour at one of the outputs significantly affects the rest of them. This may be taken into account at every sample time when computing the following control action. Every single controller can consider control actions computed by its neighbours as a measurable disturbance. This disturbance is easily included in the control action calculation using a feedforward compensation as explained in section 3.2. Figure 7 shows how this theory is applied to this research. Each GPC will have two kinds of inputs: on the one hand the measurement of its output and the corresponding reference (red arrows), and on the other hand the measured disturbances (green arrows). Disturbances could be considered both upstream and downstream, but in this case only downstream disturbances were taken into account, in order to simplify the problem. Moreover, the implementation of this feedforward compensation will be done in a sequential manner. That is, the control actions to be applied at each gate are computed sequentially, starting from the most downstream gate and proceeding upwards to the first (upstream) gate. Then, when computing the optimal aperture of a given gate, the aperture of the nearest downstream gate (which was computed in the previous step of this sequence) is considered as a measurable disturbance. This feedforward scheme is later referred in the text as a sequential feedforward. In section 5 results will show a significant improvement of the canal control performance by considering downstream couplings as disturbances when computing control actions.

Finally, two controllers can cooperate, as explained in section 3.3, to obtain an optimal control sequence, by using an algorithm based on game theory. To implement this algorithm, a communication channel between the controllers (or agents) is necessary. This is a distributed control schema. Starting with the structure presented in figure 7, the distributed control algorithm is implemented in controllers GPC1 and GPC2. A communication link is established between them and each controller takes into account the control actions performed by the other one for calculating its own control actions. The neighbour control actions will be considered as measurable disturbances.

5 RESULTS

Different scenarios and control approaches have been tested in simulation. The canal benchmark has been modeled in SIC. The operation point has been established with $12m^3/s$ at the head of the canal and gates positions of $1m$ for the first gate and $0.5m$ for all the other gates. Table 1 shows lavels and flows at significant positions of the canal for the operation point. Three predictive approaches have been tested in the different scenarios:

- Control Schema 1: Downstream local MPC in each one of the gates.
- Control Schema 2: Local MPC with sequential feedforward.
- Control Schema 3: Distributed MPC in the two first gates and local MPC in the others with sequential feedforward.

Sampling time has been fixed to 6 minutes. The duration of the simulation tests is four days. A comparison among the three approaches had been performed using the following control performance and economic indexes:

- Performance Index ($\sum J$). The sum of the cost functions J in equation 4 of each one of the controller has been used as control performance index.
- Economic Index (EI) considers lost water and unsatisfied water demand. In all the test cases, the demand of lateral off-takes has been satisfied properly, but also a flow demand at the end of each one of the canal branches has been considered. The flow demand has been considered constant along the simulation time, and different economic penalization for flows over the demand (lost water) and under demand (unsatisfied demand) are applied. Figure 8 shows the penalization that has been used in the following test cases.

$$
\begin{aligned}
&q_{LW_i}(t) = \begin{cases} q_{oi}(t) - q_{oi}^d & \text{if } q_{oi}(t) > q_{oi}^d \qquad i = 1,2 \\ 0 & \text{if} q_{oi}(t) \leq q_{oi}^d \end{cases} \\
&q_{UD_i}(t) = \begin{cases} q_{oi}^d - q_{oi}(t) & \text{if } q_{oi}(t) < q_{oi}^d \qquad i = 1,2 \\ 0 & \text{if} q_{oi}(t) \geq q_{oi}^d \end{cases} \\
&LW_i = C_{LW}(\textstyle\int_0^{t_f} q_{LW_i} dt) \\
&UD_i = C_{UD}(\textstyle\int_0^{t_f} q_{UD_i} dt) \\
&EI = LW_1 + LW_2 + UD_1 + UD_2
\end{aligned}
\tag{9}
$$

Where:

- EI: Economic index
- qo_i: Flow at the tail of branch i ($i = 1$, La Pedrera branch and $i = 2$, Campo de

Cartagena branch as shown in Figure 1).

- qo_i^d: Flow demand at the tail of branch i.
- LW_i: Lost water (m^3) in branch i.
- UD_i: Unsatisfied demand (m^3) in branch i.
- C_{LW}: Cost of lost water (0.2 Euros/m^3).
- C_{LW}: Cost of water unsatisfied demand (0.5 Euros/m^3).

These two indexes J and EI are presented for each one of the control schemas and different tests in the following subsections.

5.1 Test 1: Set point changes

To compare the controllers under similar operation conditions a experiment is defined where a set of reference changes in the levels of reaches (ref1 and ref2) and in the flow (ref3). Reference 1 is increased $0.2m$ at the beginning of the second day, reference 2 is increased also $0.2m$ at the beginning of the third day and finally the flow reference 3 is increased $0.5m^3/s$ at the beginning of the third day. Figure 9 shows the controlled variables (level in ref1, ref2, ref4 and ref5 and flow in ref3) using the three test controllers (green dashed line for control schema 1, blue dashed-dotted for schema 2 and red for schema 3). The best performance is obtained using distributed control in reaches 1 and 2. Level zero in the figures corresponds to the operating point value (See Table 1). It can be seen that in set-point change in reach 1 (Figure 9a), damping appearing in control schemes 1 and 2 is considerably reduced. Notice that the disturbance of the setpoint change of ref1 in the behavior of the level of reach 2 is dramatically diminished (Figure 9b).

Table 3 shows the global control performance index considered as the sum of the local cost function for each of one of the controllers as defined in equation (4). The last column shows that the economic index using the control schema 3 is a third of the economic index of control schema 1.

5.2 Test 2: Off-take flow changes

This second test is devoted to analyze the behavior of the tested controllers when changes in off-take flow are produced. Off-takes are considered as perturbations since the farmers decided at any time the flow they need for their local irrigation (nevertheless, they usually follow a previous established irrigation plan). For this reason the off-take prediction is considered in the MPC control.

In the presented test, a flow of $1m^3/s$ is extracted from the canal in points off1 and off4 (See Figure 1) from the beginning of the second day to the end of the simulation period.

