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Robust economic model predictive control of a community micro-grid☆

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In this paper we propose a novel economic robust predictive controller for periodic operation. The proposed controller joins dynamic and economic trajectory planning and robust predictive controller for tracking in a single layer taking into account bounded disturbances, algebraic constraints and periodic operation. We study the closed-loop system properties of the proposed controller and provide a design procedure that guarantees that the perturbed closed-loop system converges asymptotically to the optimal economic reachable periodic trajectory, constraint satisfaction and recursive feasibility. The proposed controller has been applied to control a cluster of interconnected micro-grids. Each nano-grid is connected to an electric utility and has a renewable energy source, a cluster of batteries and a metal hydride based hydrogen storage system. The cluster must satisfy a periodic energy demand while maximizing the profit of the energy sold to the electric utility taking into account time varying prices.

1. Introduction

Nowadays, the increase of the energy demand is an important problem which is usually faced with limited fossil fuel sources and with environmental restrictions. A way to deal with this problem is the increase in the use of renewable energy sources. In this scenario, micro-grids have gained a relevant role. The definition of a micro-grid from the U.S. Department of Energy Micro-grid Exchange Group is the following: "A micro-grid is a group of interconnected loads and distributed energy resources within clearly defined electrical boundaries that acts as a single controllable entity with respect to the grid. A micro-grid can connect and disconnect from the grid to enable it to operate in both grid-connected or island-mode". Because the main aim of a micro-grid is to satisfy an internal demand and if possible sell or store the excess of produced energy, the energy storage systems have become very important in the optimal management of this type of systems. The most widely spread storage systems are batteries, however its discharge rate has motivated the development of alternative storage system such as hydrogen based systems.

The optimal management of this type of networks is a challenging problem that has received a lot of attention from the research community. In Ref. [12] a wireless data-link based power management system for a distributed hydrogen system is proposed. In Ref. [27] the important of smart control strategies for PV-hydrogen systems is demonstrated through extensive simulations. In Ref. [18] an experimental small-scaled stand-alone power system based on hydrogen is presented. In Ref. [4] a set of wind/hydrogen energy system modelling tools were validated. Model predictive control has also been applied to this class of systems, see for example [19–22,28] and [24].

Model predictive control (MPC) has demonstrated to be an excellent choice for optimal management of complex control systems, such as multivariable constrained systems, when the main objective is to guarantee closed-loop stability and constraint satisfaction while minimizing a cost function without expert intervention [3,26]. Of particular interest for the optimal operation of micro-grids is the use of an economic cost function as stage cost

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function, that is the so-called economic MPC. This control technique allows the controller to improve its performance during the transients and take into account economic issues explicitly such as the energy and/or degradation costs. Some examples are shown in Refs. [7–9] where MPC designs based on Lyapunov theory have been developed. These approaches are capable of optimizing closed-loop performance taking into account economic considerations for a broad class of complex process systems, such as those subject to asynchronous and delayed measurements and uncertain variables. In addition, in Ref. [25]; recent results on the stabilizing design of economic MPC are summarized. Another important research is presented in Ref. [6] where the terminal constraint is taken from an economic MPC without loosing the stabilizing features of the controller. In Ref. [31]; a single-layer economic MPC has been proposed integrating the RTO into the MPC.

The previous results try to regulate the system to an optimum steady state operation point however, in certain cases, the best way to operate a system from an economic point of view is to follow a non steady trajectory, usually periodic, see Refs. [10,11,14]. In renewable energy systems non-steady state operation appears naturally because of the quasi-periodic disturbances such as exogenous periodic demands, fluctuating prices (as in the electricity for instance) and energy generation profiles. A typical solution to deal with non-steady operation is the use of a predictive control structure composed by two layer in which the optimal trajectory is calculated by a dynamic real time optimizer, that is a RTO which takes into account the dynamic model of process to control, and a predictive control to move the closed-loop system to the previous optimal trajectory, see Ref. [30]. However this approach does not take into account the economic cost function during the transient which has motivated to several authors to propose the use of the economic cost function as a stage cost. Examples of it can be found in Refs. [2,10,11].

