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Distributed model predictive control of nonlinear systems subject to asynchronous and delayed measurements[☆]

Jinfeng Liu[a], David Muñoz de la Peña[b], Panagiotis D. Christofides[a,c,*]

[a] Department of Chemical and Biomolecular Engineering, University of California, Los Angeles, CA 90095-1592, USA

[b] Departamento de Ingeniería de Sistemas y Automática, Universidad de Sevilla, 41092, Sevilla, Spain

[c] Department of Electrical Engineering, University of California, Los Angeles, CA 90095-1592, USA

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ABSTRACT

In this work, we design distributed Lyapunov-based model predictive controllers for nonlinear systems that coordinate their actions and take asynchronous measurements and delays explicitly into account. Sufficient conditions under which the proposed distributed control designs guarantee that the state of the closed-loop system is ultimately bounded in a region that contains the origin are provided. The theoretical results are demonstrated through a chemical process example.

1. Introduction

Process control systems traditionally utilize dedicated, point-to-point wired communication links to measurement sensors and control actuators to regulate process variables at desired values. While this paradigm to process control has been successful, we are currently witnessing an augmentation of the existing, dedicated local control networks, with additional networked (wired and/or wireless) actuator/sensor devices which have become cheap and easy-to-install the last few years. Such an augmentation in sensor information and networked-based availability of data has the potential (Christofides et al., 2007; Neumann, 2007; Ydstie, 2002) to significantly improve: (i) the achievable closed-loop system performance, and (ii) the ability of the plant management systems to prevent or deal with abnormal situations more quickly and effectively. However, augmenting local control networks with additional networked sensors and actuators poses a number of new challenges including the feedback of additional measurements that may be asynchronous and/or delayed. Furthermore, augmenting dedicated, local control networks with additional networked sensors and actuators gives rise to the need to design/redesign and coordinate separate control systems that operate on the process.

Model predictive control (MPC) is a natural control framework to deal with the design of coordinated, distributed control systems because it can account for the actions of other actuators in computing the control action of a given set of actuators in real-time. Motivated by the lack of available methods for the design of networked control systems (NCS) for chemical processes, in a previous work (Liu, Muñoz de la Peña, Ohran, Christofides & Davis, 2008), we introduced a decentralized control architecture for nonlinear systems with continuous and asynchronous measurements. In this architecture, the pre-existing local control system (LCS) uses continuous sensing and actuation and an explicit control law. On the other hand, the NCS uses networked sensors and actuators and has access to additional measurements that are not available to the LCS. The NCS is designed via Lyapunov-based model predictive control (LMPC). Following up on this work, in another recent work (Liu, Muñoz de la Peña & Christofides, 2009), we proposed a distributed model predictive control (MPC) method for the design of networked control systems where both the pre-existing LCS and the NCS are designed via LMPC. This distributed MPC design utilizes continuous feedback, requires one-directional communication between the two distributed controllers, and may reduce

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* Corresponding author at: Department of Chemical and Biomolecular Engineering, University of California, Los Angeles, CA 90095-1592, USA. Tel.: +1 310 7941015; fax: +1 310 2064107.

E-mail addresses: jinfeng@ucla.edu (J. Liu), davidmps@cartuja.us.es (D. Muñoz de la Peña), pdc@seas.ucla.edu (P.D. Christofides).

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the computational burden in the evaluation of the optimal manipulated inputs compared with a fully centralized LMPC. The results obtained in Liu et al. (2009) are based on the assumption that continuous state feedback is available. In the present work, we consider the design of distributed MPC schemes in a more common setting for chemical processes. That is, measurements of the state are not available continuously but asynchronously and with delays. With respect to other available results on distributed MPC design, several distributed MPC methods have been proposed in the literature that deal with the coordination of separate MPC controllers (Camponogara, Jia, Krogh, & Talukdar, 2002; Dunbar, 2007; Keviczky, Borrelli, & Balas, 2006; Magni & Scattolini, 2006; Raimondo, Magni, & Scattolini, 2007; Rawlings & Stewart, 2007; Richards & How, 2007). All of the above results are based on the assumption of continuous sampling and perfect communication between the sensor and the controller. Previous work on MPC design for systems subject to asynchronous or delayed measurements has primarily focused on centralized MPC design (Liu, Muñoz de la Peña, Christofides, & Davis, 2009; Muñoz de la Peña & Christofides, 2008) and has not addressed distributed MPC with the exception of a recent paper (Franco, Magni, Parisini, Polycarpou, & Raimondo, 2008) which addresses the issue of delays in the communication between the distributed controllers.

This work focuses on the distributed MPC of nonlinear systems subject to asynchronous and delayed measurements. In the case of asynchronous feedback, under the assumption that there exists an upper bound on the interval between two successive state measurements, distributed LMPC controllers are designed that utilize one-directional communication and coordinate their actions to ensure that the state of the closed-loop system is ultimately bounded in a region that contains the origin. Subsequently, we focus on distributed MPC of nonlinear systems subject to asynchronous measurements that also involve time-delays. Under the assumption that there exists an upper bound on the maximum measurement delay, a distributed LMPC design is proposed that utilizes bi-directional communication between the distributed controllers and takes the measurement delays explicitly into account to enforce practical stability in the closed-loop system. The proposed distributed MPC designs also possess explicitly characterized sets of initial conditions starting from where they are guaranteed to be feasible and stabilizing. The theoretical results are demonstrated through a chemical process example.

2. Preliminaries

2.1. Control problem formulation

We consider nonlinear systems described by the following state-space model

$$\dot{x}(t) = f(x(t), u_1(t), u_2(t), w(t)) \tag{1}$$

where $x(t) \in R^{n_x}$ is the state, $u_1(t) \in R^{n_{u_1}}$ is the set of inputs of controller 1 (which can be thought of as corresponding to an LCS) and $u_2(t) \in R^{n_{u_2}}$ is the set of inputs of controller 2 (which can be thought of as corresponding to an NCS). The inputs u_1 and u_2 are restricted to be in two nonempty convex sets $U_1 \subseteq R^{n_{u_1}}$ and $U_2 \subseteq R^{n_{u_2}}$ containing the origin, respectively. The disturbance $w(t) \in R^{n_w}$ is bounded, i.e., $w(t) \in W$ where $W := \{w \in R^{n_w} \text{ s.t. } |w| \leq \theta, \theta > 0\}$.[1] We assume that f is a locally Lipschitz vector function and $f(0, 0, 0, 0) = 0$. This means that the origin is an equilibrium point for the nominal system.

[1] $|\cdot|$ denotes Euclidean norm of a vector.

Remark 1. In general, distributed control systems are formulated based on the assumption that the controlled systems are decoupled or partially decoupled. However, we consider a fully coupled process model with two sets of possible manipulated inputs; this is a common occurrence in process control as we illustrate in Section 5.

Remark 2. In order to simplify the notation, we consider the case of full state feedback that may be asynchronous and/or delayed. The results can be extended to controllers based on partial state measurements by introducing a state observer; the results can also be extended to the case in which measurements are corrupted by bounded noise by introducing a filter to estimate the state.

2.2. Lyapunov-based controller

We assume that there exists a Lyapunov-based controller $u_1(t) = h(x(t))$ which satisfies the input constraints on u_1 for all x inside a given stability region and renders the origin of the nominal closed-loop system asymptotically stable with $u_2(t) = 0$. The importance and justification of this requirement will be made clear in Sections 3 and 4 below. Using converse Lyapunov theorems (Lin, Sontag, & Wang, 1996; Massera, 1956), this assumption implies that there exist functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ of class $\mathcal{K}$[2] and a continuously differentiable Lyapunov function V for the nominal closed-loop system that satisfy the following inequalities

$$\begin{gathered} \alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \qquad \left|\frac{\partial V(x)}{\partial x}\right| \leq \alpha_4(|x|) \\ \frac{\partial V(x)}{\partial x} f(x, h(x), 0, 0) \leq -\alpha_3(|x|), \quad h(x) \in U_1 \end{gathered} \tag{2}$$

for all $x \in O \subseteq R^{n_x}$ where O is an open neighborhood of the origin. We denote the region $\Omega_\rho \subseteq O$[3] as the stability region of the closed-loop system under the control $u_1 = h(x)$ and $u_2 = 0$.