Figures 10 shows the controlled variables of the controllers and 11 the manipulated variables (gate position in g2, g3, ga4 and g5 and flow at the head of the canal). Notice that in gate 2 only the distributed controller is able to maintain the set point level, and with local controllers (schema 1), this gate reaches the maximum opening limit. The reason is the lack of communication between controllers 1 and 2. Controller 1 takes the decision independently of the needs of controller 2 in schema 1 (and even in schema 2) but in distributed controller both controller act in a coordinated way reaching a satisfactory performance.

Table 4 presents the performance index for test 2. Again, best results are obtained with the distributed controller in reaches 1 and 2.

5.3 Test 3: Off-take flow and references changes

This last test is a more complex situation with several simultaneous level and flow references and off-take flow changes. This test will show the coupling of the different subsystems and the effect of upstream perturbations at the downstream part of the canal.

The following reference changes and off-take flow modification have been considered:

- Change of $0.4m.$ in the level reference of gate 1 (Reference 1) at the beginning of the second day
- Increase $0.4m.$ in reference 2 at the beginning of the third day
- Change of $5m^3/s$ in reference 3 at the beginning of day 3

- Change of $0.1m.$ in the level reference of gate 4 at the beginning of the forth day
- A flow of $1m^3/s$ is extracted from the canal in points off1 and off4 since the beginning of the second day

Figures 12 show the controlled variables of the controllers. Again, the best behavior is obtained using schema 3 and the worst performance with schema 1.

Figure 12d shows the evolution of the level at the end of reach 4. Notice the effect of perturbations during the second and third day. Most of them are due to changes produced upstream. The behavior is quite oscillatory, but the amplitude of the oscillations is quite small (around $2cm.$).

Table 5 presents economic and performance indicator of the three approaches. Notice that an important decrease of both indexes is obtained when control schema 3 is applied.

6 CONCLUSIONS AND FUTURE WORKS

In this paper a distributed predictive controller has been proposed to control irrigation canals. An accurate model of a real irrigation canal in Spain has been used as a test bed for the controller. The model has been developed using the well known SIC software. This software uses the Saint-Venant equations to model the dynamics of the canal with better accuracy than other methods. The SIC software has been interfaced to the predictive controller which has been developed using Matlab. The results show that the proposed distributed control algorithm achieves better control performance than a local based controller scheme without information exchange (which is by far the most usual control scheme in automated irrigation canals). The improvements in control performance will lead to a better and more efficient management of irrigation canals that ultimately results in money and resource savings.

Future work will be focused on the development of more complex algorithms and in the validation of the controller in the actual irrigation canal. One interesting feature of the control of irrigation canals is that the dynamics are relatively slow so that complex control algorithms can be used even when the available control hardware has moderate computing capabilities. Thus, the use of nonlinear prediction models and the consideration of uncertain or non measurable disturbances are possibilities that can be explored. On the other hand, the validation of the control scheme in the actual irrigation canal imply the implementation of the control algorithms in preexistent hardware with the least possible addition of new control hardware (that for budget and reliability reasons will be based on microcontrollers).

APPENDIX I. REFERENCES

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List of Tables

u1	Flow ($m^3 \cdot s^{-1}$)	12	y1	Water level (m)	82.951
u2	Gate opening (m)	1	y2	Water level (m)	82.073
u3	Gate opening (m)	0.5	y3	Flow ($m^3 \cdot s^{-1}$)	5.41
u4	Gate opening (m)	0.5	y4	Water level (m)	81.269
u5	Gate opening (m)	0.5	y5	Water level (m)	80.643

TABLE 1: Operating point used for prediction model identification.

u1	Flow $(m^3 \cdot s^{-1})$	Head Gate	y1	Water level (m)	Branch 1/ RS 4.275
u2	Gate opening (m)	Branch 1/ RS 4.27	y2	Water level (m)	Branch 1/ RS 0
u3	Gate opening (m)	Branch 1/ RS 6.672	y3	Flow $(m^3 \cdot s^{-1})$	Branch 1/ RS 6.669
u4	Gate opening (m)	Branch 2/ RS 12.964	y4	Water level (m)	Branch 2/ RS 6.972
u5	Gate opening (m)	Branch 2/ RS 6.969	y5	Water level (m)	Branch 2/ RS 3.021

TABLE 2: Canal control: input-output pairing

Performance	$\sum J$	EI (Euros)
Control Schema 1	28.45	905
Control Schema 2	18.79	662
Control Schema 3	5.30	402

TABLE 3: Table of the performance indexes of each schema for Test 1 in a four-day simulation. Control performance in second column and economic performance in third column

Performance	$\sum J$	EI (Euros)
Control Schema 1	125.23	1674
Control Schema 2	52.20	1101
Control Schema 3	16.03	816

TABLE 4: Table of the performance indexes of each schema for Test 2 in a four-day simulation. Control performance in second column and economic performance in third column

Performance	$\sum J$	EI (Euros)
Control Schema 1	157.49	12845
Control Schema 2	116.43	7767
Control Schema 3	68.92	4660

TABLE 5: Table of the performance indexes of each schema for Test 3. Control performance in second column and economic performance in third column