A relevant issue that the control system should deal with is the dependence of the economic cost function on exogenous parameters that may be changed along the system evolution, such as energy prices, expected energy demand or unitary operation costs. These changes in the economic cost function may lead to a redesign of the predictive controller, including the constraints of the optimization problem, and the loss of feasibility [5,15]. Recently, a novel MPC formulation that addresses these problems was presented in Ref. [17]. This controller steers the closed-loop system to the best economically optimal periodic trajectory that the system can reach guaranteeing the constraint satisfaction and asymptotic stability even in the case of changes on the economic cost function. This controller was applied to a micro-grid in Ref. [23]. In this work, the control of a non-isolated micro-grid was considered assuming that periodic predictions of the demand and generation profiles of the photovoltaic system were available. The proposed controller did not take into account disturbances in the model or in the predictions and in spite of the inherent robustness of the predictive controllers, certain properties, such as constraint satisfaction, may be lost if the uncertainty is large enough.

On the other hand, energy distribution networks are modeled by algebraic-differential equations, where the algebraic equations typically describe energy balances in the nodes of the grid that may depend on (possibly varying) energy demand. The control system must be designed to ensure the fulfilment of these algebraic constraints along the time together with stability and recursive feasibility of the grid in presence of uncertainty. Motivated by these issues, a robust economic model predictive controller for energy systems subject to constraints on the operation limits and energy balances under random variations of the expected demands and the economic cost function is presented. This controller extends the control scheme presented in Ref. [17] to deal with the robust case following the constraint tightening method proposed in Ref. [1] and taking into account linear differential-algebraic systems. This controller considers additional decision variables introducing an artificial periodic reference and minimizes a cost function that accounts for the economic cost of the artificial variables and the deviation of the nominal predictions from artificial trajectory. The state and input constraints are tightened taking into account a semi-feedback scheme. Due to this linear feedback, the effect of the perturbations is rejected and the controller guarantees robust satisfaction. The complexity of the resulting optimization problem to be solved on line is similar to the one of the nominal controller. The resulting control law guarantees the convergence to a neighborhood of a robust optimal trajectory that minimizes the cost function and satisfies the constraints for all possible uncertainties. Equality constraints for the uncertain system are satisfied thanks to a feed-forward policy.

The proposed controller has been used for the economic operation of a cluster of interconnected micro-grids. Each nanogrid has a renewable energy source, a cluster of batteries and a metal hydride based hydrogen storage system and its connected to an electric utility. The micro-grid system must satisfy an energy load at each sampling time and maximize the profit of the energy sold to the electric utility. It is assumed that expected profiles of the renewable energy generators and of the load are available, although there may exist mismatches between the real and the expected profiles. The mismatches are modeled with a bounded additive uncertainty. The proposed control system operates the energy system minimizing a given operation economic cost and satisfying the load and the operational constraints in spite of the variations on the produced energy, the load and the unitary prices of the cost function.

1.1. Notation

Bold letters are used to denote a sequence of T values of a trajectory, i.e. $\mathbf{z} = \{z(0), \cdots, z(T-1)\}$. $\mathbf{z}(\theta)$ denotes the sequence $\mathbf{z}(\theta) = \{z(0;\theta), \cdots, z(T-1;\theta)\}$. If the cardinality of a sequence is not T, then the sequence is denoted as $\mathbf{z}_N(\theta)$ where N is the cardinal. $\mathbb{I}_{[a,b]}$ denotes the set of integer numbers contained in the interval $[a,b]$ and $\mathbb{I}_a$ denotes the set of positive integer numbers including the origin, that is $\{0, 1, \cdots, a\}$. The notation $(i|k)$ denotes the time step to which a given variable is referred.