By continuity and the local Lipschitz property assumed for the vector field $f(x, u_1, u_2, w)$ and the fact that the manipulated inputs u_1 and u_2 are bounded in convex sets, there exists a positive constant M such that

$$|f(x, u_1, u_2, w)| \leq M \tag{3}$$

for all $x \in \Omega_\rho$, $u_1 \in U_1$, $u_2 \in U_2$ and $w \in W$. In addition, by the continuous differentiable property of the Lyapunov function V and the Lipschitz property assumed for the vector field $f(x, u_1, u_2, w)$, there exist positive constants L_x, R_x, R_w such that

$$\left|\frac{\partial V}{\partial x} f(x, u_1, u_2, 0) - \frac{\partial V}{\partial x} f(x', u_1, u_2, 0)\right| \leq L_x |x - x'| \tag{4}$$

and

$$|f(x, u_1, u_2, w) - f(x', u_1, u_2, 0)| \leq R_x |x - x'| + R_w |w| \tag{5}$$

for all $x, x' \in \Omega_\rho$, $u_1 \in U_1$, $u_2 \in U_2$ and $w \in W$.

3. Distributed LMPC with asynchronous measurements

In this section, we design distributed LMPC for systems subject to asynchronous measurements. In Section 4, we will extend the results to systems subject to delayed measurements.

[2] A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

[3] We use Ω_ρ to denote the set $\Omega_\rho := \{x \in R^{n_x} | V(x) \leq \rho\}$.

3.1. Modeling of asynchronous measurements

We assume that the state of the system of Eq. (1), $x(t)$, is available asynchronously at time instants t_k where $\{t_{k\geq 0}\}$ is a random increasing sequence. The distribution of $\{t_{k\geq 0}\}$ characterizes the time needed to obtain a new measurement. In general, if there exists a possibility of arbitrarily large periods of time in which a new measurement is not available, then it is not possible to provide guaranteed stability properties. This is because there exists a non-zero probability that the system may operate in an open-loop for a period of time large enough for the state to leave the stability region. In order to study the stability properties in a deterministic framework, in the present work, we assume that there exists an upper bound T_m on the interval between two successive measurements, i.e., $\max_k\{t_{k+1} - t_k\} \leq T_m$. This assumption is reasonable from a process control perspective.

3.2. Distributed LMPC formulations

In Liu et al. (2009), we introduced a distributed MPC design where both controller 1 and controller 2 were designed via LMPC. Under the assumption of continuous measurements in Liu et al. (2009), it was proved that this control scheme guarantees practical stability of the closed-loop system and has the potential to maintain the closed-loop stability and performance in the face of new or failing controllers/actuators and to reduce computational burden in the evaluation of the optimal manipulated inputs compared with a centralized LMPC controller. However, when asynchronous measurements are present, the results obtained in Liu et al. (2009) no longer hold. In order to simplify (but without loss of generality) the notations and description of the proposed distributed LMPC for system subject to asynchronous measurements (as well as asynchronous and delayed measurements discussed in Section 4), we will adopt the same strategy as used in Liu et al. (2009); that is, to design one LMPC controller for controller 1, and one for controller 2. The LMPC controllers computing the input trajectories of u_1 and u_2 are referred to as LMPC 1 and LMPC 2, respectively. In this section, we extend the results of Liu et al. (2009) to take into account asynchronous measurements explicitly, both in the constraints imposed on the LMPC controllers and in the implementation strategy. A schematic diagram of the considered closed-loop system is shown in Fig. 1.

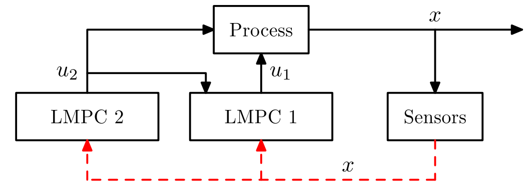

Fig. 1. Distributed LMPC design for systems subject to asynchronous measurements.

In the presence of asynchronous measurements, the controllers need to operate in an open-loop between successive state measurements. We propose taking advantage of the MPC scheme to update the inputs based on a prediction obtained using the model. This is achieved by having the control actuators to store and implement the last computed optimal input trajectories. The proposed implementation strategy is as follows

(1) When a measurement is available at t_k, LMPC 2 computes the optimal input trajectory of u_2.
(2) LMPC 2 sends the entire optimal input trajectory to its actuators and also sends the entire optimal input trajectory to LMPC 1.
(3) Once LMPC 1 receives the entire optimal input trajectory for u_2, it evaluates the optimal input trajectory of u_1.
(4) LMPC 1 sends the entire optimal input trajectory to its actuators.
(5) When a new measurement is received ($k \leftarrow k+1$), go to step 1.

We first design the optimization problem of LMPC 2. This optimization problem depends on the latest state measurement $x(t_k)$, however, LMPC 2 does not have any information about the value that u_1 will take. In order to make a decision, LMPC 2 must assume a trajectory for u_1 along the prediction horizon. To this end, the Lyapunov-based controller $u_1 = h(x)$ is used. The LMPC 2 is based on the following optimization problem

$$\min_{u_{a2}\in S(\Delta)} \int_{t_k}^{t_k+N\Delta} (\tilde{x}(t)^T Q_c \tilde{x}(t) + u_{a1}(t)^T R_{c1} u_{a1}(t) + u_{a2}(t)^T R_{c2} u_{a2}(t))\mathrm{d}t \tag{6a}$$

$$\text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u_{a1}(t), u_{a2}(t), 0), \quad \forall t \in [t_k, t_k + N\Delta) \tag{6b}$$

$$u_{a1}(t) = h(\tilde{x}(t_k + j\Delta)), \quad \forall t \in [t_k + j\Delta, t_k + (j+1)\Delta) \tag{6c}$$

$$u_{a2}(t) \in U_2, \quad \forall t \in [t_k, t_k + N\Delta) \tag{6d}$$

$$\dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_k + j\Delta)), 0, 0), \quad \forall t \in [t_k + j\Delta, t_k + (j+1)\Delta) \tag{6e}$$

$$\tilde{x}(t_k) = \hat{x}(t_k) = x(t_k) \tag{6f}$$

$$V(\tilde{x}(t)) \leq V(\hat{x}(t)), \quad \forall t \in [t_k, t_k + N_R\Delta) \tag{6g}$$

where $S(\Delta)$ is the family of piece-wise constant functions with a sampling time Δ, N is the prediction horizon, Q_c, R_{c1} and R_{c2} are positive definite weight matrices, $\tilde{x}$ is the predicted trajectory of the nominal system with u_2 being the input trajectory computed by the LMPC of Eq. (6) (i.e., LMPC 2) and u_1 being the Lyapunov-based controller $h(x)$ applied in a sample-and-hold fashion with $j = 0, \ldots, N-1$, $\hat{x}$ is the predicted trajectory of the nominal system with u_1 being $h(x)$ applied in a sample-and-hold fashion and $u_2 = 0$, and N_R is the smallest integer that satisfies the inequality $T_m \leq N_R\Delta$. To take full advantage of the nominal model in the computation of the control action, we take $N \geq N_R$. The optimal solution to this optimization problem is denoted by $u_{a2}^*(t|t_k)$ which is defined for $t \in [t_k, t_k + N\Delta)$. Once the optimal input trajectory of u_2 is available, it is sent to LMPC 1 as well as to its corresponding control actuators.

Note that the constraints of Eqs. (6e)–(6f) generate a reference state trajectory (i.e., a reference Lyapunov function trajectory) of the closed-loop system; and the constraint of Eq. (6g) ensures that the predicted decrease of the Lyapunov function from t_k to $t_k+N_R\Delta$, if $u_1 = h(x)$ and $u_2 = u_{a2}^*(t)$ are applied, is at least equal to the one obtained from the constraint of Eq. (6e). By imposing the constraint of Eq. (6g) (as well as the constraint of Eq. (7g)), we can prove that the proposed distributed control system inherits the stability properties of the Lyapunov-based controller $h(x)$ when it is implemented in a sample-and-hold fashion. Note also that we have considered input constraints, see Eq. (6d).

