

# Oracle-Based Economic Predictive Control

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**Abstract**—This paper deals with an economic predictive controller for the optimal operation of a plant under the assumption that the only available information of the system is the economic cost function to be minimized. In order to predict the evolution of the economic cost for a given input trajectory, an oracle with a NARX structure is proposed. Sufficient conditions to ensure the existence of such oracle are given, and based on this oracle, a novel predictive controller is proposed. The proposed controller is only based on input-output data and no further information about the inner dynamics of the plant is necessary. Under certain assumptions, it is proven that the proposed oracle-based economic predictive controller provides the same solution of a standard economic MPC based on the model plant (in the ideal case of accurate estimation), inheriting the properties of this class of controllers. The proposed oracle-based economic predictive controller is applied to a quadruple-tank process example.

## I. INTRODUCTION

Often, control systems have to simultaneously consider both performance and safety requirements. This objective has been typically addressed by means of a hierarchical structure where a real time optimization layer calculates the equilibrium point that minimizes the operation cost, while a control layer regulates the system to this equilibrium point. Recently, in the model predictive control (MPC) framework, this hierarchical control structure has been united in a single layer, aimed to minimize the operation cost during the transient, instead of a tracking cost, often designed to provide robustness and stability properties. This is the so-called economic MPC, whose properties have been studied in several works [1], [2]. The main difference between economic MPC and regulation MPC is that the former relaxes the architecture of the optimization problem, being able to minimize an economic cost function which is not necessarily positive definite.

Economic predictive controllers are based on the availability of a model of the plant in order to predict the evolution of the states of the system, and based on this, the expected economic cost to be minimized is calculated. However, there may exist situations in which no measurements of the inner variables of the plant are available, for example to maintain privacy of operation. Consider for instance a data center in which the operation cost accounts for the cost of the electric consumption of the refrigeration system and the consumption of the servers. In order to design a controller to optimize the operation cost, sharing inner information of the state of the servers could be limited due to security reasons,

while sharing only the operation cost may not jeopardize the security of the system.

In this paper we study the case in which the only available measurement from the plant is the value of the economic performance index, and the model of the plant is unknown. Based on the historical data of the inputs and the signal of the economic cost, we derive a data-based predictive controller that ensures asymptotic stability of the plant and its economically optimal operation.

The prediction of the behaviour of the plant is done by means of an oracle that forecasts the economic performance of the plant from a historical data set of the pairs inputs-economic costs, using a nonlinear autoregressive exogenous model (NARX) structure. Based on this oracle, an optimization control problem is proposed in such a way that the resulting controller inherits the properties of the ideal economic MPC based on the process model.

The proposed controller has been applied in simulation to an economic minimization problem of a quadruple-tank process, using Gaussian processes to obtain the oracle.

The rest of the paper is structured as follows: section II presents the problem formulation and the standard economic MPC. Section III states the conditions under which it is possible to define an oracle to predict the future evolution of the economic cost. Section IV describes the proposed oracle-based economic predictive controller applied and section V presents the case study.

## Notation

Given two column vectors  $v$  and  $w$ ,  $(v, w)$  stands for  $[v^T, w^T]^T$ . The set  $\mathbb{I}_a^b$  stands for the set of integers from  $a$  to  $b$ . A function  $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  is a  $\mathcal{K}$ -function if it is strictly increasing and  $\alpha(0) = 0$ . Besides, if a  $\mathcal{K}$ -function is such that  $\lim_{s \rightarrow \infty} \alpha(s) = \infty$  then it is called a  $\mathcal{K}_\infty$ -function.

## II. PROBLEM FORMULATION

In this paper we consider that the system to be controlled is a sampled continuous time system described by an unknown discrete time model

$$x(k+1) = f(x(k), u(k)), \quad (1)$$

where  $x(k) \in \mathbb{R}^n$  is the state of the plant and  $u(k) \in \mathbb{R}^m$  is the control input. It is assumed that the inputs are subject to (hard) constraints  $u(k) \in \mathcal{U}$ , where  $\mathcal{U} \subset \mathbb{R}^m$  is a compact set.