2. Problem formulation

In this work we consider the optimal operation of systems defined by the following set of linear differential-algebraic equations subject to both measurable and unknown disturbances:

$$x(k+1) = Ax(k) + B_u u(k) + B_d d(k) + B_w w(k) \tag{1}$$

$$0 = E_x x(k) + E_u u(k) + E_d d(k) + E_w w(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $d(k) \in \mathbb{R}^s$ and $w(k) \in \mathbb{R}^q$ are the state, input, measurable and unknown disturbance vectors of the system at time step k respectively. The evolution of the measurable disturbance signal $d(k) \in \mathbb{R}^s$ is assumed periodic with period T and known, i.e. $d(k) = d(k+T)$. The disturbance signal $w(k) \in \mathbb{R}^q$ is considered unknown but bounded. It is assumed that the equality constraints (2) have a solution, that is, they are linearly independent and the number of equations is less than the number of inputs.

This class of systems can be used to model distribution networks that must satisfy a demand for which an uncertain prediction is available. The algebraic equations can model the network topology constraints, with energy and/or mass balance equations for instance, while the demand prediction error is modeled with the disturbance.

We consider a set of coupled state and input constraints

$$(x(k), u(k)) \in \mathscr{Z} \subseteq \mathbb{R}^{n+m} \tag{3}$$

where $\mathscr{Z}(k)$ is a closed convex polyhedron that contains the origin in its interior. It is also assumed that the uncertain signal $w(k)$ is bounded as follows:

$$w(k) \in \mathscr{W} \subseteq \mathbb{R}^{q} \tag{4}$$

being $\mathscr{W}$ a known compact polyhedron that contains the origin.

The performance of the evolution of the system (1) is measured by an economic time varying stage cost function $l(k,x,u,p)$ that depends on the state, input and a parameter $p \in R^{np}$, of any dimension, which could describe the prices and other contract with the electric utility information. This function and/or the parameter p may be changed during the operation of the system and this variation is not known a priori.

Assumption 1. The stage cost function $l(k,x,u,p)$ satisfy the following conditions:

1. $l(\cdot)$ is positive, i.e. $l(k,x,u,p) \geq 0$ for all (k,x,u,p).
2. $l(\cdot)$ is convex in (x,u) for all k.
3. $l(\cdot)$ is periodic; that is, $l(k,x,u,p) = l(k+T,x,u,p)$.

2.1. Controller formulation

Usually the search of the optimal economic performance is the main control objective of a process. The performance of a system can be expressed as the average of the economic cost function of the closed-loop trajectories and it can be posed as follows:

$$L_\infty(x, \mathbf{u}_\infty, p) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=0}^{M-1} l(k, x(k), u(k), p)$$

where x is the initial state and $\mathbf{u}_\infty$ is the set of the corresponding closed-loop input trajectories.

In theory, a way to obtain the optimal trajectory to operate the system (1) is derived from the solution of an infinite horizon optimal control problem that minimizes the average cost. In general this problem cannot be solved, but in particular given the periodic nature of the cost function and the known disturbances $d(k)$, and under the assumption of uniqueness of the solution, the optimal trajectories can be obtained for the nominal system; that is with $w(k) = 0$, solving a finite horizon open-loop problem that optimizes the average cost of a period [2,17].

For the uncertain case, however, the calculation of the optimal trajectory has to take into account the effect of the unknown disturbance in the cost and to guarantee constraint satisfaction for the worst possible case. There are different strategies to deal with the effect of uncertainty in the predictions, such as mini-max optimization or the use of stochastic programming. In general, these strategies are computationally very demanding. In this paper we propose to follow a different path based on minimizing the cost for the nominal system, while guaranteeing robust constraint satisfaction by means of reducing the constraints to account for the worst possible perturbation.