List of Figures

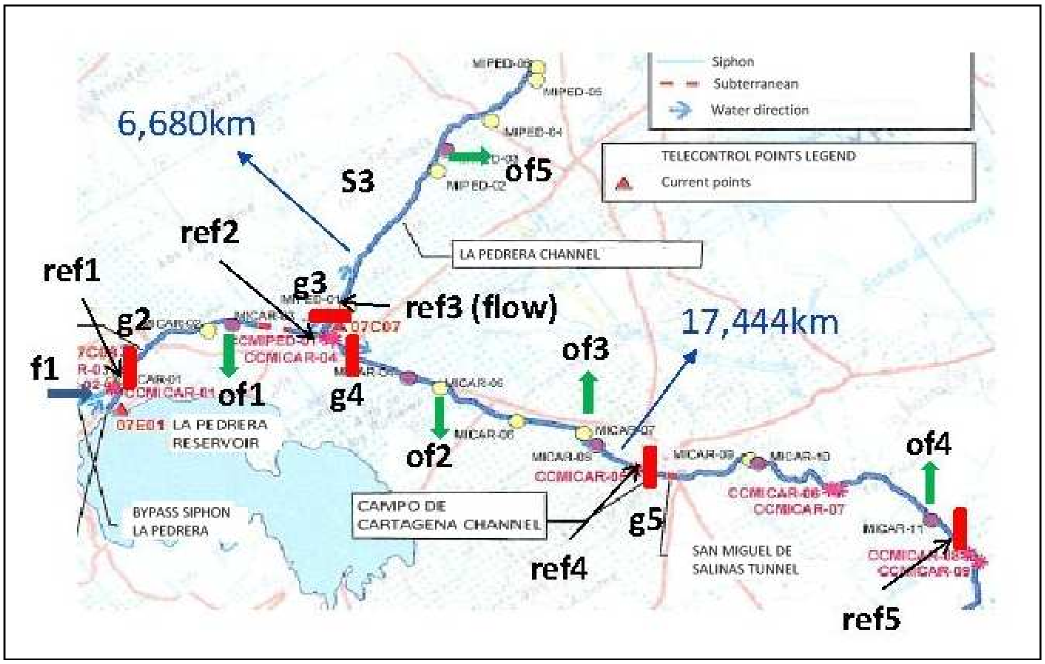


FIG. 1: Section of the Postrasvase Tajo-Segura.

