

# Energy-efficiency-oriented Gradient-based Economic Predictive Control of Multiple-Chiller Cooling Systems<sup>\*</sup>

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## Abstract:

The growing use of air conditioning systems has become one of the main drivers of energy consumption in buildings. Many efforts are being made to develop new designs and control strategies to improve energy efficiency and minimise electricity consumption. In this work, a model for a case study of multiple-chiller-based cooling system is presented, based on surrogate models derived from information provided by manufacturers, and the study of the economic performance index. Then, an economic predictive control strategy will aim to operate the system optimizing the efficiency of the plant. Instead of the classical two-layer economic predictive control structure, where the reference to be tracked by the controller is given by a real-time optimizer, here we consider a single-layer control strategy where the gradients with respect to the manipulated inputs of the economic performance index are included in the cost function of the model predictive controller. The resulting optimization problem to be solved on line is a QP, which considerably eases the optimization problem, while also avoiding discrepancies between layers that could lead to loss of feasibility.

Keywords: model predictive control, multiple-chiller system, economic control, HVAC

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## 1. INTRODUCTION

Recent studies of energy efficiency in developed regions claim that up to a 40% of the total energy is consumed in buildings. Pérez-Lombard et al. (2008) finds that heating, ventilation and air conditioning (HVAC) systems consume approximately 50% of this energy. Thus, it is not surprising the efforts made by researchers during last years to improve the performance of this kind of systems.

In this work the main focus of attention will be multiple-chiller-based cooling systems where the coordinated operation and sequencing of the involved chiller units are optimized. In Torzhkov et al. (2010) an approach for optimizing the sequencing of multiples chillers is presented taking future predicted values as inputs to determine the optimal choice of various set-points. In Hure et al. (2019) the minimization of the system performance index in terms electrical energy prices is performed by employing a successive linear programming algorithm and the gradient algorithm for finding the initial feasible point. In Deng et al. (2014) the MPC problem that finds the chiller system optimal scheduling is presented as a large-scale mixed-integer nonlinear programming problem for whom an heuristic algorithm to obtain sub-optimal solutions is proposed.

The objective of this work involves the design of an economic model predictive control strategy that optimizes the operation of the plant while meeting power demands. A common approach to this problem is the design of a two-layer control strategy based on a real-time optimizer (RTO) that finds an economically optimal operating point that is provided as set point to a model predictive control that drives the system towards this point (Rawlings et al., 2018). This approach might lead to possible loss of feasibility if the setpoints provided by the RTO are unreachable or if the optimal operation point changes driven by variations in the unitary costs or in the air conditioning demand. Besides, this requires the solution of two optimization problems, which makes its implementation in embedded systems difficult. In this work, we propose a gradient-based economic MPC aimed to the optimal operation of the HVAC system in presence of possible variations of the demand or parameters of the economic performance index. Furthermore, this controller only requires the solution of a quadratic programming problem provided the gradient of the economic cost function, which facilitates its implementation.

## 2. MULTIPLE CHILLER COOLING SYSTEM

A water cooling system featuring a constant flow topology and two chillers is studied. A general scheme of a double-chiller cooling system is shown in Fig. 2.

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Taking into account that the inner control loops of the chillers provide an over-damped behaviour of the output water temperature  $T_{wo}$  of the closed-loop system, we propose a Hammerstein model to describe the dynamics of the chiller. This is composed by the static nonlinearity described by (3) in series with a first-order system. In this case we have considered a time constant of 2 minutes so that the steady state is reached at around 10 minutes.

The electric consumption  $C_E$  has also been identified using manufacturer information. This model has the form

$$C_E = C_{EN} \cdot f_2(T_{aa}, T_{wo}) \cdot f_3(PLR). \quad (6)$$

This expression depends on the chiller nominal consumption  $C_{EN}$ , associated to the machine dimensioning, and two modifying factors described by

$$f_2(T_{aa}, T_{wo}) = p_0 + p_1 T_{wo} + p_2 T_{wo}^2 + p_3 T_{aa} + p_4 T_{aa}^2 + p_5 T_{aa} T_{wo}, \quad (7)$$

$$f_3(PLR) = k_0 + k_1 PLR + k_2 PLR^2 + k_3 PLR^3. \quad (8)$$

Again, coefficients  $p_i$  and  $k_i$  have been obtained by system identification using manufacturers test training data. Figure 3 shows the electricity consumption  $C_E$  as a function of  $PLR$  and  $T_{wo}$ . Ambient air temperature  $T_{aa}$  has been kept constant at 35 °C in the plot. However, the model exhibits that the higher  $T_{aa}$ , the larger  $C_E$  is, in a similar way than  $T_{wo}$ .