The optimization problem of LMPC 1 depends on $x(t_k)$ and the decision taken by LMPC 2 (i.e., u_{a2}^*). This allows LMPC 1 to compute a u_1 such that the closed-loop performance is optimized, while guaranteeing that the stability properties of the Lyapunov-based controller are preserved. Specifically, LMPC 1 is based on the following optimization problem

$$\min_{u_{a1}\in S(\Delta)} \int_{t_k}^{t_k+N\Delta} (\check{x}(t)^T Q_c \check{x}(t) + u_{a1}(t)^T R_{c1} u_{a1}(t) + u_{a2}(t)^T R_{c2} u_{a2}(t))\mathrm{d}t \tag{7a}$$

$$\text{s.t.} \quad \dot{\check{x}}(t) = f(\check{x}(t), u_{a1}(t), u_{a2}(t), 0), \quad \forall t \in [t_k, t_k + N\Delta) \tag{7b}$$

$$\dot{\tilde{x}}(t) = f(\tilde{x}(t), h(\tilde{x}(t_k + j\Delta)), u_{a2}(t), 0),$$
$$\forall t \in [t_k + j\Delta, t_k + (j+1)\Delta) \quad (7c)$$

$$u_{a2}(t) = u^*_{a2}(t|t_k), \quad \forall t \in [t_k, t_k + N\Delta) \quad (7d)$$

$$u_{a1}(t) \in U_1, \quad \forall t \in [t_k, t_k + N\Delta) \quad (7e)$$

$$\check{x}(t_k) = \tilde{x}(t_k) = x(t_k) \quad (7f)$$

$$V(\check{x}(t)) \le V(\tilde{x}(t)), \quad \forall t \in [t_k, t_k + N_R\Delta) \quad (7g)$$

where $\check{x}$ is the predicted trajectory of the nominal system if $u_2 = u^*_{a2}(t)$ and $u_1 = u_{a1}(t)$ are applied, and $\tilde{x}$ is the predicted trajectory of the nominal system if $u_2 = u^*_{a2}(t)$ and the Lyapunov-based controller $h(x)$ are applied in a sample-and-hold fashion with $j = 0, \ldots, N-1$. The optimal solution to this optimization problem is denoted by $u^*_{a1}(t|t_k)$ which is defined for $t \in [t_k, t_k + N\Delta)$. The constraint of Eq. (7g) guarantees that the predicted decrease of the Lyapunov function from t_k to $t_k + N_R\Delta$, if $u_1 = u^*_{a1}(t)$ and $u_2 = u^*_{a2}(t)$ are applied, is at least equal to the one obtained when $u_1 = h(x)$ and $u_2 = u^*_{a2}(t)$ are applied. Note that the trajectory $\tilde{x}(t)$ predicted by the constraint of Eq. (7c) is the same as the optimal trajectory predicted by LMPC 2 of Eq. (6). This trajectory will be used in the proof of the closed-loop stability properties of the proposed controller. The manipulated inputs of the proposed control scheme of Eqs. (6)–(7) are defined as follows

$$\begin{aligned} u_1(t) &= u^*_{a1}(t|t_k), \quad \forall t \in [t_k, t_{k+1}) \\ u_2(t) &= u^*_{a2}(t|t_k), \quad \forall t \in [t_k, t_{k+1}). \end{aligned} \quad (8)$$

Note that, as explained before, the actuators apply the last evaluated optimal input trajectories between two successive state measurements.

3.3. Stability properties

In this subsection, we prove that the proposed distributed control scheme of Eqs. (6)–(7) inherits the stability properties of the Lyapunov-based controller $h(x)$ implemented in a sample-and-hold fashion. This property is presented in Theorem 1 below. To state this theorem, we need the following propositions.

Proposition 1 (*c.f.* *Muñoz de la Peña and Christofides (2008)*). *Consider the nominal sampled trajectory $\hat{x}$ of the system of Eq. (1) in a closed-loop with the Lyapunov-based controller $h(x)$ applied in a sample-and-hold fashion and $u_2(t) = 0$. Let $\Delta, \epsilon_s > 0$ and $\rho > \rho_s > 0$ satisfy*

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + \alpha_4(\alpha_1^{-1}(\rho))L_x M\Delta \le -\epsilon_s/\Delta. \quad (9)$$

Then, if $\rho_{\min} < \rho$ where

$$\rho_{\min} = \max\{V(\hat{x}(t+\Delta)) : V(\hat{x}(t)) \le \rho_s\} \quad (10)$$

and $\hat{x}(0) \in \Omega_\rho$, the following inequality holds

$$V(\hat{x}(k\Delta)) \le \max\{V(\hat{x}(0)) - k\epsilon_s, \rho_{\min}\}. \quad (11)$$

Proposition 1 ensures that if the nominal system under the control $u_1 = h(x)$ implemented in a sample-and-hold fashion and $u_2 = 0$ starts in Ω_ρ, then it is ultimately bounded in $\Omega_{\rho_{\min}}$. The following proposition provides an upper bound on the deviation of the state trajectory obtained using the nominal model, from the real-state trajectory when the same control actions are applied.

Proposition 2 (*c.f.* *Liu et al. (2008)*). *Consider the systems*

$$\begin{aligned} \dot{x}_a(t) &= f(x_a(t), u_1(t), u_2(t), w(t)) \\ \dot{x}_b(t) &= f(x_b(t), u_1(t), u_2(t), 0) \end{aligned} \quad (12)$$

with initial states $x_a(t_0) = x_b(t_0) \in \Omega_\rho$. There exists a class $\mathcal{K}$ function $f_W(\cdot)$ such that

$$|x_a(t) - x_b(t)| \le f_W(t - t_0), \quad (13)$$

for all $x_a(t), x_b(t) \in \Omega_\rho$ and all $w(t) \in W$ with

$$f_W(\tau) = \frac{R_w\theta}{R_x}(e^{R_x\tau} - 1).$$

Proposition 3 bounds the difference between the magnitudes of the Lyapunov function of two states in Ω_ρ.

Proposition 3 (*c.f.* *Liu et al. (2008)*). *Consider the Lyapunov function $V(\cdot)$ of the system of Eq. (1). There exists a quadratic function $f_V(\cdot)$ such that*

$$V(x) \le V(\hat{x}) + f_V(|x - \hat{x}|) \quad (14)$$

for all $x, \hat{x} \in \Omega_\rho$ with $f_V(s) = \alpha_4(\alpha_1^{-1}(\rho))s + Ms^2$ and $M > 0$.

In Theorem 1 below, we provide sufficient conditions under which the distributed LMPC design of Eqs. (6)–(7) guarantees that the state of the closed-loop system is ultimately bounded in a region that contains the origin.

Theorem 1. *Consider the system of Eq. (1) in a closed-loop with the distributed LMPC of Eqs. (6)–(7) based on a controller $h(x)$ that satisfies the condition of Eq. (2). Let $\Delta, \epsilon_s > 0$, $\rho > \rho_{\min} > 0$, $\rho > \rho_s > 0$ and $N \ge N_R \ge 1$ satisfy the conditions of Eqs. (9) and (10) and the following inequality*

$$-N_R\epsilon_s + f_V(f_W(N_R\Delta)) < 0 \quad (15)$$

with N_R being the smallest integer satisfying $N_R\Delta \ge T_m$. If $x(t_0) \in \Omega_\rho$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_a} \subseteq \Omega_\rho$ where

$$\rho_a = \rho_{\min} + f_V(f_W(N_R\Delta)).$$

Proof. In order to prove that the closed-loop system is ultimately bounded in a region that contains the origin, we prove that $V(x(t_k))$ is a decreasing sequence of values with a lower bound.