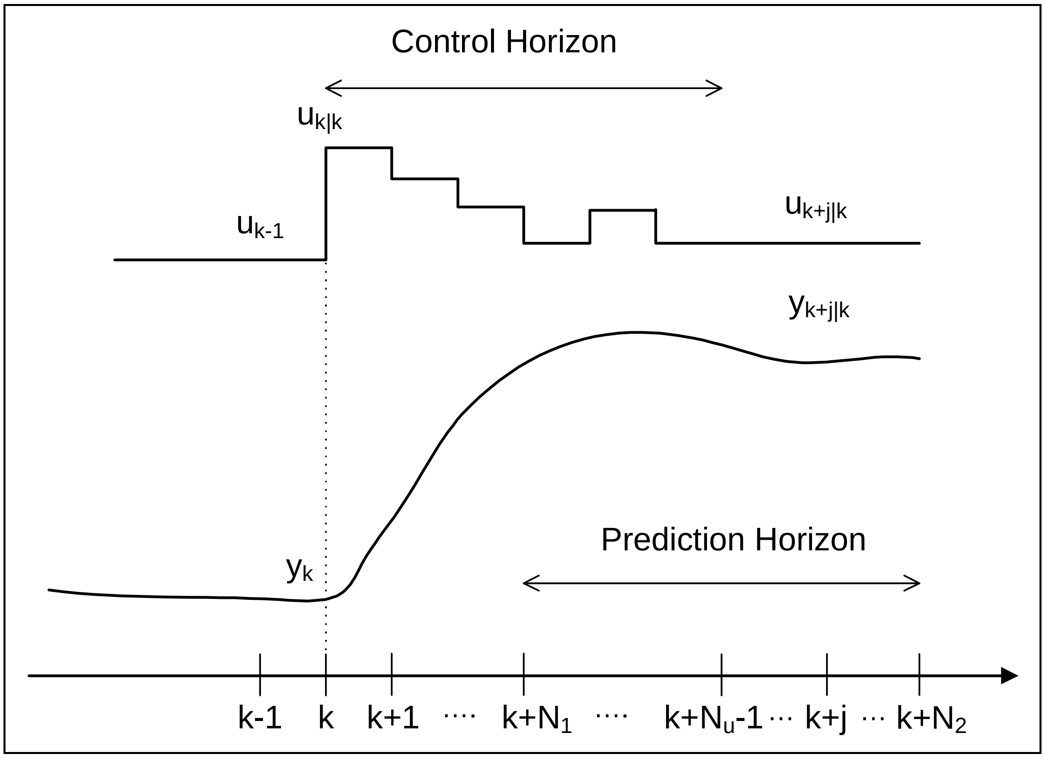


FIG. 2: Prediction in Model Predictive Control

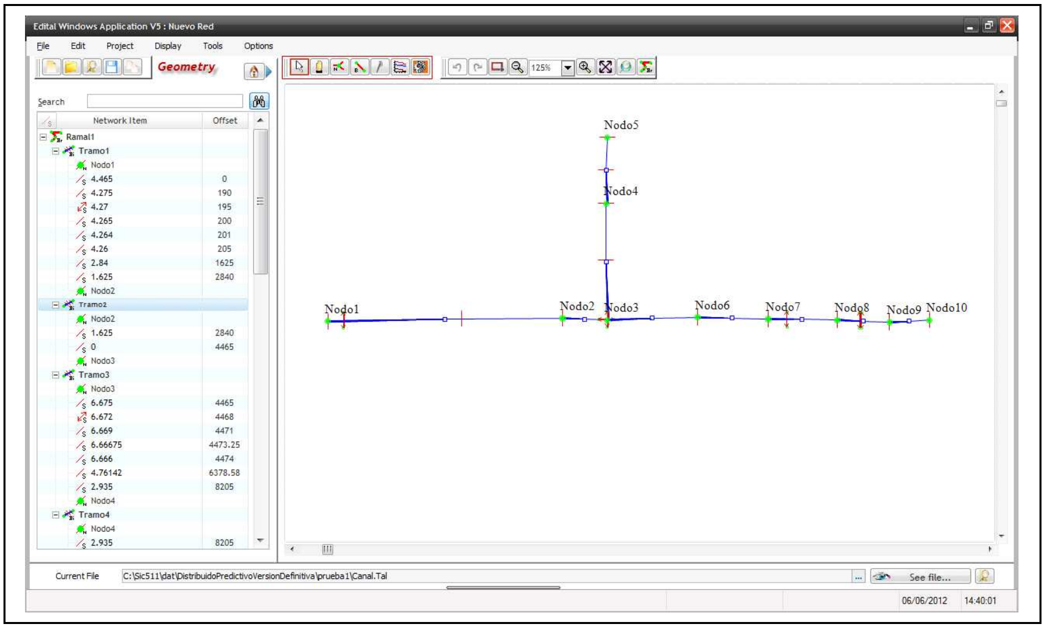


FIG. 3: View of the SIC tool for editing the canal hydraulic model

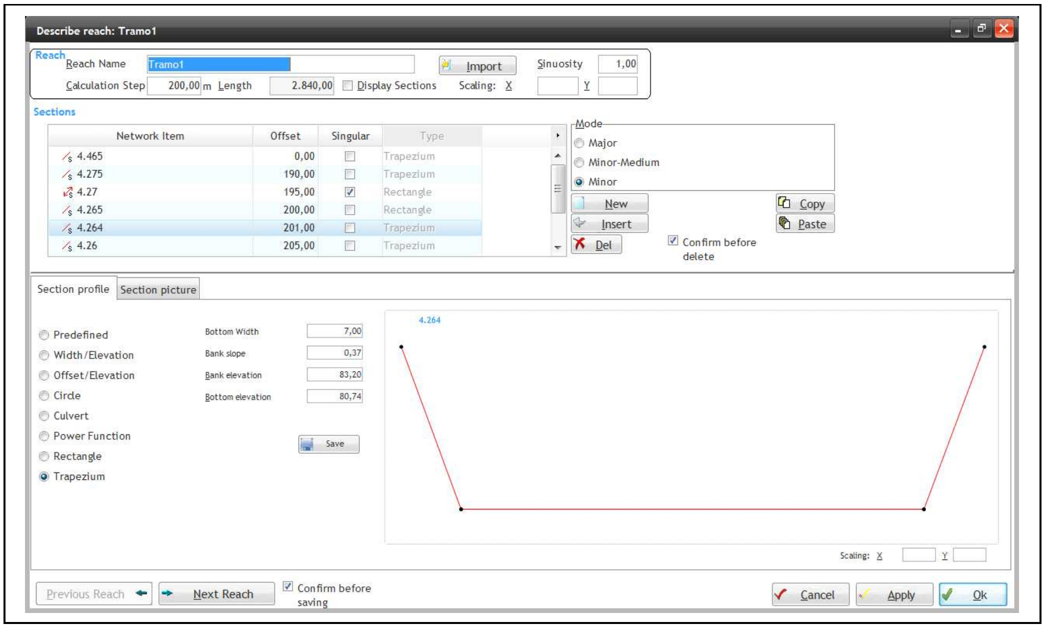


FIG. 4: Introducing data related to a cross-section

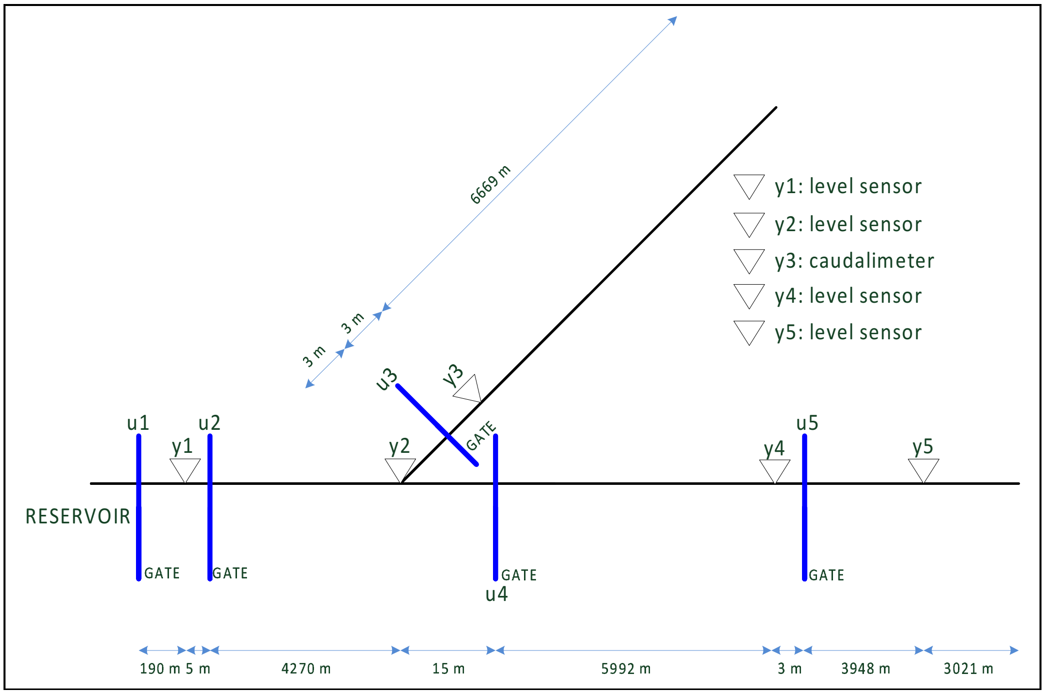