The aim of this paper is to present a robust model predictive control technique based on nominal predictions that steers the plant to (a neighborhood of) the optimal trajectory. One important issue to take into account in this approach is that the controller is implemented in a receding horizon scheme, so the controller can compensate at each time step the effect of the disturbance at the previous sample time. Including this idea into the optimization problem leads to nested (or multistage) optimization problems which are also very demanding. To obtain a suboptimal solution the controller is designed assuming that there exists a local linear controller that can stabilize the nominal system while satisfying the algebraic constraints. In particular, this controller is used to include some degree of feedback in the open-loop predictions, reducing the effect of the worst possible uncertainties in the constraints.

To design this local controller, an auxiliary control input $v(k)$ is introduced from the explicit solution of equation (2) in order to satisfy that equality constraint for any disturbance $w(k)$. The value of input vector $u(k)$ that satisfies the equality constraint (2) is given by

$$u(k) = M_x x(k) + M_v v(k) + M_d d(k) + M_w w(k) \tag{5}$$

where $v(k) \in \mathrm{IR}^{m_v}$ denotes the new set of control inputs. Matrices $M_x \in \mathrm{IR}^{m\times n}$, $M_v \in \mathrm{IR}^{m\times m_v}$, $M_d \in \mathrm{IR}^{m\times s}$ and $M_w \in \mathrm{IR}^{m\times q}$ are obtained from the solution of (2). Matrix M_v is an orthonormal basis for the null space of E_u obtained from the singular value decomposition. Matrices M_x, M_d and M_w provide a particular solution to the equation that depend on $x(k), d(k)$ and $w(k)$. There are infinite solutions which provide different ways of distributing the power to satisfy the equation (2) for a given predicted demand. An inappropriate selection of these matrices may lead to optimization problems with a reduced feasibility region.

Using this variable change, the following equivalent model is obtained

$$x(k+1) = \widehat{A}x(k) + \widehat{B}_v v(k) + \widehat{B}_d d(k) + \widehat{B}_w w(k) \tag{6}$$

where

$$\begin{aligned}
\widehat{A} &= A + B_u M_x \\
\widehat{B}_v &= B_u M_v \\
\widehat{B}_d &= B_d + B_u M_d \\
\widehat{B}_w &= B_w + B_u M_w
\end{aligned}$$

It is assumed that the pair $(\widehat{A}, \widehat{B}_v)$ is controllable. This model will be used to define the reduced set of constraints for the MPC optimization problem.

In order to reject the effect of the uncertainty in the predictions, a local linear control law $v(k) = \widehat{K}e(k)$ is proposed, where $e(k)$ is the deviation of the perturbed predictions from the nominal predictions obtained the previous sampling time (this will be detailed in the following sections). This control law is designed for the system

$$e(k+1) = \widehat{A}e(k) + \widehat{B}_v v(k)$$

which is obtained taking into account the definition of the auxiliary control variable and assuming that $d(k) = 0$. The local control law $\widehat{K}$ is designed to ensure that the dynamics of the deviation of system in closed-loop with this linear controller $e(k+1) = \widehat{A}_K e(k)$, where

$$\widehat{A}_K = \widehat{A} + \widehat{B}_v \widehat{K},$$

it is asymptotically stable.

On the other hand, the solution of the algebraic equation for $w(k)$ given by (5) is also used to guarantee that the input applied by the proposed controller guarantees the satisfaction of the equality constraint (2) taking into account the current value of the disturbance $w(k)$. This is only possible if the current value of $w(k)$ can be measured to compensate its effect by means of a feed-forward scheme. To this end at each sampling time, the MPC controller will calculate the optimal control input $u^*(k)$ based on the nominal predictions of the plant (that is, assuming that $w(j)=0$), and then the real value of $u(k)$ is updated taking into account the measured value $w(k)$ as follows,

$$u(k) = u^*(k) + M_w w(k)$$

Considering the previous aspects, the robust predictive controller can be formulated. The parameters that define the optimization problem at time step k are the current state x and the (a priori known) values of the measurable disturbance $\mathbf{d}$. As it is usual in predictive controllers, the predicted N-step control input trajectory $\mathbf{u}_N$ is a decision variable optimization control problem but, following [17]; an artificial reference trajectory given by the initial state x_0^r and the sequence of future T-step reference inputs $\mathbf{u}^r$ is considered as additional decision variable. The cost function of the proposed controller is defined as follows:

$$V_N(x,\mathbf{d};x_0^r,\mathbf{u}^r,\mathbf{u}_N) = V_t(x,\mathbf{d};x_0^r,\mathbf{u}^r,\mathbf{u}_N) + V_p(\mathbf{d};x_0^r,\mathbf{u}^r)$$

where

$$V_t(x,\mathbf{d};x_0^r,\mathbf{u}^r,\mathbf{u}_N) = \sum_{i=0}^{N-1} ||x(i)-x^r(i)||_Q^2 + \|u(i)-u^r(i)\|_R^2$$

$$V_p(\mathbf{d};x_0^r,\mathbf{u}^r) = \frac{1}{T}\sum_{i=0}^{T-1} l(k+i,x^r(i),u^r(i),p)$$

The term $V_t(x,\mathbf{d};x_0^r,\mathbf{u}^r,\mathbf{u}_N)$ penalizes the tracking error of the open-loop predicted trajectories with respect to the artificial reference along the prediction horizon N. The term $V_p(\mathbf{d};x_0^r,\mathbf{u}^r)$ penalizes the nominal economic cost function of the artificial reference.

The optimal trajectories of the proposed robust MPC for tracking periodic signals can be obtained from the solution of the following finite horizon optimal control problem $\mathscr{P}_N(x,\mathbf{d})$

$$\min_{x_0^r,\mathbf{u}^r,\mathbf{u}} V_N(x,\mathbf{d};x_0^r,\mathbf{u}^r,\mathbf{u}_N)$$

$$s.t \quad x(0) = x \tag{7a}$$

$$x(i+1) = Ax(i) + B_u u(i) + B_d d(i) \tag{7b}$$

$$0 = E_x x(i) + E_u u(i) + E_d d(i) \tag{7c}$$

$$(x(i),u(i)) \in \mathscr{Z}_i \tag{7d}$$

$$x(N) = x^r(N) \tag{7e}$$

$$x^r(i+1) = Ax^r(i) + B_u u^r(i) + B_d d(i) \tag{7f}$$

$$0 = E_x x(i) + E_u u(i) + E_d d(i) \tag{7g}$$

$$(x^r(i),u^r(i)) \in \mathscr{Z}_N \tag{7h}$$

$$x^r(T) = x^r(0) = x_0^r \tag{7i}$$

where the set $\mathscr{Z}_0 = \mathscr{Z} \ominus \mathscr{Z}_W$ and for $i \geq 1$, the set $\mathscr{Z}_i$ is defined as follows

$$\begin{aligned}
&\mathscr{Z}_i = \mathscr{Z} \ominus \mathscr{Z}_W \ominus R_z(i)\\
&\mathscr{Z}_W = \{z : (0, M_w w), \forall w \in \mathscr{W}\}\\
&R_z(i) = \bigoplus_{0}^{i-1} Q_z(j)\\
&Q_z(j) = \left\{z : \left(\widehat{A}_K^j \widehat{B}_W w, M_v \widehat{K} \widehat{A}_K^j \widehat{B}_W w\right), \forall w \in \mathscr{W}\right\}
\end{aligned}$$

It is important to remark that the calculation of these sets is trivial, even for large scale systems [13].

The optimal solution of this optimization problem at time step k is denoted $(x_0^{r*}(k), \mathbf{u}^{r*}(k), \mathbf{u}_N^*(k))$. The value of the control variables is obtained as follows:

$$u(k) = u^*(0|k) + M_w w(k) \tag{8}$$

where $u^*(0|k)$ is the optimum value for the first input at time step k. Notice that in order to ensure that the real control action $u(k)$ is admissible, i.e. $(x(k),u(k)) \in \mathscr{Z}$, the MPC is designed for a tighter set of constraints $\mathscr{Z} \ominus \mathscr{Z}_W$.