Part 1: In this part, we prove that the stability results stated in Theorem 1 hold in the case that $t_{k+1} - t_k = T_m$ for all k and $T_m = N_R\Delta$. This case corresponds to the worst possible situation in the sense that LMPC 1 and LMPC 2 need to operate in an open-loop for the maximum possible amount of time. In order to simplify the notation, we assume that all the signals used in this proof refer to the different optimization variables of the problems solved at time step t_k; that is, $\hat{x}(t_{k+1})$ is obtained from the nominal closed-loop trajectory of system of Eq. (1) under the Lyapunov-based controller $u_1 = h(x)$ implemented in a sample-and-hold fashion and $u_2 = 0$ starting from $x(t_k)$. By Proposition 1 and the fact that $t_{k+1} = t_k + N_R\Delta$, the following inequality can be obtained

$$V(\hat{x}(t_{k+1})) \le \max\{V(\hat{x}(t_k)) - N_R\epsilon_s, \rho_{\min}\}. \quad (16)$$

From the constraints of Eqs. (6g) and (7g) in LMPC 2 and LMPC 1, the following inequality can be written

$$V(\check{x}(t)) \le V(\tilde{x}(t)) \le V(\hat{x}(t)), \quad \forall t \in [t_k, t_k + N_R\Delta). \quad (17)$$

From inequalities of Eqs. (16) and (17) and taking into account that $\hat{x}(t_k) = \tilde{x}(t_k) = \check{x}(t_k) = x(t_k)$, the following inequality is obtained

$$V(\check{x}(t_{k+1})) \le \max\{V(x(t_k)) - N_R\epsilon_s, \rho_{\min}\}. \quad (18)$$

When $x(t) \in \Omega_\rho$ for all times (this point will be proved below), we can apply Proposition 3 to obtain the following inequality

$$V(x(t_{k+1})) \le V(\check{x}(t_{k+1})) + f_V(|\check{x}(t_{k+1}) - x(t_{k+1})|). \quad (19)$$

Applying Proposition 2 we obtain the following upper bound on the deviation of $\check{x}(t)$ from $x(t)$

$$|x(t_{k+1}) - \check{x}(t_{k+1})| \leq f_W(N_R\Delta). \tag{20}$$

From inequalities of Eqs. (19) and (20), the following upper bound on $V(x(t_{k+1}))$ can be written

$$V(x(t_{k+1})) \leq V(\check{x}(t_{k+1})) + f_V(f_W(N_R\Delta)). \tag{21}$$

Using the inequality of Eq. (18), we can re-write the inequality of Eq. (21) as follows

$$V(x(t_{k+1})) \leq \max\{V(x(t_k)) - N_R\epsilon_s, \rho_{\min}\} + f_V(f_W(N_R\Delta)). \tag{22}$$

If the conditions of Eq. (15) is satisfied, from the inequality of Eq. (22), we know that there exists $\epsilon_w > 0$ such that the following inequality holds

$$V(x(t_{k+1})) \leq \max\{V(x(t_k)) - \epsilon_w, \rho_a\} \tag{23}$$

which implies that if $x(t_k) \in \Omega_\rho/\Omega_{\rho_a}$, then $V(x(t_{k+1})) < V(x(t_k))$, and if $x(t_k) \in \Omega_{\rho_a}$, then $V(x(t_{k+1})) \leq \rho_a$.

Because the upper bound on the difference between the Lyapunov function of the actual trajectory x and the nominal trajectory $\check{x}$ is a strictly increasing function of time (see Propositions 2 and 3 for the expressions of f_V and f_W), the inequality of Eq. (23) also implies that

$$V(x(t)) \leq \max\{V(x(t_k)), \rho_a\}, \quad \forall t \in [t_k, t_{k+1}]. \tag{24}$$

Using the inequality of Eq. (24) recursively, it can be proved that if $x(t_0) \in \Omega_\rho$, then the closed-loop trajectories of the system of Eq. (1) under the proposed distributed LMPC design of Eqs. (6)–(7) stay in Ω_ρ for all times (i.e., $x(t) \in \Omega_\rho, \ \forall t$). Moreover, using the inequality of Eq. (23) recursively, it can be proved that if $x(t_0) \in \Omega_\rho$, the closed-loop trajectories of the system of Eq. (1) under the proposed distributed LMPC design of Eqs. (6)–(7) satisfy

$$\limsup_{t\to\infty} V(x(t)) \leq \rho_a.$$

This proves that $x(t) \in \Omega_\rho$ for all times and $x(t)$ is ultimately bounded in Ω_{ρ_a} for the case when $t_{k+1} - t_k = T_m$ for all k and $T_m = N_R\Delta$.

Part 2: In this part, we extend the results proved in Part 1 to the general case, that is, $t_{k+1} - t_k \leq T_m$ for all k and $T_m \leq N_R\Delta$ which implies that $t_{k+1} - t_k \leq N_R\Delta$. Because f_V and f_W are strictly increasing functions of their arguments and f_V is convex, following similar steps as in Part 1, it can be shown that the inequality of Eq. (22) still holds. This proves that the stability results stated in Theorem 1 hold. □

Remark 3. The distributed LMPC design proposed in this subsection can be extended to include multiple LMPC controllers by using a one direction sequential communication strategy (i.e., LMPC k sends information to LMPC $k-1$) and by letting each LMPC send along with its trajectory, all the trajectories received from previous controllers to its successor LMPC (i.e., LMPC k sends both its trajectory and the trajectories received from LMPC $k+1$ to LMPC $k-1$). A similar extension of the distributed LMPC design for systems subject to asynchronous and delayed measurements in Section 4 is also possible.

4. Distributed LMPC with delayed measurements

In this section, we consider distributed LMPC of systems subject to asynchronous and delayed measurements.

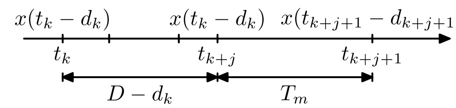

Fig. 2. A possible sequence of delayed measurements.

4.1. Modeling of delayed measurements

We assume that the state of the system of Eq. (1) is received by the controllers at asynchronous time instants t_k where $\{t_{k\geq 0}\}$ is a random increasing sequence of times and that there exists an upper bound T_m on the interval between two successive measurements. We also assume that there are delays in the measurements received by the controllers due to delays in the sampling process and data transmission. In order to model delays in measurements, another auxiliary variable d_k is introduced to indicate the delay corresponding to the measurement received at time t_k, that is, at time t_k, the measurement $x(t_k - d_k)$ is received. In general, if the sequence $\{d_{k\geq 0}\}$ is modeled using a random process, there exists the possibility of arbitrarily large delays. In this case, it is improper to use all the delayed measurements to estimate the current state and decide the control inputs, because when the delays are too large, they may introduce enough errors to destroy the stability of the closed-loop system. In order to study the stability properties in a deterministic framework, we assume that the delays associated with the measurements are smaller than an upper bound D, which is, in general, relevant to measurement sensors and data transmission networks.

Note that because the delays are time-varying, it is possible that at a time instant t_k, the controllers may receive a measurement $x(t_k - d_k)$ which does not provide new information (i.e., $t_k - d_k < t_{k-1} - d_{k-1}$); that is, the controller has already received a measurement of the state after time $t_k - d_k$. We assume that each measurement is time-labeled and hence the controllers are able to discard a newly received measurement if $t_k - d_k < t_{k-1} - d_{k-1}$. Fig. 2 shows part of a possible sequence of $\{t_{k\geq 0}\}$. At time t_k, the state measurement $x(t_k - d_k)$ is received. There exists a possibility that between t_k and t_{k+j}, with $t_{k+j} - t_k = D - d_k$ and j being an unknown integer, all the measurements received do not provide new information. Note that any measurements received after t_{k+j} provide new information because the maximum delay is D and the latest received measurement was $x(t_k - d_k)$. The maximum possible time interval between t_{k+j} and t_{k+j+1} is T_m. Therefore, the maximum amount of time the system might operate in open-loop following t_k is $D + T_m - d_k$. This upper bound will be used in the formulation of distributed LMPC design for systems subject to delayed measurements below.