FIG. 5: Canal control: location for inputs and outputs

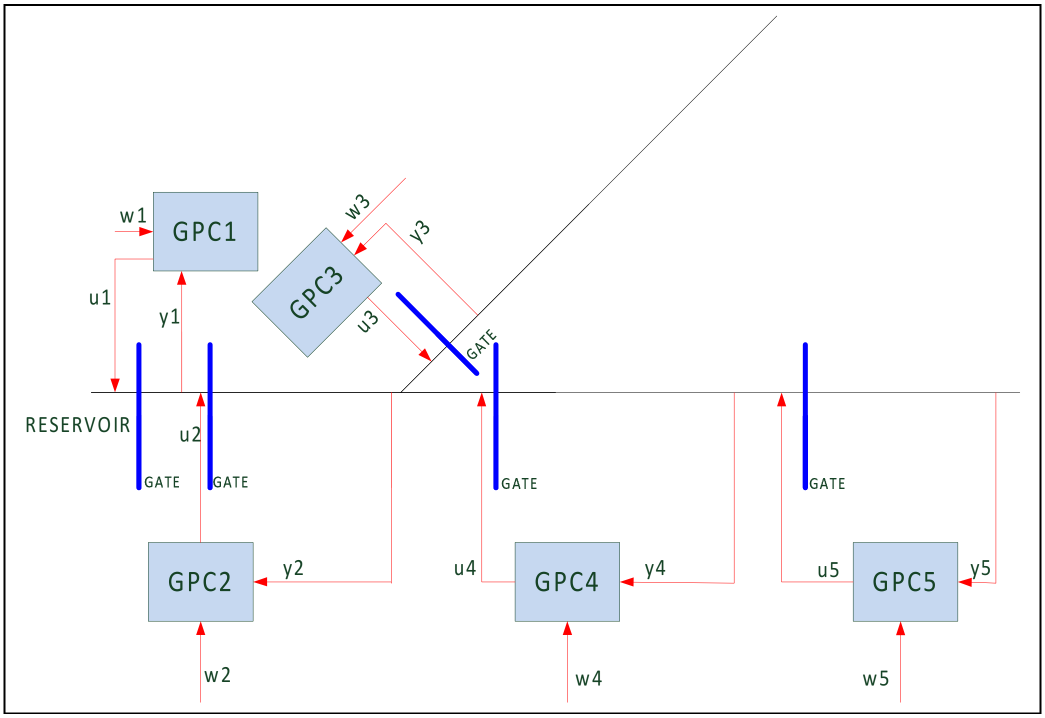


FIG. 6: Canal control structure based on decentralized GPC predictive controllers

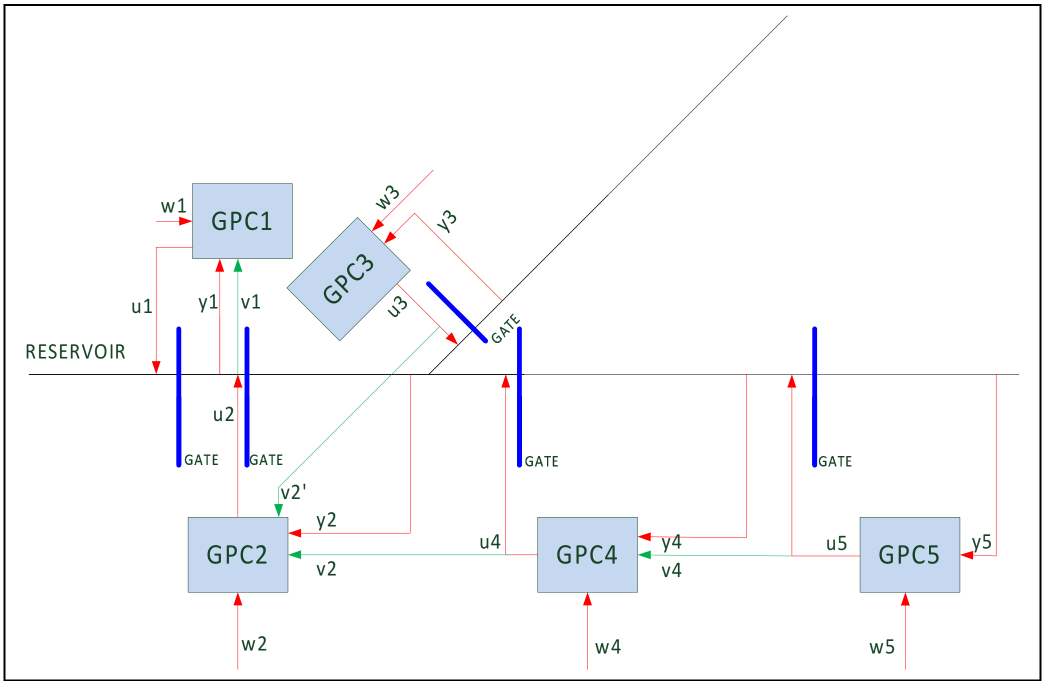


FIG. 7: Canal control structure based on decentralized GPC predictive controllers. Consideration of measurable disturbances