The objective of the control system to be designed is to guarantee that the closed-loop system is stable while a certain economic cost function is minimized during the transient. This cost function is said to be economic because

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it measures the performance of the evolution of the system according to a generic function that does not necessarily penalize only the tracking error w.r.t. a given target. The economic cost function to be considered in this paper is given by an appropriate economic stage cost function of the form  $L(x, u)$ , where no assumption is made on its sign.

*Remark 1:* There may exist a collection of variables of the system  $y_c(k) = h(x(k), u(k)) \in \mathbb{R}^p$  which are subject to (soft) constraints  $y_c(k) \in \mathcal{Y}_c$ , being  $\mathcal{Y}_c \subset \mathbb{R}^p$  a closed set. To cope with this case, the stage cost function  $L(x, u)$  can be used to take into account the constraints in the states or outputs by adding a term to  $L$  that behaves as a barrier function, penalizing the violations of the constraints. This term can be thought of as a term that measures the economic cost of not fulfilling the constraints.

*Assumption 1:* The model function  $f(x, u)$  and the economic cost function  $L(x, u)$  are continuous.

According to the given economic cost function, the optimal equilibrium point in which the plant can be operated is obtained from the solution of the following optimization problem

$$(x_s, u_s) = \arg \min_{x_s, u_s \in \mathcal{U}} L(x, u) \quad (2a)$$

$$\text{s.t.} \quad x_s = f(x_s, u_s). \quad (2b)$$

The economically optimal operation of a system is a very complex problem that has been thoroughly studied recently. See for instance the excellent survey paper [1] and the references in there. For the asymptotic stabilization of economically optimal controllers, the dissipativity property plays an important role. This has also been related to the turnpike property previously used in optimal control [3]. In this work, this condition is stated in the following assumption.

*Assumption 2:* The system  $f$  is strictly dissipative with respect to the supply rate  $s(x, u) = L(x, u) - L(x_s, u_s)$ , i.e. there exists a storage function  $\lambda : \mathbb{R}^n \rightarrow \mathbb{R}$  and a  $\mathcal{K}_\infty$  function  $\rho$  such that

$$\begin{aligned} \lambda(f(x, u, d)) - \lambda(x) \leq \\ -\rho(\|x - x_s\|) + L(x, u) - L(x_s, u_s). \end{aligned} \quad (3)$$

It is also assumed that the storage function is bounded below for any admissible trajectory of the system.

Using the results in [1], an asymptotically stabilizing economic MPC can be designed for this system derived from the optimization problem  $P_N(x(k))$  given by:

$$\min_{\hat{\mathbf{u}}} \sum_{j=0}^{N-1} L(\hat{x}(j|k), \hat{u}(j)) \quad (4a)$$

$$\text{s.t.} \quad \hat{x}(0|k) = x(k) \quad (4b)$$

$$\hat{x}(j+1|k) = f(\hat{x}(j|k), \hat{u}(j)), j \in \mathbb{I}_0^N \quad (4c)$$

$$\hat{u}(i) \in U \quad (4d)$$

$$\hat{x}(N|k) = x_s. \quad (4e)$$

*Assumption 3:* It is assumed that the optimal solution of the problem  $P_N(x(k))$  is unique.

The control law  $u(k) = \kappa_{\text{eco}}(x(k))$  is derived from the solution of (4) applied in a receding horizon manner, i.e.  $u(k) = u^*(0; x(k))$ .

This control law requires prior knowledge of the model of the plant,  $f$ , and the measurement of the state at each sampling time ( $x(k)$ ). However, there exists scenarios where obtaining this information may not be available. For instance, there may exist systems for which an accurate model to be used in the predictions is not available, or the available model is too complex to be used in a predictive controller. Instead, historical input-output data from the plant may be the only available information that can be used for the derivation of the model. In addition, the state may not be accessible, so the only signal that can be used for feedback is the measured economic cost.