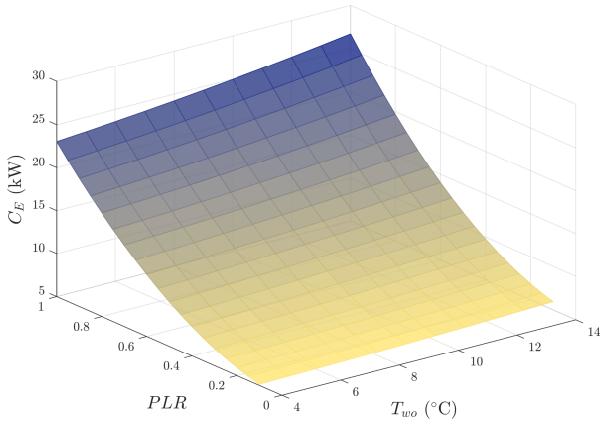


Fig. 3. Electric consumption of the chiller.

Once the system behavior has been described by the above-detailed expressions, the energy efficiency ratio  $EER$  is used as standard measure of the system performance (Leff and Teeters, 1978). This coefficient allows finding operating points where the machine efficiency is maximized. This  $EER$  can be defined as the effective cooling power divided by the machine electricity consumption

$$EER = \frac{PLR \cdot CAP}{C_E}. \quad (9)$$

Figure 4 shows the  $EER$  coefficient as a function of  $PLR$  and  $T_{wo}$ .

### 3.2 Installation Model

The total cooling power demand of the building  $Q_B$  can be calculated as a function of the input and output flow temperatures of the distribution network

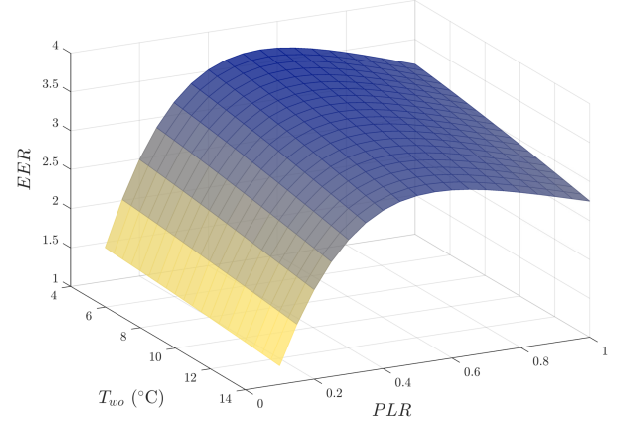


Fig. 4. Energy efficiency ratio.

$$Q_B = \sum_{i=1}^7 Q_i = c_p \cdot q \cdot (T_{wi} - T_{wo}). \quad (10)$$

The cooling power given to each room  $Q_i$  depends on the positions of the valves  $v_i$  and the model of the fan coils. First, the model of the fan coil has been obtained using the same method of curve adjusting from manufacturer data, following the function

$$Q_i = f_4(T_{wo}, v_i \cdot \frac{q}{7}), \quad (11)$$

where  $T_{wo}$  is the water temperature at the input of the fan coil assuming negligible heat losses, and  $v_i \cdot q/7$  is the flow going through the fan coil. The function  $f_4$  is

$$f_4(T, q) = d_{00} + d_{10}T + d_{01}q + d_{20}T^2 + d_{11}Tq + d_{02}q^2 + d_{30}T^3 + d_{21}T^2q + d_{12}Tq^2 + d_{03}q^3. \quad (12)$$