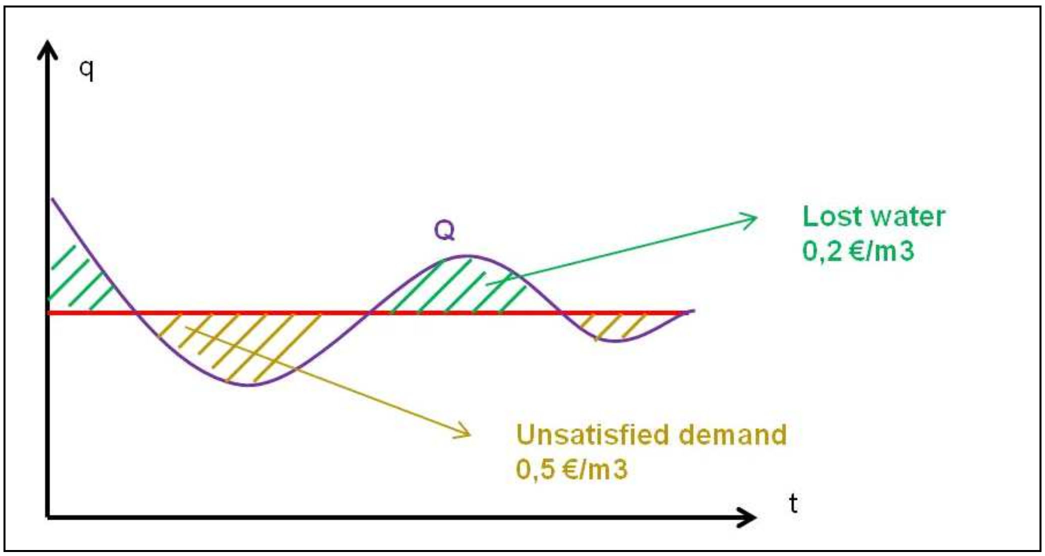


FIG. 8: Economic index computation

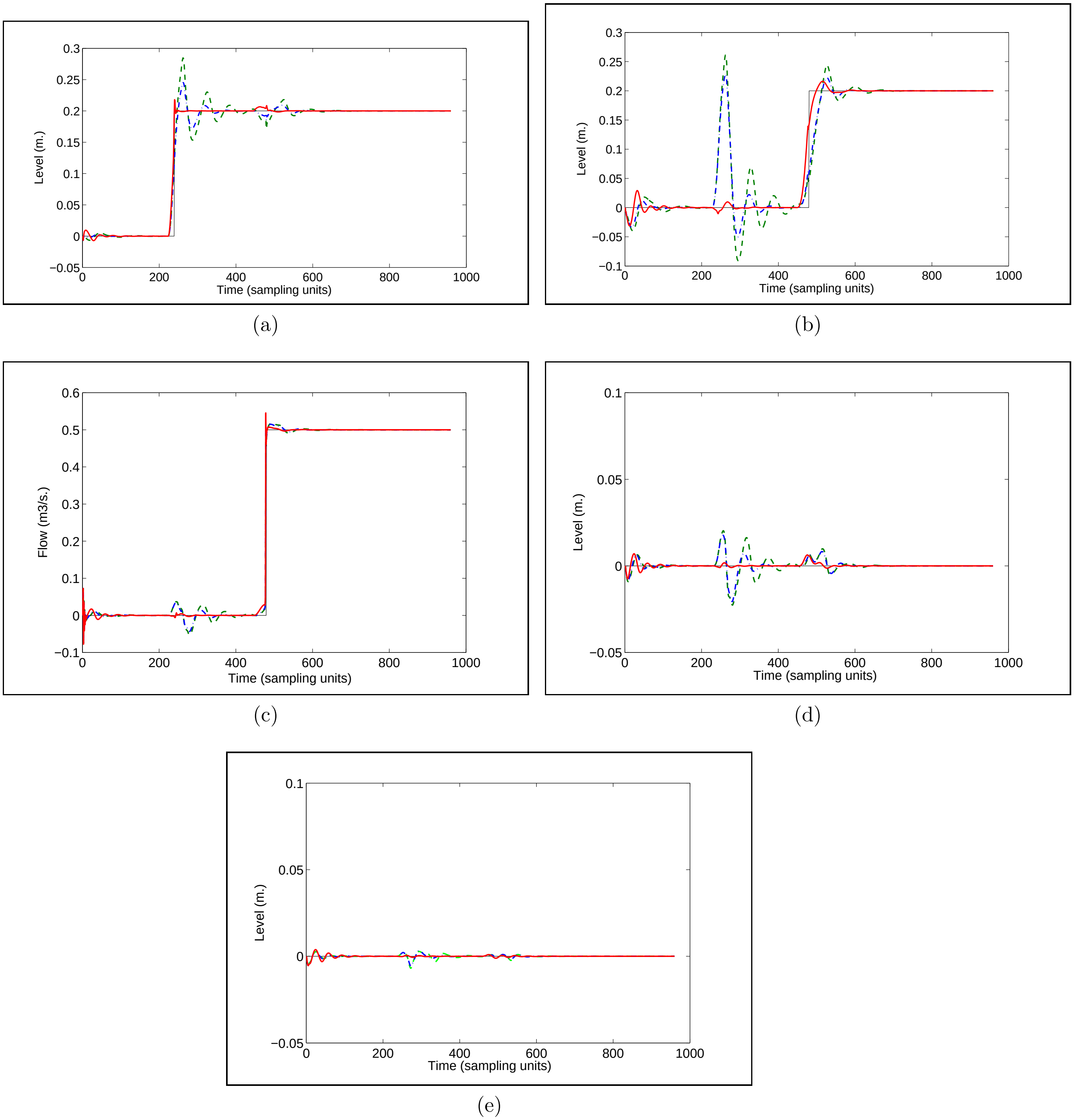
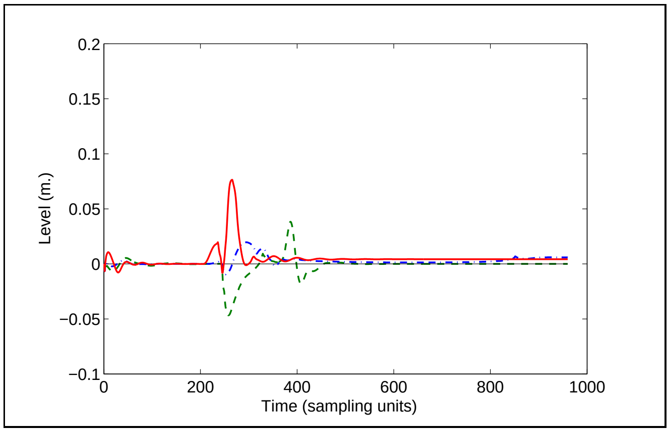
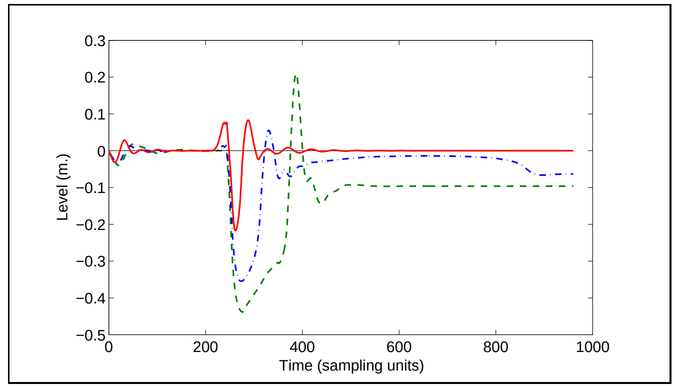


FIG. 9: Test 1: Level at ref1 to ref5 ((a) to (e) figures) position with control schema 1 (dashed green), schema 2 (dotted-dashed blue) and schema 3 (solid red)



(a)