The main objective of this work is to design a controller that stabilizes the plant and minimizes its economic performance under the assumption that the model of the plant is not known and that the only output signal of the plant that is measured is the value of the economic cost at each sampling time

$$y(k) = L(x(k), u(k)) \in \mathbb{R}. \quad (5)$$

Using a data base of past inputs and economic cost trajectories, a function used to predict the evolution of the output will be obtained. This function is called an oracle, since it forecasts the economic performance of the plant. Once that this oracle is obtained, a suitable predictive controller will be designed.

It will be demonstrated that under some standard conditions, it is possible to solve the problem inheriting the closed-loop properties of the economic MPC based on the model of the system.

### III. THE ORACLE

In this section the oracle, that is, the procedure to calculate the predictions, is presented, and the conditions on the system that allow its determination are given. The oracle to be used in this work has the form of a nonlinear auto-regressive model with exogenous signals (NARX), which has been extensively used in nonlinear systems identification [4].

The structure of the oracle is given by the following nonlinear differential equation

$$\hat{y}(k) = \mathcal{O}(z(k), u(k)), \quad (6)$$

where  $\hat{y}(k)$  is the estimated output at sampling time  $k$  and  $z(k)$  is the state vector, given by the following collection of past inputs and outputs

$$\begin{aligned} z(k) &= (y(k-1), \dots, y(k-n_a), \\ &\quad u(k-1), \dots, u(k-n_b)). \end{aligned} \quad (7)$$

Hence, the dimension of the state vector  $z$  is  $n_z = n_a + m \cdot n_b$ , and the oracle is a function  $\mathcal{O} : \mathbb{R}^{n_z} \times \mathbb{R}^m \rightarrow \mathbb{R}$ , since the output is a real number.

Notice that the output considered at time instant  $k$ ,  $y(k)$ , is the value of the economic cost function at that time instant, which in general depends on the value of the input at the

same time instant  $u(k)$ , leading to an inner feed-forward structure. This implies that the state vector  $z(k)$  can only depend on the sequence of past outputs up to  $k - 1$ , i.e.  $y(k - 1), \dots, y(k - n_a)$ . Then, the state feedback controller to be designed with the form  $u(k) = \kappa_{\text{MPC}}(z(k))$  is such that the current control action  $u(k)$  depends on the information of the plant available up to  $k - 1$ , given the set-up defined by (6) and (7).

The conditions under which a system can be described as a NARX have been widely studied during the last 30 years. One of the first results on this topic was given by Sontag [5], relating the existence of this model to the observability property of the system. Later, Chen and Billings [6] proved that local NARX models can be obtained if the system is locally observable. A comprehensive study of this problem, for local and global estimators, was presented by Levin and Narendra [7].

In [7, Theorem 3] it was proven that if the linearised model at the equilibrium point  $(x_s, u_s)$  is observable, then the dynamics of the system can be locally described by a NARX model. When a global NARX model is required, the strong observability property must be ensured, from the linearised system, for instance. However, in [7, Theorem 6] the authors proved that only generic observability is necessary, which is a property that almost every system enjoys in practice. From this result, the conditions required to ensure the existence of the oracle can be derived:

*Theorem 1:* Consider that Assumption 1 holds and consider also that the economic cost function is such that for all  $\tilde{x}$  where the gradient  $\nabla_x L(\tilde{x}, u) = 0$ , the Hessian matrix  $\nabla_{xx} L(\tilde{x}, u)$  is non singular. Then an oracle (6) can be determined as valid for almost every input sequence. Besides, the oracle function is continuous and the horizons can be taken as  $n_a = n_b = 2n$ , where  $n$  is the minimum number of the states of the system.

Once that the existence of the oracle has been demonstrated, the procedure to derive the oracle can be obtained using the estimation and learning theory methods. There exists a number of methods capable of approximating the real function (the so-called ground truth function) from possibly noisy sampled data, such as support vector machines, neural networks or direct weight optimization [8]. More recently, other methods such as Gaussian processes [9] or Lipschitz interpolation [10], [11] have gained a lot of attention thanks to their capability to provide estimations of the prediction error.