The constraints of the optimization variables are contracted with every step of the prediction horizon. Constraint (7d) shows that as the prediction step i increases, the set $\mathscr{Z}_i$ reduced taking into account the possible effect to a perturbation on the predicted system in closed-loop with the auxiliary controller. This contraction is time invariant and can be calculated off-line. Constraints (7b)-(7c) are defined by the nominal model, i.e. considering $w(i)=0$, and provides the predicted state and input trajectories. Constraint (7b) imposes that the initial state of the predicted trajectory is equal to the state of the system at time step k. Constraint (7e) states that the predicted state must reach the artificial reference in N steps. Constraints (7f)-(7g) are defined by the nominal model and provides the artificial references state and input trajectories. Note that the initial state of the artificial reference is a free variable, however, it is constrained to be a periodic trajectory in constraint (7i). The artificial references must satisfy the state and input constraints, but because in order to guarantee recursive feasibility, the artificial reference is used to define the shifted input trajectory at prediction time $N-1$, the constraint set is contracted by the same set for all steps which depends on the prediction horizon N. In particular, the artificial references must satisfy (7d) for $i=N$. These constraints guarantee under certain assumptions recursive feasibility using an appropriately defined shifted solution.

It is important to note that the constraints of the optimization problem do not depend on the economic cost function. This implies that a sudden change in the parameters of this function cannot cause a loss of feasibility of the optimization problem. This will be shown in the simulation example. In addition, the proposed controller only takes into account in an explicit manner the effect of the uncertainty by tightening the set of constraints. This implies that the computational complexity is equal to the nominal counterpart controller presented in Ref. [17]. In order to reduce the computational burden, triggered evaluation could be taken into account as in Ref. [32].

3. Robust stability of the proposed controller

In this section we prove that under certain assumptions the controlled system converges asymptotically to a neighborhood of the trajectory obtained solving the following optimization problem

$$\min_{x_0^r,\mathbf{u}^r} V_p(\mathbf{d};x_0^r,\mathbf{u}^r) = \frac{1}{T}\sum_{i=0}^{T-1} l(k+i,x^r(i),u^r(i),p) \tag{9a}$$

$$s.t.\quad x^r(i+1) = Ax^r(i) + B_u u^r(i) + B_d d(i) \tag{9b}$$

$$0 = E_x x(i) + E_u u(i) + E_d d(i) \tag{9c}$$

$$(x^r(i),u^r(i))\in\mathscr{Z}_N \tag{9d}$$

$$x^r(T) = x^r(0) = x_0^r \tag{9e}$$

This optimization problem is denoted the robust planner throughout this paper. The optimal input and state trajectories provided by (9) are denoted as robust planner trajectories, $(\mathbf{x}^\circ,\mathbf{u}^\circ)$. Because the known disturbance and the parameters of the cost function are periodic, and the cost function is strictly convex, the optimal periodic trajectories do not depend on the time step k in which the optimization problem is formulated. The resulting trajectory takes into account the effect of the uncertainty in the constraints in order to guarantee robust constraint satisfaction. In this case, the robust planner is not independent of the prediction horizon of the corresponding robust MPC, because the reduction of the constraint set depends on N. It is important to remark that it is not necessary to solve the planner optimization problem to define the MPC controller.