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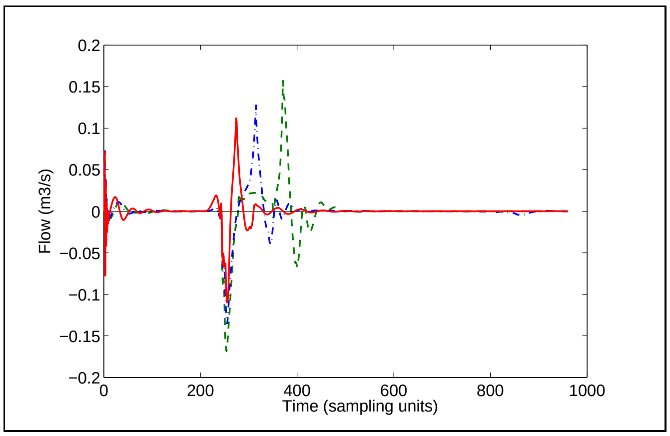
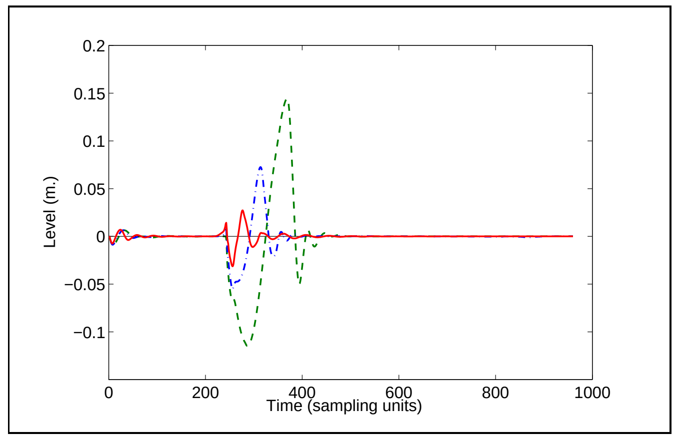


(c)



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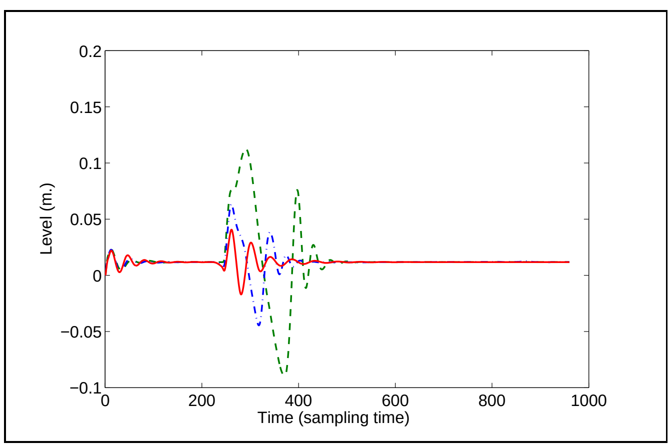


(e)

FIG. 10: Test 2: Controlled variables at ref1 to ref5 ((a) to (e) figures) position with control schema 1 (dashed green), schema 2 (dotted-dashed blue) and schema 3 (solid red)

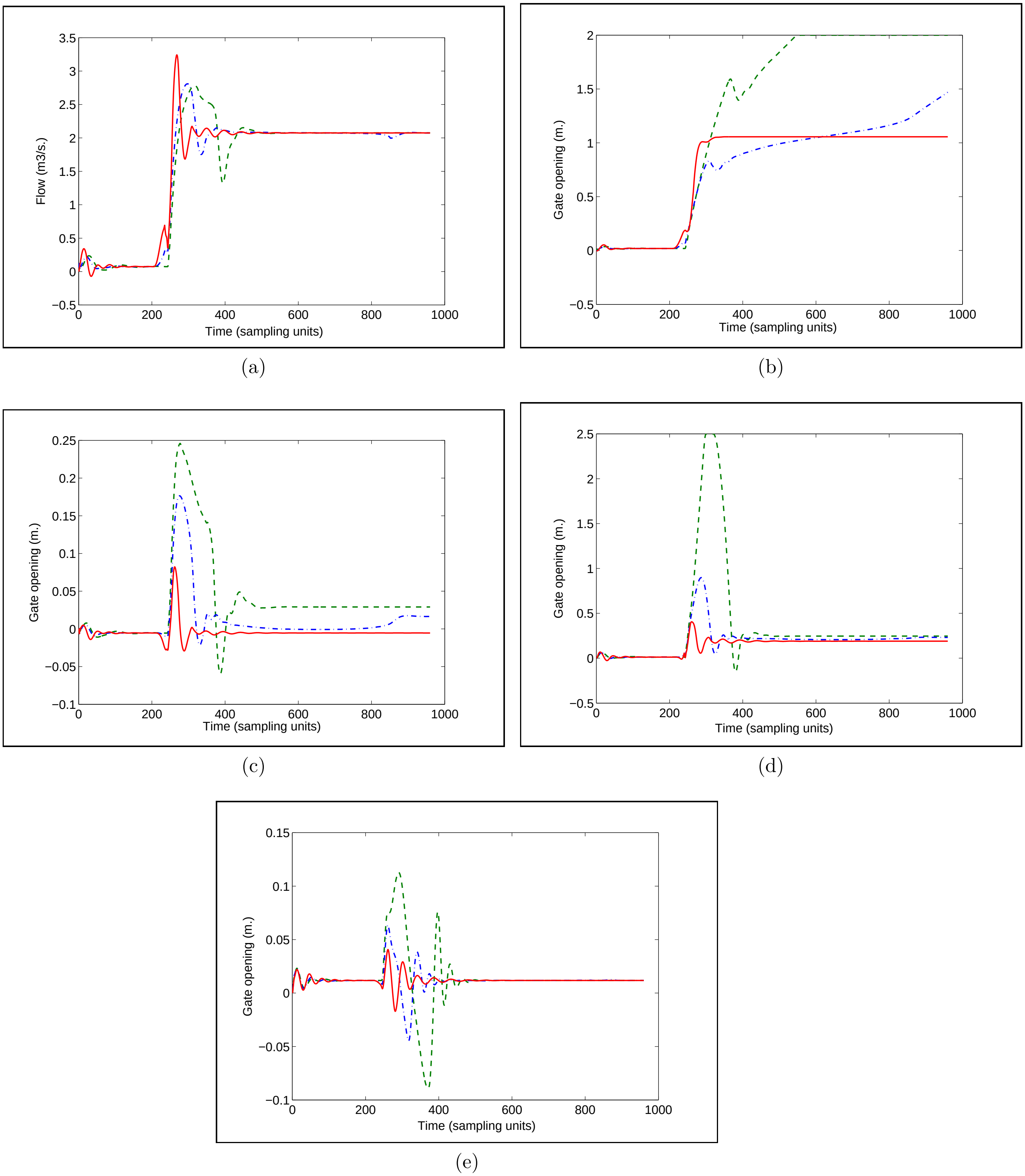
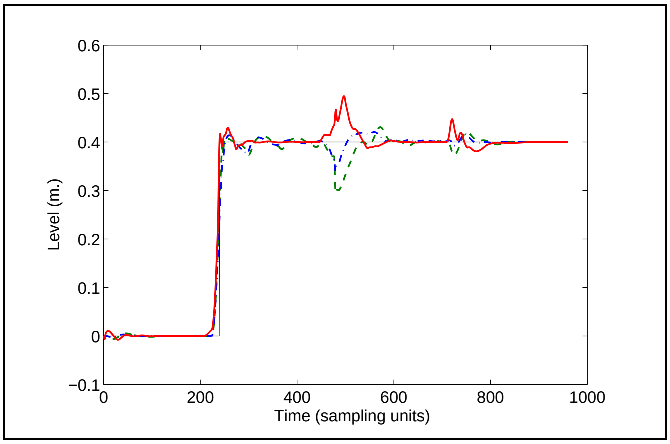
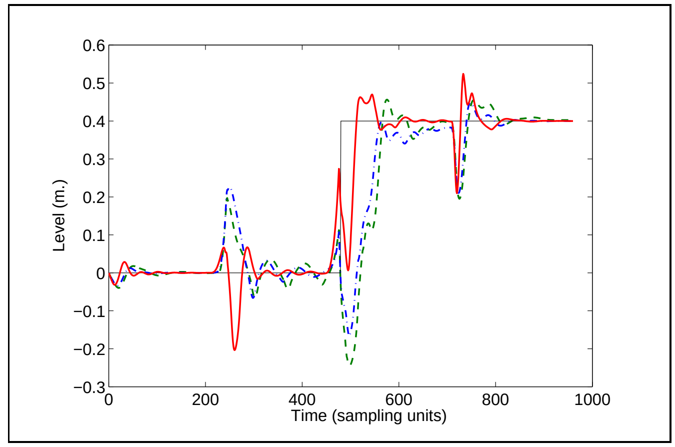