The model order  $n$  may not be known a priori<sup>1</sup>. Depending on the estimation method chosen, the model function will have certain structure. The parameters of this estimator, including the memory horizons  $n_a$  and  $n_b$ , are calculated from a data base of historical inputs and outputs, namely the *training data set*. In addition, a different collection of data points is used for validation of the proposed estimator. This cross-validation methodology allows one to derive the best

<sup>1</sup>Later, in Theorem 2, it will be proven that no assumption on  $n$  is required.

structure of the estimator, as well as the best horizons  $n_a$  and  $n_b$ , from the real data.

#### IV. ORACLE-BASED ECONOMIC PREDICTIVE CONTROL

In this section, the proposed oracle-based predictive controller is presented and its stability and optimality properties are studied under the assumption of perfect estimation.

Assuming that an oracle is available, the system can be posed as a state-space prediction model as follows

$$\hat{z}(j+1|k) = \hat{F}(\hat{z}(j|k), \hat{u}(j)) \quad (8)$$

$$\hat{y}(j|k) = \mathcal{O}(\hat{z}(j|k), \hat{u}(j)), \quad (9)$$

where the predicted state  $\hat{z}(j|k) \in \mathbb{R}^{n_z}$  is given by

$$\hat{z}(j|k) = (\hat{y}(j-1|k), \dots, \hat{y}(k), \dots, y(k+j-n_a), \hat{u}(j-1), \dots, \hat{u}(0), \dots, u(j-n_b))$$

for  $j \geq 1$ . It includes past measurements  $y$  and  $u$  if  $n_a \geq j$  or  $n_b \geq j$  respectively, and only estimated values  $\hat{y}$  or  $\hat{u}$  otherwise.

Thus, the prediction model is

$$\begin{aligned} \hat{F}(\hat{z}(j|k), \hat{u}(j)) &= (\mathcal{O}(\hat{z}(j|k), \hat{u}(j)), \\ &\hat{y}(j-1|k), \dots, \hat{y}(1|k), \\ &y(k), \dots, y(k+j-n_a+1), \\ &\hat{u}(j), \dots, \hat{u}(j-n_b+1)). \end{aligned}$$

From the current value of the regressors  $z(k)$ , the control law is derived from the solution of the following optimization problem  $P_{\mathcal{O}}(z(k))$

$$\min_{\hat{\mathbf{u}}} V_N = \sum_{j=0}^{N-1} \hat{y}(j|k) \quad (10a)$$

$$\text{s.t. } \hat{z}(0|k) = z(k) \quad (10b)$$

$$\hat{z}(j+1|k) = \hat{F}(\hat{z}(j|k), \hat{u}(j)), j \in \mathbb{I}_0^{N_p-1} \quad (10c)$$

$$\hat{y}(j|k) = \mathcal{O}(z(j|k), u(j)) \quad (10d)$$

$$\hat{u}(j) \in U \quad (10e)$$

$$\hat{u}(j) = u_s, j \in \mathbb{I}_N^{N_p-2} \quad (10f)$$

$$\hat{y}(j|k) = y_s, j \in \mathbb{I}_N^{N_p-1}. \quad (10g)$$

where  $N_p = N + \max(n_a, n_b)$ .

In order to obtain a controller that provides, under certain assumptions, the same control action as the ideal model-based MPC (equations (4)), a suitable terminal constraint has to be considered. In (10), the terminal constraint is defined by (10f) and (10g). Note that these constraints require that both the input and the output maintain certain value for a given time period, defined by the memory horizons of the NARX model. The values  $u_s$  and  $y_s$  have to be calculated offline, following a procedure similar to the one used to define the optimal steady state of the model-based economic formulation ( $x_s$  in equation (2)). To this end, the following

optimization problem, based on the oracle, has to be solved:

$$(u_s, y_s) = \arg \min_{u, y} y \quad (11a)$$

$$\text{s.t.} \quad y = \mathcal{O}(z, u) \quad (11b)$$

$$z = (y, \dots, y, u, \dots, u). \quad (11c)$$

The control law is then given by

$$u(k) = \kappa_{\text{eco}}(z(k)) = \hat{u}^*(0).$$

In the following theorem it is stated that this controller, under some ideal assumptions, is equivalent to the model-based economic MPC, and hence it renders the controlled system asymptotically stable and minimizes the economic performance.