4.2. Distributed LMPC formulations

As in the previous section, we propose taking advantage of the system model both to estimate the current system state from a delayed measurement and to control the system in open-loop when new information is not available. To this end, when a delayed measurement is received the controllers use the system model and the manipulated inputs that have been applied to the system to get an estimate of the current state and then a standard MPC optimization problem is solved in order to decide the optimal future input trajectory that will be applied until new measurements are received. However, in the distributed schemes previously proposed (see Fig. 1), LMPC 2 does not know the input trajectory which has been implemented by LMPC 1 because there is only one-directional communication from LMPC 2 to LMPC 1. In order to get a good estimate of the current state from a delayed measurement, the distributed LMPC structure of Eqs. (6)–(7) needs to be modified to have bi-directional communication

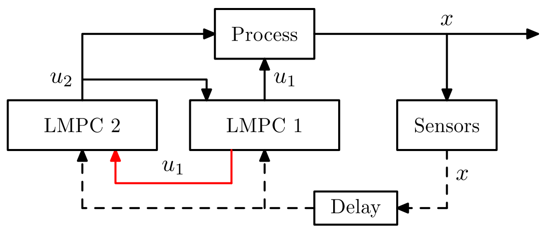

Fig. 3. Distributed LMPC design for systems subject to delayed measurements.

so that LMPC 1 can send its optimal input trajectory to LMPC 2. A schematic of the distributed LMPC scheme for systems subject to asynchronous and delayed measurements considered in this section is shown in Fig. 3. When at t_k, a delayed measurement $x(t_k - d_k)$ is received, the information sent from LMPC 1 to LMPC 2 allows LMPC 2 to estimate the current state by using the system model of Eq. (1) and the input trajectories $u_1(t)$ (which has received from LMPC 1) and $u_2(t)$ (which LMPC 2 has stored in memory) applied in $t \in [t_k - d_k, t_k)$. The proposed implementation strategy in the presence of delayed measurements is as follows

(1) When a measurement $x(t_k - d_k)$ is available at t_k, LMPC 2 checks whether the measurement provides new information. If $t_k - d_k > \max_{l<k} t_l - d_l$, go to step 2. If the measurement does not contain new information and is discarded, go to step 6.
(2) LMPC 2 estimates the current state of the system $\tilde{x}(t_k)$ and computes the optimal input trajectory of u_2 based on $\tilde{x}(t_k)$.
(3) LMPC 2 sends the entire optimal input trajectory to its actuators and also sends $\tilde{x}(t_k)$ and the entire optimal input trajectory to LMPC 1.
(4) Once LMPC 1 receives $\tilde{x}(t_k)$ and the entire optimal input trajectory for u_2, it evaluates the optimal input trajectory of u_1 based on $\tilde{x}(t_k)$.
(5) LMPC 1 sends the entire optimal input trajectory to its actuators and LMPC 2.
(6) When a new measurement is received ($k \leftarrow k+1$), go to step 1.

The proposed LMPC 2 for systems subject to delayed measurements is based on the following optimization problem

$$\min_{u_{d2} \in S(\Delta)} \int_{t_k}^{t_k+N\Delta} (\tilde{x}(t)^T Q_c \tilde{x}(t) + u_{d1}(t)^T R_{c1} u_{d1}(t)$$
$$+ u_{d2}(t)^T R_{c2} u_{d2}(t))\mathrm{d}t \tag{25a}$$
$$\text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u_{d1}^*(t), u_{d2}^*(t), 0),$$
$$\forall t \in [t_k - d_k, t_k) \tag{25b}$$
$$\dot{\tilde{x}}(t) = f(\tilde{x}(t), u_{d1}(t), u_{d2}(t), 0),$$
$$\forall t \in [t_k, t_k + N\Delta) \tag{25c}$$
$$u_{d1}(t) = h(\tilde{x}(t_k + j\Delta)),$$
$$\forall t \in [t_k + j\Delta, t_k + (j+1)\Delta) \tag{25d}$$
$$u_{d2}(t) \in U_2, \forall t \in [t_k, t_k + N\Delta) \tag{25e}$$
$$\tilde{x}(t_k - d_k) = x(t_k - d_k) \tag{25f}$$
$$\dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_k + j\Delta)), 0, 0),$$
$$\forall t \in [t_k + j\Delta, t_k + (j+1)\Delta) \tag{25g}$$
$$\hat{x}(t_k) = \tilde{x}(t_k) \tag{25h}$$
$$V(\tilde{x}(t)) \le V(\hat{x}(t)), \quad \forall t \in [t_k, t_k + N_{Dk}\Delta) \tag{25i}$$

where $j = 0, \ldots, N-1$, and N_{Dk} is the minimum integer satisfying $N_{Dk}\Delta \ge T_m + D - d_k$ and $u_{d1}^*(t), u_{d2}^*(t)$ are the latest input trajectories sent by the controllers to the actuators. The optimal solution to this optimization problem is denoted by $u_{d2}^*(t|t_k)$ which is defined for $t \in [t_k, t_k + N\Delta)$. Once this optimal input trajectory of u_2 is available, it is sent to the control actuators controlled by LMPC 2 and to LMPC 1 together with the estimate of the current state $\tilde{x}(t_k)$.

There are two types of calculations in the optimization problem of Eq. (25). The first type of calculation is to estimate the current state $\tilde{x}(t_k)$ based on the delayed measurement $x(t_k - d_k)$ and input values have applied to the system from $t_k - d_k$ to t_k (constraints of Eqs. (25b) and (25f)). The second type of calculation is to evaluate the optimal input trajectory of u_2 based on $\tilde{x}(t_k)$ while satisfying the input constraint of Eq. (25e) and the constraint of Eq. (25i). The constraint of Eq. (25i) is required to ensure the practical closed-loop stability. Note that the length of the constraint N_{Dk} depends on the current delay d_k, so it may have different values at different time instants and has to be updated before solving the optimization problem of Eq. (25).

The proposed LMPC 1 for systems subject to delayed measurements depends on $\tilde{x}(t_k)$ and $u_{d2}^*(t|t_k)$. Specifically, it is based on the following optimization problem

$$\min_{u_{d1} \in S(\Delta)} \int_{t_k}^{t_k+N\Delta} (\check{x}(t)^T Q_c \check{x}(t) + u_{d1}(t)^T R_{c1} u_{d1}(t)$$
$$+ u_{d2}(t)^T R_{c2} u_{d2}(\tau))\mathrm{d}t \tag{26a}$$
$$\text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), h(\tilde{x}(t_k + j\Delta)), u_{d2}(t), 0),$$
$$\forall t \in [t_k + j\Delta, t_k + (j+1)\Delta) \tag{26b}$$
$$\dot{\check{x}}(t) = f(\check{x}(t), u_{d1}(t), u_{d2}(t), 0),$$
$$\forall t \in [t_k, t_k + N\Delta) \tag{26c}$$
$$u_{d2}(t) = u_{d2}^*(t|t_k), \quad \forall t \in [t_k, t_k + N\Delta) \tag{26d}$$
$$u_{d1}(t) \in U_1, \quad \forall t \in [t_k, t_k + N\Delta) \tag{26e}$$
$$\check{x}(t_k) = \tilde{x}(t_k) \tag{26f}$$
$$V(\check{x}(t)) \le V(\tilde{x}(t)), \quad \forall t \in [t_k, t_k + N_{Dk}\Delta). \tag{26g}$$

The optimal solution to this optimization problem is denoted by $u_{d2}^*(t|t_k)$ which is defined for $t \in [t_k, t_k + N\Delta)$ and it is send to the control actuators controlled by LMPC 1 and LMPC 2. Note that LMPC 1 gets $\tilde{x}(t_k)$ from LMPC 2, so it does not need to estimate the current state and only needs to evaluate the optimal input trajectory of u_1 based on $\tilde{x}(t_k)$ while satisfying the input constraint of Eq. (26e) and the constraint of Eq. (26g). The constraint of Eq. (26g) is required to ensure closed-loop practical stability. The manipulated inputs of the distributed control scheme of Eqs. (25)–(26) for systems subject to asynchronous and delayed measurements are defined as follows

$$\begin{aligned} u_1(t) &= u_{d1}^*(t|t_k), \quad \forall t \in [t_k, t_{k+i}) \\ u_2(t) &= u_{d2}^*(t|t_k), \quad \forall t \in [t_k, t_{k+i}) \end{aligned} \tag{27}$$

for all t_k such that $t_k - d_k > \max_{l<k} t_l - d_l$ and for a given t_k, the variable i denotes the smaller integer that satisfies $t_{k+i} - d_{k+i} > t_k - d_k$.

4.3. Stability properties

In this subsection, we present the stability property of the proposed distributed control scheme of Eqs. (25)–(26). This property is presented in Theorem 2 below.