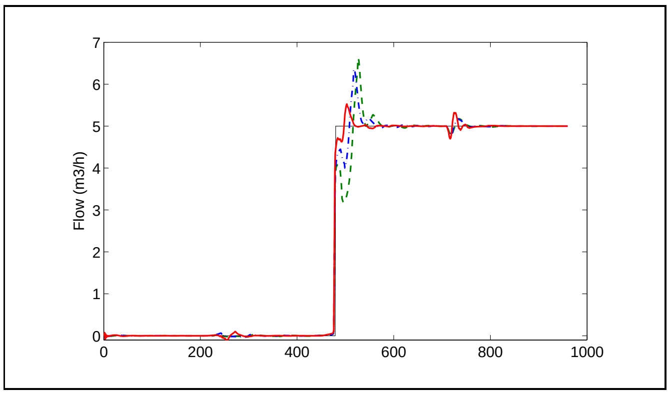
FIG. 11: Test 2: Manipulated variables. Flow at the head a) and gate positions at points ref2 to ref5 ((b) to (e) figures) with control schema 1 (dashed green), schema 2 (dotted-dashed blue) and schema 3 (solid red)



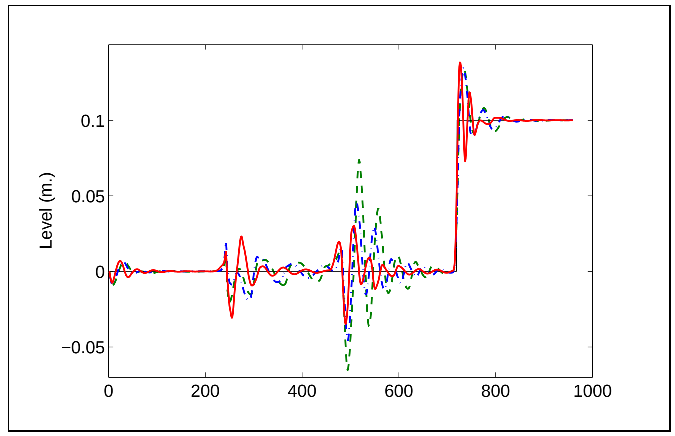
(a)



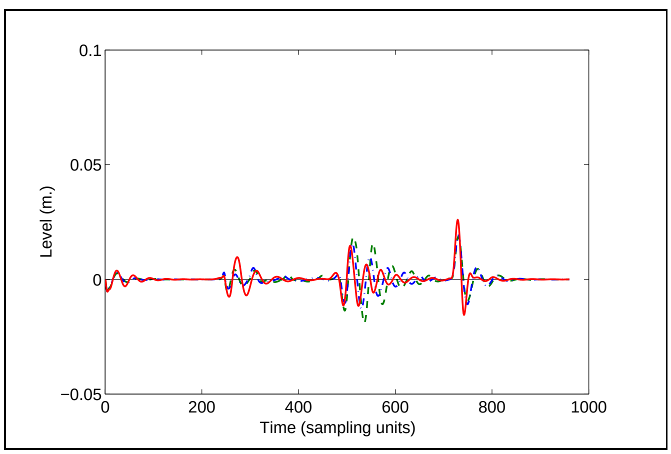
(b)



(c)



(d)



(e)

FIG. 12: Test 3: Controlled variables at ref1 to ref5 ((a) to (e) figures) position with control schema 1 (dashed green), schema 2 (dotted-dashed blue) and schema 3 (solid red)