*Theorem 2:* Assume that the oracle provides an exact estimation of the value of the economic cost function, and that the initial state is such that  $x(0)$  is feasible for  $P_N(x(k))$ , so the corresponding state vector  $z(0)$  is feasible for  $P_{\mathcal{O}}(z(k))$ . Then, the evolution of the system controlled by the economic control law derived from  $P_{\mathcal{O}}(z(k))$  is equal to the one resulting from the control law derived from  $P_N(x(k))$ .

*Proof:* Assume that both  $P_N(x(k))$  and  $P_{\mathcal{O}}(z(k))$  begin with the same feasible solution, where  $z(0)$  is the regression equivalence of  $x(0)$ . Provided that no estimation error implies that  $y(k) = L(x(k), u(k)) = \hat{y}(k) = \mathcal{O}(z(k), u(k))$ , the only fact that could make both problems (and solutions) differ is the difference between the equality terminal constraints. It must be proven that (i) the optimal solution of  $P_N(x(k))$  is a solution of  $P_{\mathcal{O}}(z(k))$  and (ii) the opposite.

(i) The terminal constraint in  $P_{\mathcal{O}}(z(k))$  (equations (10f) and (10g)) consists in maintaining  $u = u_s$  while forcing  $y$  to equal  $y_s$  from  $N$  to  $N_p = N + \max(n_a, n_b)$ . Hence, if the optimal solution of  $P_N(x(k))$  is  $x(N) = x_s$ , then  $y(N) = y_s$ , and since  $u(j) = u_s$ ,  $j \in \mathbb{I}_N^{N_p}$ , then  $z(N_p) = z_s$ , which is a feasible solution of  $P_{\mathcal{O}}(z(k))$ . This implies that the solution of  $P_N(x(k))$  is suboptimal for  $P_{\mathcal{O}}(z(k))$ , so  $V_N^* \geq V_{\mathcal{O}}^*$ .

(ii) On the other hand, in order to prove the opposite, it is considered without loss of generality that the state of the oracle-based system,  $z$ , contains, at least, the last  $2n$  measurements of the output  $y$  and the input  $u$ .<sup>2</sup>

Under the assumptions of Theorem 1, from [7, Thm 5] it can be stated that there exists a continuous and bijective function  $\Phi$  such that for every state  $x(j-2n|k)$  and sequence of inputs  $\mathbf{u} = (u(j-2n), \dots, u(j-1))$ , the state of the oracle  $z(j|k)$  satisfies that  $x(j-2n|k) = \Phi(z(j|k))$ , since the sequence  $\mathbf{u}$  is part of the vector  $z(k)$ .

Assume that  $z(j|k) = z(j+1|k) = z_s$ . Then  $x(j-2n|k) = \Phi(z(j|k))$  must be equal to  $x(j-2n+1|k) = \Phi(z(j+1|k))$ , which implies that  $x(j-2n|k) = f(x(j-2n|k), u_s) = x_s$ . And hence if the optimal solution of  $P_{\mathcal{O}}(z(k))$  leads to  $z(N_p) = z_s$ , then  $x(N) = x(N_p - 2n) = \Phi(z(N_p)) = x_s$ , which is a suboptimal solution of  $P_N(x(k))$ , so  $V_{\mathcal{O}}^* \geq V_N^*$ .

<sup>2</sup>See that if  $n_a < 2n$  the oracle can be extended to  $\hat{n}_a = 2n$ , with some of its components not affecting the output.