The output water of each room fan coil is finally mixed into the total flow  $q_i$ , and its temperature  $T_{wi}$  can be calculated as

$$T_{wi} = \frac{1}{7} \sum_{i=1}^7 (v_i T_{fco,i} + (1 - v_i) T_{wo}), \quad (13)$$

where  $T_{fco,i}$  is the water temperature at the output of each fan coil, which is given by the expression

$$T_{fco,i} = T_{wo} + \frac{Q_i}{c_p v_i q/7}. \quad (14)$$

### 3.3 Building Model

Modeling the thermodynamics of a building is a very complex problem with many approaches. There are many simulation software packages with a large number of predefined libraries like EnergyPlus or TRNSYS. In this work a resistance-capacitance modeling library for MATLAB introduced by Sturzenegger et al. (2014) has been the chosen option. This toolbox for building modeling was specially developed for its use in model predictive control applications. A total of seven rooms have been included in the model. The temperature of each room  $T_{r,i}$  depends on a wide variety of factors considered in the toolbox, and each of the rooms has its own unique configuration, so that no two of them are alike. The control of this

temperatures is done by the heat flow  $Q_i$  that goes into the fan coil. The building model is also subject to the ambient air temperature  $T_{aa}$ , which acts as a disturbance to the system.

#### 4. OPTIMAL OPERATION STRATEGY

Figure 4 shows that the  $PLR$  that achieves the maximum  $EER$  for any  $T_{wo}$  is between 0.4 and 0.6. In this range, the cooling power to electric consumption ratio of the chiller is at its highest. However, the  $PLR$  also determines how much cooling power is delivered to the rooms, which makes it dependant on the cooling power demand of the building. In addition, Fig. 3 shows that electric consumption always increases when  $PRL$  does. All of this implies that to minimize power consumption,  $PRL$  has to be as low as possible while satisfying the cooling power demand. However, this problem has a new caveat when using multiple chiller units. Since the objective of this work is to find a controller that operates the system in the most efficient possible way, this energy problem has to be defined first so that it can be optimized.

The optimal value of the partial load ratios  $PLR$  of each of the chillers and the positions  $v_i$  of each of the valves can be calculated by solving the following optimization problem

$$\mathcal{P}(Q_i) = \min_{PLR_i, v_i} C_E^*(T_{aa}, v_i, PLR_1, PLR_2) \quad (15a)$$

$$\text{s.t.} \quad (4), (5), (10), (11), (13), (14) \quad (15b)$$

where  $C_E^* = C_{E,1} + C_{E,2}$  is the sum of the electric consumption of two chiller units with the same model (6) for both.

Figure 5 shows the total cost of chiller operation according to the power demand for both cases when a single chiller unit and two chiller units are working. The electric consumption for the case of one chiller saturates near 70 kW, which corresponds to the maximum capacity point for a single chiller where  $PLR_1 = 1$ . With 2 chillers, demands higher than 100 kW can be satisfied. It can be seen that there is an intersection between the cost of the two cases. This point around 42 kW is  $Q_B^*$ . Below this demand, it is more efficient to work with one chiller instead of two. On the other hand, for demands higher than  $Q_B^*$ , it is more efficient to work with two chillers, even before the point when the single working chiller unit saturates. For

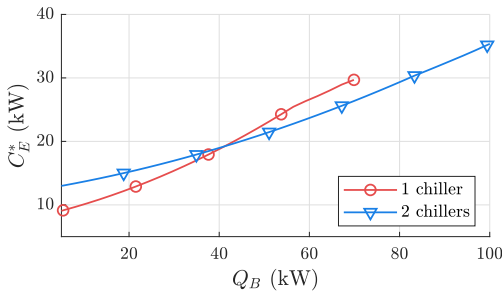


Fig. 5. Optimal energy consumption.

example, if  $Q_B < Q_B^*$ , then the operating point of chillers module is  $\{PLR_1, PLR_2\} = \{PLR_{op}, 0\}$ . Otherwise, if  $Q_B > Q_B^*$ , this operating point is  $\{PLR_1, PLR_2\} = \{PLR_{op}, PLR_{op}\}$ . With  $0 < PLR_{op} < 1$ .

Now, we are ready to present the function that will serve as economic performance index to be optimized. This cost function ponders the economic cost in terms of energy consumption and a comfort term measured as the difference between the temperature of each room  $T$  and its corresponding reference signal  $T_{ref}$

$$f_{eco}(PLR_i, v_i, \alpha, T_{ref}, T_{aa}) = C_E^*(T_{aa}, v_i, PLR_1, PLR_2) + \alpha \|T_r - T_{ref}\|^2, \quad (16)$$

where  $\alpha$  is the weighting factor that indicates how much importance we give to the comfort term.