Theorem 2. *Consider the system of Eq. (1) in a closed-loop with the distributed LMPC design of Eqs. (25)–(26) based on a controller $h(x)$ that satisfies the condition of Eq. (2). Let $\Delta, \epsilon_s > 0$, $\rho > \rho_{\min} > 0$, $\rho > \rho_s > 0$, $N \ge 1$ and $D \ge 0$ satisfy the conditions of Eqs. (9) and (10) and the following inequality*

$$-N_R\epsilon_s + f_V(f_W(N_D\Delta)) + f_V(f_W(D)) < 0 \tag{28}$$

with N_D being the smallest integer satisfying $N_D\Delta \ge T_m + D$, and N_R being the smallest integer satisfying $N_R\Delta \ge T_m$. If $N \ge N_D$,

$x(t_0) \in \Omega_\rho$ and $d_0 = 0$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_d} \subseteq \Omega_\rho$ where

$$\rho_d = \rho_{\min} + f_V(f_W(N_D\Delta)) + f_V(f_W(D)).$$

Proof. We assume that at t_k, a delayed measurement containing new information $x(t_k - d_k)$ is received, and that the next measurement with new state information is not received until t_{k+i}. This implies that $t_{k+i} - d_{k+i} > t_k - d_k$ and that the distributed LMPC problem of Eqs. (25)–(26) is solved at t_k and the optimal input trajectories $u^*_{d1}(t|t_k)$ and $u^*_{d2}(t|t_k)$ are applied from t_k to t_{k+i} (see constraint of Eqs. (25b) and (27)). We follow a similar approach as the one used in the proof of Theorem 1; that is, to prove that $V(x(t_k))$ is a decreasing sequence of values with a lower bound.

Part 1: In this part we prove that the stability results stated in Theorem 2 hold for $t_{k+i} - t_k = N_{Dk}\Delta$ and all $d_k \leq D$. By Proposition 1, the following inequality can be obtained (see constraint of Eq. (25g))

$$V(\hat{x}(t_{k+i})) \leq \max\{V(\hat{x}(t_k)) - N_{Dk}\epsilon_s, \rho_{\min}\}. \tag{29}$$

From the constraints of Eqs. (25i) and (26g) in LMPC 2 of Eq. (25) and LMPC 1 of Eq. (26), the following inequality can be written

$$V(\check{x}(t)) \leq V(\tilde{x}(t)) \leq V(\hat{x}(t)), \quad \forall t \in [t_k, t_k + N_{Dk}\Delta). \tag{30}$$

From the inequalities of Eqs. (29), (30) and taking into account that $\hat{x}(t_k) = \check{x}(t_k) = \tilde{x}(t_k)$, the following inequality is obtained

$$V(\check{x}(t_{k+i})) \leq \max\{V(\tilde{x}(t_k)) - N_{Dk}\epsilon_s, \rho_{\min}\}. \tag{31}$$

When $x(t) \in \Omega_\rho$ for all times (this point will be proved below), we can apply Proposition 3 to obtain the following inequalities

$$\begin{aligned} &V(\tilde{x}(t_k)) \leq V(x(t_k)) + f_V(|x(t_k) - \tilde{x}(t_k)|) \\ &V(x(t_{k+i})) \leq V(\check{x}(t_{k+i})) + f_V(|x(t_{k+i}) - \check{x}(t_{k+i})|). \end{aligned} \tag{32}$$

Applying Proposition 2 we obtain the following bounds on the deviation of $\tilde{x}(t)$ and $\check{x}(t)$ from $x(t)$

$$\begin{aligned} &|x(t_k) - \tilde{x}(t_k)| \leq f_W(d_k) \\ &|x(t_{k+i}) - \check{x}(t_{k+i})| \leq f_W(N_D\Delta). \end{aligned} \tag{33}$$

Note that Proposition 2 can be applied because (25b), (26c), (26d), (26f) and the implementation procedure guarantee that $\tilde{x}(t_k)$ and $\check{x}(t_{k+i})$ have been estimated using the same inputs applied to the system. We also have taken into account that $N_D\Delta \geq N_{Dk} + d_k$ for all d_k. Using inequalities of Eqs. (31)–(33), the following upper bound on $V(x(t_{k+i}))$ is obtained

$$\begin{aligned} V(x(t_{k+i})) \leq\ &\max\{V(x(t_k)) - N_{Dk}\epsilon_s, \rho_{\min}\} \\ &+ f_V(f_W(N_D\Delta)) + f_V(f_W(d_k)). \end{aligned} \tag{34}$$

In order to prove that the Lyapunov function is decreasing between two consecutive new measurements, the following inequality must hold

$$N_{Dk}\epsilon_s > f_V(f_W(N_D\Delta)) + f_V(f_W(d_k)) \tag{35}$$

for all possible $0 \leq d_k \leq D$. Taking into account that f_W and f_V are strictly increasing functions of their arguments, that N_{Dk} is a decreasing function of the delay d_k and that if $d_k = D$ then $N_{D_k} = N_R$, if condition of Eq. (28) is satisfied, condition of Eq. (35) holds for all possible d_k and there exists $\epsilon_w > 0$ such that the following inequality holds

$$V(x(t_{k+i})) \leq \max\{V(x(t_k)) - \epsilon_w, \rho_d\} \tag{36}$$

which implies that if $x(t_k) \in \Omega_\rho/\Omega_{\rho_d}$, then $V(x(t_{k+i})) < V(x(t_k))$, and if $x(t_k) \in \Omega_{\rho_d}$, then $V(x(t_{k+i})) \leq \rho_d$.

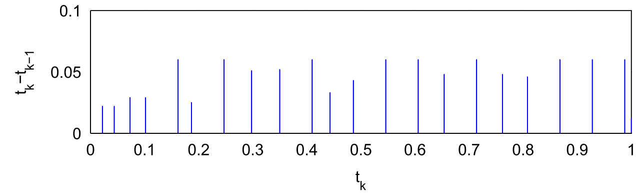

Fig. 4. Asynchronous measurement sampling times $\{t_{k\geq 0}\}$ with $T_m = 3\Delta$: the x-axis indicates $\{t_{k\geq 0}\}$ and the y-axis indicates the size of the interval between t_k and t_{k-1}.

Because the upper bound on the difference between the Lyapunov function of the actual trajectory x and the nominal trajectory $\check{x}$ is a strictly increasing function of time, the inequality of Eq. (36) also implies that

$$V(x(t)) \leq \max\{V(x(t_k)), \rho_d\}, \quad \forall t \in [t_k, t_{k+i}]. \tag{37}$$

Using the inequality of Eq. (37) recursively, it can be proved that if $x(t_0) \in \Omega_\rho$, then the closed-loop trajectories of system of Eq. (1) under the proposed distributed LMPC design of Eqs. (25)–(26) stay in Ω_ρ for all times (i.e., $x(t) \in \Omega_\rho, \ \forall t$). Moreover, using the inequality of Eq. (36) recursively, it can be proved that if $x(t_0) \in \Omega_\rho$, the closed-loop trajectories of system of Eq. (1) under the proposed distributed LMPC design of Eqs. (25)–(26) satisfy

$$\limsup_{t\to\infty} V(x(t)) \leq \rho_d.$$

This proves that $x(t) \in \Omega_\rho$ for all times and $x(t)$ is ultimately bounded in Ω_{ρ_d} for the case when $t_{k+i} - t_k = N_{Dk}\Delta$.

Part 2: In this part, we extend the results proved in Part 1 to the general case, that is, $t_{k+i} - t_k \leq N_{Dk}\Delta$. Taking into account that f_V and f_W are strictly increasing functions of their arguments and following similar steps in Part 1, it is easy to prove that the inequality of Eq. (35) holds for all possible $d_k \leq D$ and $t_{k+i} - t_k \leq N_{Dk}\Delta$. Using this inequality and following the same line of argument as in the previous part, the stability results stated in Theorem 2 can be proved. □

Remark 4. The sufficient conditions presented in Theorem 2 state that in order to guarantee practical stability, $V(x(t_k))$ must be a decreasing sequence of values with a lower bound for the worst possible case from a feedback control point of view; that is, the measurements are received every T_m (the maximum time between successive measurements) with a delay equal to the maximum delay D.

Remark 5. Although the proofs of Theorems 1 and 2 provided are constructive, the constants obtained are conservative. This is the case with most of the results presented in the literature, see for example Neŝić, Teel, and Kokotovic (1999) for further discussion on this issue. In practice, the maximum time that the system can operate in an open-loop is better estimated through closed-loop simulations. The various inequalities proved in Theorems 1 and 2 are more useful as guidelines on the interaction between the various parameters that define the system and the controllers and may be used as guidelines to design the controllers.