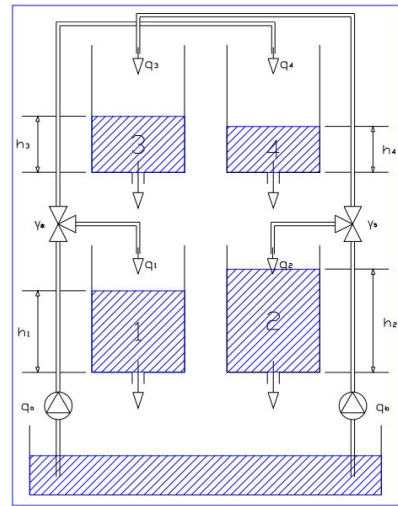


Fig. 1: Quadruple-tank process scheme, reproduced from [13]

Then, it is straight forward that  $V_N^* = V_{\mathcal{O}}^*$ , so the solution is the same for both problems, provided that Assumption 3 holds. ■

*Remark 2:* This theorem only states a hypothetical result since it is unlikely that the oracle provides an exact estimation of the evolution of the economic cost function. The estimation error would add mismatches between the real plant and the prediction model that might induce undesired closed-loop behaviour. This would require a robust design of the proposed controller, by means of the input-to-state stability theory, [12].

Note that a common requirement for robust MPC designs is to guarantee nominal stability of the controller, which is indeed the contribution of this paper. The next step would be to prove the bounded effect of the estimation error on the predicted trajectory. This topic is the subject of future work to be done in this research line.

## V. CASE STUDY

The system taken into consideration is the quadruple-tank process described in [13]. Figure 1 represents the system's scheme. It consists of four tanks, where the two on top discharge on the inferior ones. They are filled with two pumps, which send two flows represented by  $q_a$  and  $q_b$ . These flows enter the three-ways valves, which divide each flow into two branches, determined by the fraction  $\gamma_a$  and  $\gamma_b$ . Thus,  $q_a \gamma_a$  goes into the tank number one,  $q_a(1 - \gamma_a)$  into tank four,  $q_b \gamma_b$  into the tank number two and  $q_b(1 - \gamma_b)$  into tank number three. Tank number three discharges into tank number one, and the fourth one into the second one, as represented in Figure 1.

There are two control inputs, the flows  $q_a$  and  $q_b$  ( $\text{m}^3 \text{s}^{-1}$ ). The heights of the tanks are denoted as  $h_i, i \in \mathbb{I}_1^4$  (m). The dynamics of the plant are nonlinear, and are well modelled by the following set of differential equations:

TABLE I: Parameters of the system

Param.	Definition	Value	Units
$A$	Area of the four tanks	0.03	$\text{m}^2$
$a_1$	Eq. area of the hole of tank 1	$1.31 \times 10^{-4}$	$\text{m}^2$
$a_2$	Eq. area of the hole of tank 2	$1.51 \times 10^{-4}$	$\text{m}^2$
$a_3$	Eq. area of the hole of tank 3	$9.27 \times 10^{-5}$	$\text{m}^2$
$a_4$	Eq. area of the hole of tank 4	$8.82 \times 10^{-5}$	$\text{m}^2$
$\gamma_a$	Fraction of three-ways valve $a$	0.3	-
$\gamma_b$	Fraction of three-ways valve $b$	0.4	-
$g$	Gravity acceleration	9.8	$\text{m s}^{-2}$
$c$	Unitary cost of pumping	1	-
$p$	Unitary cost of storage	20	€
$V_{\min}$	Minimum storable volume	0.012	$\text{m}^3$

$$\begin{aligned}
 A_1 \frac{dh_1(t)}{dt} &= -a_1 \sqrt{2gh_1(t)} + a_3 \sqrt{2gh_3(t)} + \gamma_a \frac{q_a(t)}{3600} \\
 A_2 \frac{dh_2(t)}{dt} &= -a_2 \sqrt{2gh_2(t)} + a_4 \sqrt{2gh_4(t)} + \gamma_b \frac{q_b(t)}{3600} \\
 A_3 \frac{dh_3(t)}{dt} &= -a_3 \sqrt{2gh_3(t)} + (1 - \gamma_b) \frac{q_b(t)}{3600} \\
 A_4 \frac{dh_4(t)}{dt} &= -a_4 \sqrt{2gh_4(t)} + (1 - \gamma_a) \frac{q_a(t)}{3600},
 \end{aligned}$$

where  $A_i$  ( $\text{m}^2$ ) denotes the area of tank  $i$  and  $a_i$  ( $\text{m}^2$ ) ti equivalent area of the hole of tank  $i$ .