Using this performance index, the problem of designing a real-time optimizer (RTO) that finds the optimal control action to be applied to the system can be formulated as

$$u_{ref} = \arg \min_{PLR_i, v_i} f_{eco} \quad (17a)$$

$$\text{s.t.} \quad (4), (5), (10), (11), (13), (14) \quad (17b)$$

Notice that this is a parametric optimization problem that depends on ambient air temperature  $T_{aa}$ , the reference of the temperature setpoint  $T_{ref}$ , and the weighting factor  $\alpha$  of the comfort term. Since temperature of the ambient air can vary and the reference of temperature or the comfort weight can be freely chosen by the users, the optimal operation point may be varying.

#### 5. ECONOMIC CONTROL STRATEGY

A classical approach to solve the economic control problem is the use of two-layer control structure. In the first layer the optimal input signal reference is calculated by a real time optimizer (RTO) where the economic cost function is minimised according to some economic criteria. In the second layer an advance control strategy like model predictive control (MPC) is adopted to regulate the plant to the optimal operation point provided by the RTO. Although this control structure has widely been used, discrepancies between the RTO and the MPC models or changes in the cost function because of variations of the reference signal or the economic strategy could lead to the loss of feasibility, or to a set point that is not reachable. Moreover, notice that two different optimization problems must be solved at each sampling period which may be computationally demanding.

To overcome these problems, this work proposes the design of a single-layer control strategy where the functionality of the RTO is integrated into the MPC layer by adding the gradients of the economic performance index with respect to the manipulated inputs to the MPC cost function as explained in Alamo et al. (2014). Doing this, feasibility is guaranteed despite changes in the economic cost function. In addition, a single quadratic optimization problem has to be solved at each sampling period. This problem can now be solved using quadratic programming methods, making it easier to implement this control strategy using embedded systems like conventional industrial PLCs (Krupa et al. (2018)).

The model of the plant to be controlled used by the MPC controller is a discrete-time linearization of the nonlinear model explained in Section 3. In the studied system, the manipulated inputs are the partial load ratio of each chiller unit  $PLR_j$  and the position of each valve  $v_i$ , and the measured outputs are the temperatures of each room  $T_{r,i}$

as well as the output flow temperature of the chiller units module  $T_{w,o}$ . The state of the system has been obtained by identification methods, so it does not match with any physical variable. To consider the effects of disturbances, in this case the external ambient air temperature  $T_{aa}$ , which is not measured, and following the strategy proposed in Pannocchia and Rawlings (2003), the state of the system has been augmented so that the new state is given by

$$\mathbf{x} = [x \ d]^\top. \quad (18)$$

where  $x \in \mathbb{R}^{nx}$  is the state of the system and  $d \in \mathbb{R}^{ny}$  are the disturbances on the outputs. The model of the system can now be represented as

$$\mathbf{x}(k+1) = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \mathbf{x} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \quad (19a)$$

$$y(k) = [C \ I] \mathbf{x}. \quad (19b)$$

where  $\mathbf{x} \in \mathbb{R}^{nx+nd}$  is the new state of the system,  $u \in \mathbb{R}^{nu}$  the manipulated inputs, and  $y \in \mathbb{R}^{ny}$  the measured outputs.

At each sampling period the augmented state is estimated. For that purpose a linear observer with gain  $K$  has been designed by solving a LQR problem. The model of the observer is given by

$$\hat{\mathbf{x}}(k+1) = \left( \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} - L[C \ I] \right) \hat{\mathbf{x}}(k) + \left( \begin{bmatrix} B \\ 0 \end{bmatrix} L \right) \mathbf{u}, \quad (20)$$

where  $\hat{\mathbf{x}} = [\hat{x} \ \hat{d}]^\top$  and  $\mathbf{u} = [u \ y]^\top$ . The state observer determines the most probable current state of the system  $\hat{\mathbf{x}}$  from the applied inputs and the measured outputs.