Remark 6. In this work, state constraints have not been considered but the proposed distributed LMPC approaches can be extended to handle such constraints by restricting the closed-loop stability region further; please see Mhaskar, El-Farra, and Christofides (2006) for more results on this issue.

Remark 7. In this work, we do not explicitly consider delays introduced in the system by the communication network or by the time needed to solve each of the LMPC optimization problems. Such delays are usually small compared to the measurement delays and can be modeled as part of an overall measurement delay.

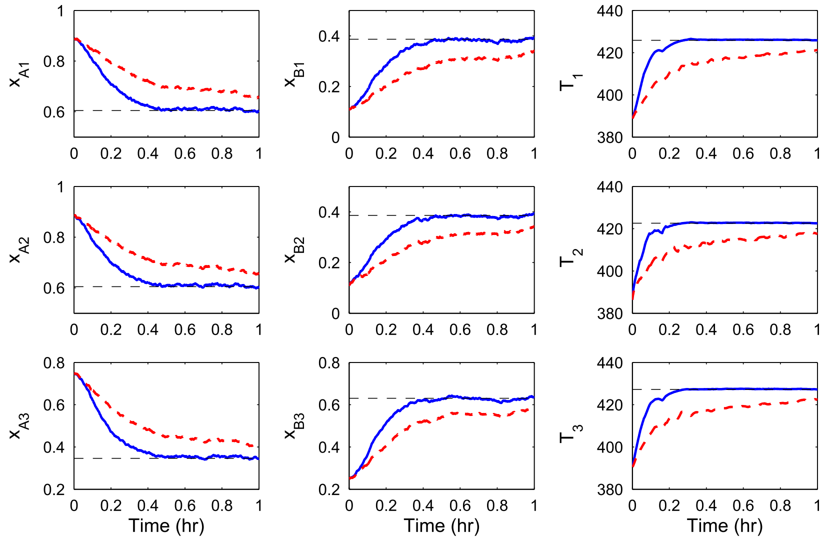

Fig. 5. State trajectories under the distributed LMPC design of Eqs. (6)–(7) (solid lines) and the distributed LMPC design proposed in Liu et al. (2009) (dashed lines) in the presence of asynchronous measurements.

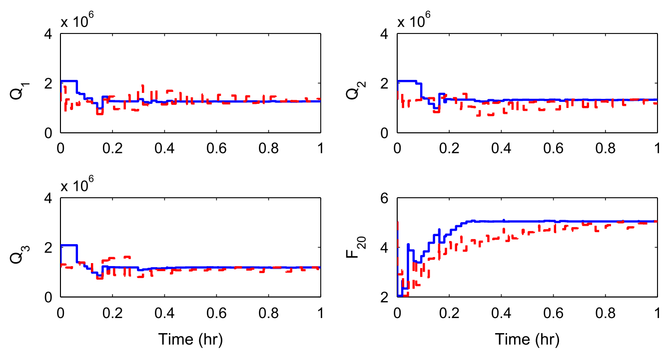

Fig. 6. Input trajectories under the distributed LMPC design of Eqs. (6)–(7) (solid lines) and the distributed LMPC design proposed in Liu et al. (2009) (dashed lines) in the presence of asynchronous measurements.

5. Application to a chemical process

5.1. Process and control problem description

The process considered in this example is a three vessel, reactor-separator process consisting of two continuously stirred tank reactors and a flash tank separator. The description and modeling of the process can be found in Liu et al. (2009). The process was numerically simulated using a standard Euler integration method, and bounded process noise was added to all the simulations in this work to simulate disturbances/model uncertainty.

Each of the vessels in the process has an external heat input. The manipulated inputs to the system are the heat inputs, Q_1, Q_2 and Q_3, and the feed stream flow rate to vessel 2, F_{20}. For each set of steady-state inputs Q_{1s}, Q_{2s}, Q_{3s} and F_{20s} corresponding to a different operating condition, the process has one steady-state x_s. The control objective is to steer the process from the initial state $x_0^T = [0.89\ \ 0.11\ \ 388.7\ \ 0.11\ \ 386.3\ \ 0.75\ \ 0.25\ \ 390.6]$ to $x_s^T = [0.61\ 0.39\ 425.9\ 0.61\ 0.39\ 422.6\ 0.35\ 0.63\ 427.3]$ which is the steady state corresponding to the operating condition: $Q_{1s} = 12.6 \times 10^5$ KJ/h, $Q_{3s} = 11.88 \times 10^5$ KJ/h, $Q_{2s} = 13.32 \times 10^5$ KJ/h and $F_{20s} = 5.04\ \mathrm{m}^3/\mathrm{h}$.

The process belongs to the following class of nonlinear systems: $\dot{x}(t) = f(x(t)) + g_1(x(t))u_1(t) + g_2(x(t))u_2(t) + w(t)$ where $x^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9] = [x_{A1} - x_{A1s}\ x_{B1} - x_{B1s}\ T_1 - T_{1s}\ x_{A2} - x_{A2s}\ x_{B2} - x_{B2s}\ T_2 - T_{2s}\ x_{A3} - x_{A3s}\ x_{B3} - x_{B3s}\ T_3 - T_{3s}]$ is the state, $u_1^T = [u_{11}\ u_{12}\ u_{13}] = [Q_1 - Q_{1s}\ Q_2 - Q_{2s}\ Q_3 - Q_{3s}]$ and $u_2 = F_{20} - F_{20s}$ are the manipulated inputs which are subject to the constraints $|u_{1i}| \le 10^6$ KJ/h ($i = 1, 2, 3$) and $|u_2| \le 3\ \mathrm{m}^3/\mathrm{h}$, and w is a bounded noise.

We use the same design of $h(x)$ as in Liu et al. (2009), and we consider a Lyapunov function $V(x) = x^T P x$ with $P = diag(5.2 \times 10^{12}[4\ 4\ 10^{-4}\ 4\ 4\ 10^{-4}\ 4\ 4\ 10^{-4}])$.[4] The values of the weights in P have been chosen in a way such that the Lyapunov-based controller $h(x)$ satisfies the input constraints, stabilizes the closed-loop system asymptotically with continuous state feedback and provides good closed-loop performance.

5.2. Asynchronous measurements without delay

For this set of simulations, it is assumed that the state measurement of the process are available asynchronously at time instants $\{t_{k\ge0}\}$ with an upper bound $T_m = 3\Delta$ on the maximum interval between two successive asynchronous state measurements, where Δ is the controller and sensor sampling time and is chosen to be $\Delta = 0.02$ h $= 1.2$ min. Based on the Lyapunov-based controller $h(x)$, we design LMPC 1 and LMPC 2. The prediction horizons of both LMPC 1 and LMPC 2 are chosen to be $N = 6$ and N_R is chosen to be 3 so that $N_R\Delta \ge T_m$. The weight matrices for the LMPC designs are chosen in a way that the distributed LMPC design proposed in Liu et al. (2009) and the design of Eqs. (6)–(7) can both stabilize the closed-loop system with continuous state measurements. Specifically, the weight matrices are chosen as follows: $Q_c = diag(10^3[2\ 2\ 0.0025\ 2\ 2\ 0.0025\ 2\ 2\ 0.0025])$ and $R_{c1} = diag([5 \times 10^{-12}\ 5 \times 10^{-12}\ 5 \times 10^{-12}])$ and $R_{c2} = 100$.

To model the time sequence $\{t_{k\ge0}\}$, we use an upper bounded random Poisson process. The Poisson process is defined by the number of events per unit time W. The interval between two successive concentration sampling times (events of the Poisson process) is given by $\Delta_a = \min\{-\ln\chi/W, T_m\}$, where χ is a random variable with a uniform probability distribution between 0 and 1. This generation ensures that $\max_k\{t_{k+1} - t_k\} \le T_m$. In this example, W is chosen to be $W = 20$. The generated time sequence $\{t_{k\ge0}\}$ for a simulation length of 1.0 h is shown in Fig. 4 and the average time interval between two successive time instants is 0.046 h.

[4] $diag(v)$ denotes a matrix with its diagonal elements being the elements of vector v and all the other elements being zeros.