The parameters of the model are given in Table I. Note that the model is only used to carry out simulations, no information is used to design the controller.

The constraints in the inputs are  $1 \leq q_a \leq 2.1 \text{ m}^3 \text{ s}^{-1}$  and  $1.2 \leq q_b \leq 2.5 \text{ m}^3 \text{ s}^{-1}$ . The economic cost, that is, our plant's only measurable output is

$$y = q_a^2 + c q_b^2 + \frac{p V_{\min}}{A_1 (h_1 + h_2)}, \quad (12)$$

measured in euros, where  $c$  is the unitary cost of water pumping,  $p$  is the unitary cost of storing water and  $V_{\min}$  the minimum volume of liquid that can be stored.

#### A. Obtaining the data set

The workspace is bounded by  $q^{\min} = [1 \ 1.2](\text{m}^3 \text{ s}^{-1})$  and  $q^{\max} = [2.1 \ 2.5](\text{m}^3 \text{ s}^{-1})$ . First, the static characteristic is estimated using a grid of steps in the inputs from  $q^{\min}$  to  $q^{\max}$  with increments of  $0.1 \text{ m}^3 \text{ s}^{-1}$ , and each step long enough to reach a steady state, since the system is stable. The result is shown in Figure 2. In addition to obtaining the equilibrium points of the system, this test is used to adjust the sampling time, which is set to  $\frac{\tau}{5} = 30 \text{ s}$ , where  $\tau$  stands for the mean settling time of the sequence of steps applied.

After defining the static characteristic, a set of experiments are carried out to obtain the data used to train the oracle. The experiments are designed using the methodologies presented in [14] to identify the dynamics of a system within a workspace: a sequence of chirp signals covering the workspace are applied to generate the raw data set containing the trajectories of costs and flows,  $\mathcal{D}_{\text{raw}}$ .

In addition, several tests with random input signals are carried out in order to obtain data sets for cross-validation.

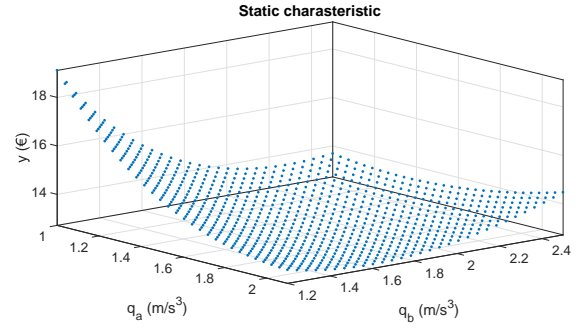


Fig. 2: Static characteristics

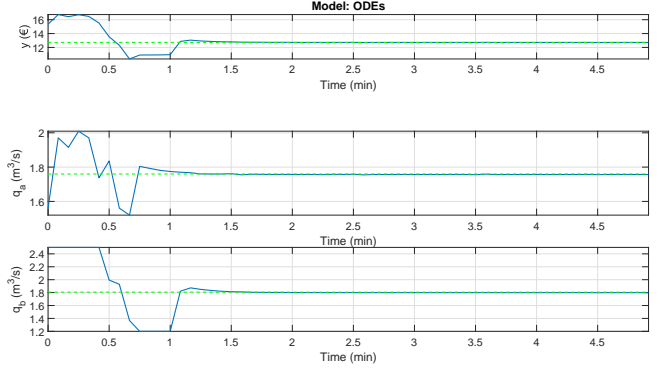


Fig. 3: Economic MPC in which the prediction model is ideal

The values of all the signals are scaled between 0 and 1. The regression state vector  $z(k)$  is constructed for different values of  $n_a$  and  $n_b$ . With these data sets, an oracle is built in the form of a Gaussian process regression [9]. The chosen covariance function (kernel) of the model is the squared-exponential kernel.

Cross validation tests are used to estimate the prediction error, which is minimized for  $n_a = 2$  and  $n_b = 3$ .