The optimization problem formulation is based on Limon and Alamo (2013),

$$\begin{aligned} \min_{x, u, x_{ss}, u_{ss}} \quad & \sum_{k=0}^{N-1} \left( \|x(k) - x_{ss}\|_Q^2 + \|u(k) - u_{ss}\|_R^2 \right) \\ & + \|x(N) - x_{ss}\|_P^2 + \|y_{ss} - y_{ref} + \hat{d}\|_T^2 \\ & + \nabla_u f_{eco}(u(-1)) (u_{ss} - u_{ref}) \end{aligned} \quad (21a)$$

$$\text{s.t.} \quad x(0) = \hat{x} \quad (21b)$$

$$x(k+1) = Ax(k) + Bu(k), \quad (21c)$$

$$y(k) = Cx(k), \quad k = 0 \dots N-1 \quad (21d)$$

$$x_{ss} = Ax_{ss} + Bu_{ss} \quad (21e)$$

$$y_{ss} = Cx_{ss} \quad (21f)$$

$$y(k), y_{ss} \in \mathcal{Y}, \quad u(k), u_{ss} \in \mathcal{U} \quad (21g)$$

where  $x_{ss} \in \mathbb{R}^{nx}$ ,  $u_{ss} \in \mathbb{R}^{nu}$ , and  $y_{ss} \in \mathbb{R}^{ny}$  are the stationary values of the state, inputs and outputs of the system respectively,  $y_{ref} \in \mathbb{R}^{ny}$  is the output reference, and  $N$  is the prediction horizon of the finite optimization problem. The cost function is composed of the regulation terms weighted by matrices  $Q \in \mathbb{R}^{nx \times nx}$  and  $R \in \mathbb{R}^{nu \times nu}$ , the terminal cost penalty weighted by  $P \in \mathbb{R}^{nx \times nx}$  that ensures stability, the offset cost weighted by  $T \in \mathbb{R}^{ny \times ny}$  that makes the controller track the output reference, and the gradients  $\nabla_u f_{eco}(u(k))$  of the economic performance index  $f_{eco}(u(k))$ , whose term depends on  $u_{ref}$ , which is given by the cooling demand of the building  $Q_B$  with respect to the intersection point  $Q_B^x$  in Fig. 5. See that at each sampling time  $Q_B$  can be estimated thanks to the fact that the temperatures  $T_{wo}$  and  $T_{wi}$  are measured and the flow  $q$  is known. The components in  $u_{ref}$  give a starting point for the gradient term to find the optimal solution.

The optimization problem (21) was solved by using Yalmip optimization library (Löfberg, 2004) in MATLAB with a *quadprog* method. Please feel free to contact the two first authors in case any value is required to prove simulation results.

## 6. SIMULATION RESULTS

We ran closed-loop simulations on the nonlinear system presented in Section 2 using the MPC formulation described in Section 3. The case example that will be presented aims to simulate the performance of the controller on the 7-room building during a summer day. The system will be subject to a varying ambient air temperature, and the cooling demand will be increased during normal working hours by forcing smaller room temperature references.

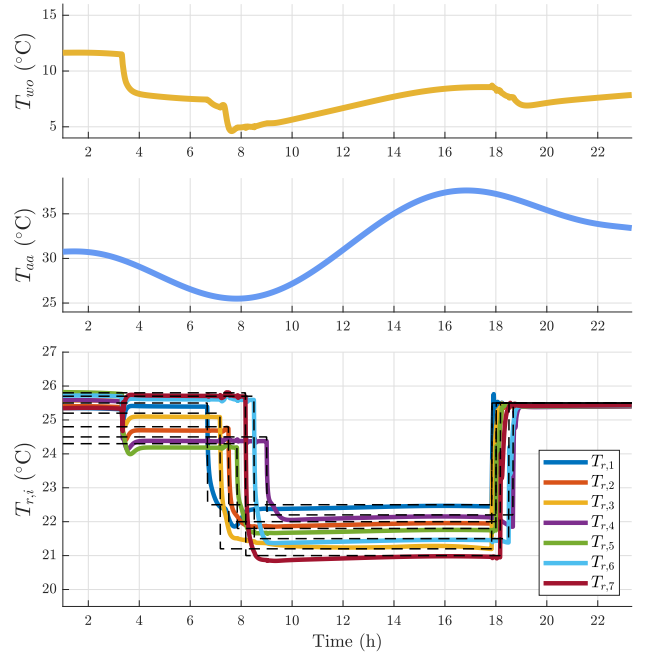


Fig. 6. Temperatures in the system. (Top) Output water temperature. (Middle) Ambient air temperature. (Bottom) Room temperatures.