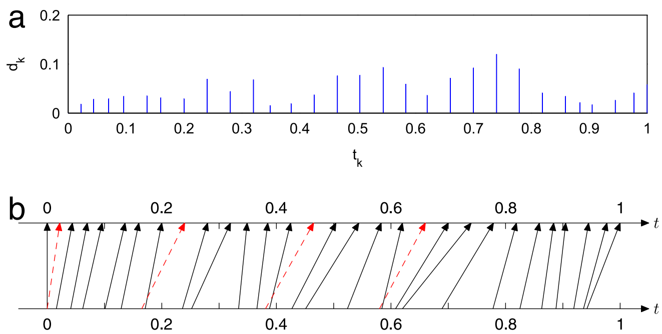

Fig. 7. Asynchronous time sequence $\{t_{k\geq 0}\}$ and corresponding delay sequence $\{d_{k\geq 0}\}$ with $T_m = 0.04$ h and $D = 0.12$ h: (a) the x-axis indicates $\{t_{k\geq 0}\}$ and the y-axis indicates the size of d_k; (b) the upper axis indicates $\{t_{k\geq 0}\}$, the lower axis indicates $t_k - d_k$, each arrow points from $t_k - d_k$ to corresponding t_k and the dashed arrows indicate the measurements which do not contain new information.

In this set of simulations, when the system operates in an open-loop, all the control designs to be tested use their last evaluated optimal input trajectories. The state and input trajectories of the system in closed-loop under the distributed LMPC design of Eqs. (6)–(7) and the one in Liu et al. (2009) are shown in Figs. 5 and 6. In Fig. 5, it can be seen that the distributed LMPC design of Eqs. (6)–(7) provides a better performance and is able to stabilize the process at the desired steady state in about 0.5 h; the design proposed in Liu et al. (2009) fails to drive the state of the process to the desired steady state because it does not account for the use of asynchronous measurements.

5.3. Asynchronous measurements subject to delays

In this subsection, we compare the performance of the distributed LMPC design of Eqs. (25)–(26) with that of the design of Eqs. (6)–(7) in the case where the delayed state measurements of the process are available asynchronously at time instants $\{t_{k\geq 0}\}$. The same sampling time Δ and weight matrices Q_c, R_c and R_{c2} used in Section 5.2 are used. The prediction horizons of both LMPC 1 and LMPC 2 are chosen to be $N = 8$ in this set of simulations so that the horizon covers the maximum possible open-loop operation interval. Note that the same estimated current state is used to evaluate both of the controllers. The same Poisson process is used to generate $\{t_{k\geq 0}\}$ with $W = 30$ and $T_m = 0.04$ h and another random

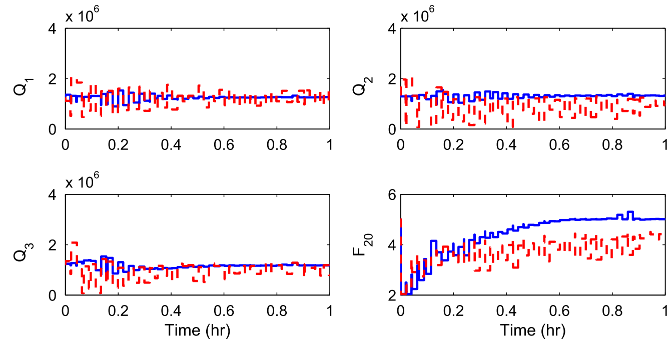

Fig. 9. Input trajectories under the distributed LMPC design of Eqs. (25)–(26) (solid lines) and the distributed LMPC design of Eqs. (6)–(7) (dashed lines) in the presence of asynchronous and delayed measurements.

process is used to generate the associated delay sequence $\{d_{k\geq 0}\}$ with $D = 0.12$ h. Fig. 7 shows the time instants when new state measurements are received, the associated delay sizes and the instants when the received measurements do not contain new information (which are discarded). The average time interval between two successive sampling times is 0.035 h and the average time delay is 0.057 h.

The state and input trajectories of the system in closed-loop under the proposed distributed LMPC design of Eqs. (25)–(26) and the distributed LMPC design of Eqs. (6)–(7) are shown in Figs. 8 and 9. In Fig. 8, we see that the proposed design of Eqs. (25)–(26) is able to stabilize the process at the desired steady state in about 0.6 h, but the control design of Eqs. (6)–(7) which does not account for measurement delays fails to drive the state to the desired steady state.

Remark 8. We have also done simulations to evaluate the computational time of the LMPCs. The simulations have been carried out using *Matlab* in a *Pentium* 3.20 GHz. The optimization problems have been solved using the built-in nonlinear programming function *fmincom* of *Matlab*. For 50 evaluations, the mean time to solve LMPC 2 of Eq. (6) and LMPC 1 of Eq. (7) are 5.52 s and 2.90 s, respectively, with the prediction horizon $N = 6$; the mean time to solve LMPC 2 of Eq. (25) and LMPC 1 of Eq. (26) are 13.95 s and 6.83 s, respectively, with the prediction horizon $N = 8$. These computational times can be reduced significantly by using a compiled nonlinear programming solver implemented in C or other programming languages.

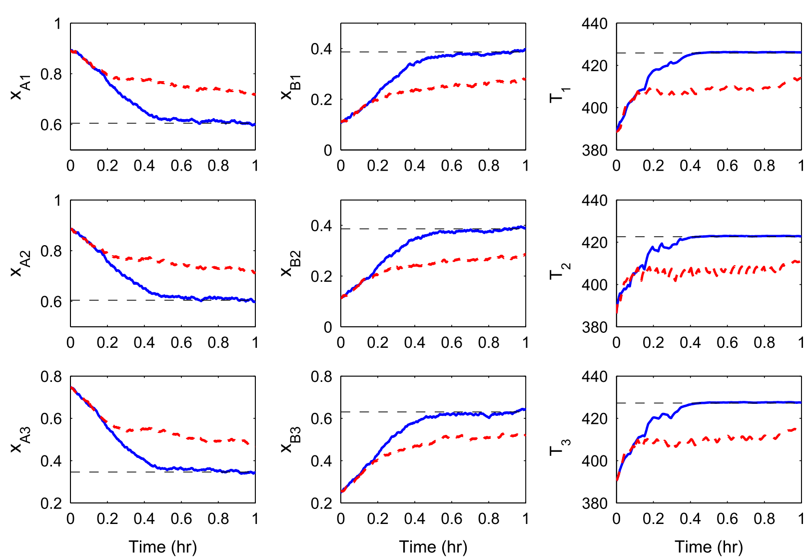

Fig. 8. State trajectories under the distributed LMPC design of Eqs. (25)–(26) (solid lines) and the distributed LMPC design of Eqs. (6)–(7) (dashed lines) in the presence of asynchronous and delayed measurements.

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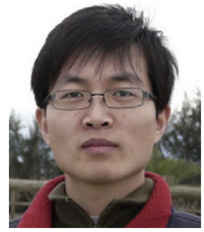

Jinfeng Liu was born in Wuhan, China, in 1982. He received the B.S. and M.S. degrees in Control Science and Engineering in 2003 and 2006, respectively, from Zhejiang University. He is currently a Ph.D. candidate in Chemical Engineering at the University of California, Los Angeles. His research interests include model predictive control, fault detection and isolation, and fault-tolerant control of nonlinear systems.

David Muñoz de la Peña was born in Badajoz, Spain, in 1978. He received the master degree in Telecommunication Engineering in 2001 and the Ph.D. in Control Engineering in 2005 from the University of Seville, Spain. In 2006–2007, he held a postdoctoral position at the Chemical and Biomolecular Engineering Department at the University of California, Los Angeles. Since 2007 he has been with the Escuela Superior de Ingenieros of the University of Seville, where he is currently an Assistant Professor. His main research interests are model predictive control, nonlinear systems and optimization.

Panagiotis D. Christofides was born in Athens, Greece, in 1970. He received the Diploma in Chemical Engineering degree in 1992, from the University of Patras, Greece, the M.S. degrees in Electrical Engineering and Mathematics in 1995 and 1996, respectively, and the Ph.D. degree in Chemical Engineering in 1996, all from the University of Minnesota. Since July 1996 he has been with the University of California, Los Angeles, where he is currently a Professor in the Department of Chemical and Biomolecular Engineering and the Department of Electrical Engineering. A description of his research interests, list of distinctions, and a list of his publications can be found at http://www.chemeng.ucla.edu/pchristo/index.html.