#### B. Control of the system

The solver chosen to solve the optimization problem is Matlab's *fmincon*. As stated in the previous section, the solver needs an equilibrium point of the systems as reference,  $(u_s, y_s)$ , which is obtained solving the optimization problem (11), which yields  $u_s = [1.76 \ 1.81]$  and  $y_s = 12.72 \text{ €}$ .

The stage cost is calculated as in (10a). The control horizon is set to  $N = 5$ . Hard constraints in the inputs and the equality terminal constraint are also considered.

Two different controllers are applied to the system, yielding the results presented below. The initial state is set to  $h = [0.55 \ 0.65 \ 0.50 \ 0.75] \text{ m}$ . In Figure 3, the model used for prediction is the ideal one, defined by the set of ODEs of the model. The oracle-based MPC is applied in Figure 4. Note that the data-based control problem is able to perform as well as the economic MPC.

Unlike in Theorem 2, there exists some prediction error between the real system and the oracle-based one, as one can appreciate in the results. Considering this difference, in order to compare the performance between both sets, one

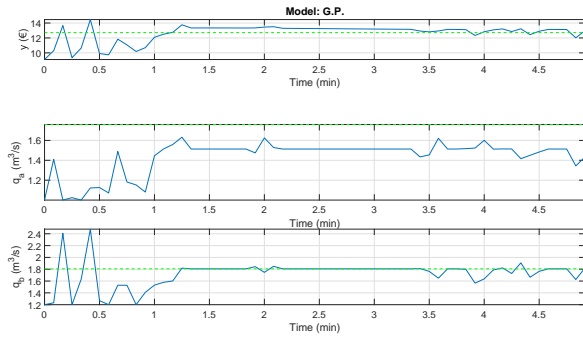


Fig. 4: Oracle-based predictive controller, in which the model is a Gaussian process

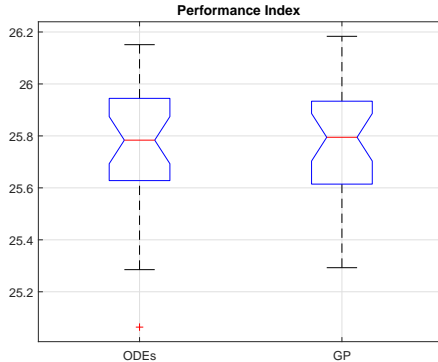


Fig. 5: Comparison of the performance of the EMPC with two different models: the ideal set of ODEs and the oracle model based on data, for 100 simulations

hundred simulations are carried out. In these tests the initial state is chosen randomly within  $[0.54 \ 0.61 \ 0.48 \ 0.71] \leq h \leq [0.650 \ 0.720 \ 0.590 \ 0.82]m$ , and the duration of each test is 5 min.

The results are shown in Figure 5, illustrating that the oracle-based predictive controller is able to emulate the behaviour of the real MPC. The performance index of each simulation is calculated as the sum of the economic cost all along each trajectory:

$$\Phi = \sum_{i=1}^{t_{sim}} y(i).$$

This performance analysis shows that the oracle-based MPC provides in this example similar behaviour, which support the theoretical results presented.

## VI. CONCLUSION

The objective of this paper was to prove that an economic predictive controller can be designed for a system whose model is unknown, under the assumption that the states are not accessible. On the contrary, only the economic cost of the plant is observed as the unique output along the operation of the system.

This set-up can be encountered in many current situations in which the client may not be keen on providing further details of the intern operation of his plant, or provided that the system is too complex to model. The evolution of the economic cost is predicted using an oracle, i.e., a data-based model of the output is learned from a set of historical input-output data points of past trajectories. The structure of the oracle is a NARX regressor, and the prediction is done with certain machine learning technique.

In the case study presented, the prediction technique chosen are Gaussian processes. The problem is applied to the economic control of a quadruple-tank process, proving to be able to perform as well as the standard economic MPC.

Future work lines include the extension of the proposed approach to the robust case, considering stability and feasibility issues in the presence of noise and estimation errors.

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