Figure 6 shows the relevant temperatures of the system. The controller is turned on at  $t = 3.3$  hours. We observe a relatively stable output water temperature  $T_{wo}$  around 8 °C, with small down peaks that happen at big demand changes. This temperature should be sufficiently low so that the fan coils can extract the required heat from the rooms, but it is soft-constrained at 5 °C to prevent freezing effects. The room temperatures are plotted together with the set-points. In the time between 6 and 10 hours, the system undergoes a peak in cooling demand where all room temperature references are decreased several degrees. The transient response time varies for each room since they are different. Nonetheless, tracking is satisfied for all of them. Around 18 hours, the set-points change to reduce the cooling demand, simulating a wind-down of the system.

Figure 7 shows the control inputs. Top and middle graphics show the  $PLR_j$  of the two chiller units, together with the solution that would be calculated in the RTO (17). It can be seen that this gradient-based MPC drives the



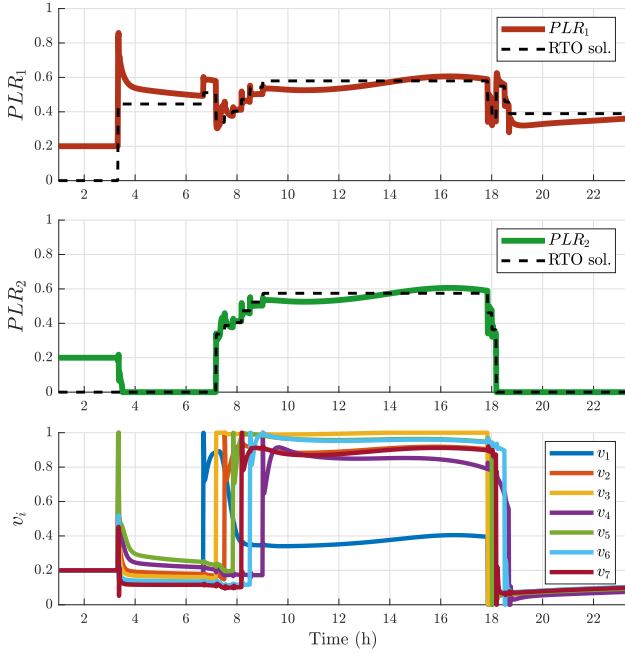


Fig. 7. Control inputs of the system. (Top) Partial load ratio of first chiller. The dashed line is the solution of the RTO (17). (Middle) Partial load ratio of second chiller. (Bottom) Aperture of room valves.

system towards the solution of the RTO. There is some offset due to the disturbances ( $T_{aa}$ ) and the uncertainty of the prediction model, but the controller still tries to converge to the reference. The controller is turned on at  $t = 3.3$  hours. At that moment, one chiller is turned down, leaving the task of generating the cooling power to the other machine. After the 6-hour mark, rooms start to set lower temperature references one by one. The first chiller steps up to satisfy this demand. However, when the second room lowers the set-point, the demand overcomes the threshold  $Q_B^x$  that indicates that two chillers running are now more cost-efficient than one. Therefore, the first chiller turns down and the second chiller turns on and goes to the same  $PLR$  point. It can be seen during this time that signals  $PRL_j$  have a soft wave form due to the disturbance  $T_{aa}$ . Both chillers work in parallel until 18 hours when the demand is still high, and valves are also open wider during this time. Then, the system winds down, and the second chiller turns down again to letting the other chiller satisfy the small demand.

## 7. CONCLUSIONS

An economic control strategy for a multiple-chiller-based cooling system was presented. First, an economic model of the system is developed making use of technical information provided by manufactures. The main contribution of this paper is the implementation of a gradient-based economic predictive control, where the gradient of the economic performance index is added to the cost function of the MPC. This not only makes the system converge to the economically optimal operation point, but it also simplifies the control strategy into a single optimization problem, avoiding discrepancies between layers that could lead to loss of feasibility.

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