3.1. Robust stability of the robust closed-loop system

In this section we prove that the robust planner trajectory is robustly stable for the system controlled by the proposed predictive controller and besides, the equality constraints are satisfied all the time. To this aim, the proposed controller must be designed appropriately satisfying the following conditions:

Assumption 2. The weighting matrices Q, R and the controller gains $\widehat{K}$ satisfy the following conditions:

1. System (6) is controllable.
2. Matrices Q and R are positive definite.
3. The control gain $\widehat{K}$ is such that $\widehat{A}_K$ is strictly stable and

$$\widehat{A}_K^{N-1} w = 0,\quad \forall w\in\mathscr{W}$$

4. The sets $\mathscr{Z}_i$ are non-empty for $i\in\mathbb{I}_N$.
5. The optimization problem (7) is strictly convex.

It is important to remark that from this Assumption, the optimization problems (7) and (9) are strictly convex and then the solution is unique. It is also interesting from a practical point of view that no robust invariant set must be calculated in the design of the proposed controller, which is computationally untractable for large scale systems. Besides, an inappropriate design of the previous local controller may result in empty feasibility regions of the MPC optimization problems. Design procedures to guarantee that the resulting optimization problem has a non-empty feasibility region are not studied in this paper but the reader can refer to [1] for an LMI based design procedure for this problem.

In practice, the condition 3 in Assumption 2, can be relaxed to

$$\max_{w\in\mathscr{W}} \left\|\widehat{A}_K^{N-1} w\right\| \le \sigma$$

where σ is the tolerance of the optimization problem solver.

Now we prove that the optimal trajectory in an economic point of view is a robustly stable trajectory of the system in the input-to-state stability sense, which is defined as follows.

Definition 1. *The periodic trajectory $\mathbf{x}^\circ$ is an input-to-state stable trajectory for the controlled system with a domain of attraction $\mathscr{X}_N$ if for all $x(0)\in\mathscr{X}_N$, then $x(k)\in\mathscr{X}_N$ and there exists a $\mathscr{K}\mathscr{L}$ function $\beta(\cdot)$ and a $\mathscr{K}$ function $\sigma(\cdot)$ such that*

$$||x(k)-x^\circ(k)|| \le \beta(||x(0)-x^\circ(0)||,k) + \sigma(||\mathbf{w}_k||_\infty)$$

for all $k \ge 0 ||\mathbf{w}_k||_\infty$ denotes the maximum value of $\|w(i)\|$ for all $i\in\mathbb{I}_{k-1}$.

Input-to-state stability (ISS) implies that for all initial state in $\mathscr{X}_N$, the closed-loop trajectory converges asymptotically to a neighborhood of $\mathbf{x}^\circ$ where it is ultimately bounded (see [16] for more details).

In the following theorem, input-to-state stability of the robust planner trajectory is proved. This states the main result of the paper.

Theorem 1. *If the conditions given in Assumption 2 hold, then system (6) controlled by the control law (8) is recursively feasible and the robust planner trajectory $\mathbf{x}^\circ$ is input-to-state stable with a region of attraction $\mathscr{X}_N$.*

Proof. First, we prove that the optimization problem is recursively feasible throughout the evolution of the closed-loop system, that is, if the initial state is inside the feasibility region of the optimization problem (7) then the closed-loop system will remain inside this region. Then, we prove that the closed loop system is input-to-state stable by demonstrating that the function

$$W(x(k)-x^\circ(k)) = V_N^*(x,\mathbf{d};x_0^r,\mathbf{u}^r,\mathbf{u}_N) - V_p^\circ(\mathbf{d};x_0^r,\mathbf{u}^r) \tag{10}$$

serves as an input-to-state Lyapunov function, see Refs. [16,17].

Finally, it will be proved that the control law (8) ensures that the controlled system satisfies the constraints on the state and inputs as well as the equality constraint for any possible value of the uncertainty.

3.2. Recursive feasibility

Consider the following shifted solution at time $k+1$ (superscript s) based on the optimal solution at time k (superscript $*$) based on correcting the deviation caused by the uncertainty at time k, $w(k)$, using the local controller K:

$$\begin{aligned}
&x^{rs}(0|k+1) = x^{r*}(1|k)\\
&u^{rs}(i|k+1) = u^{r*}(i+1|k)\quad i\in\mathbb{I}_{T-2}\\
&u^{rs}(T-1|k+1) = u^{r*}(0|k)\\
&u^s(i|k+1) = u^*(i+1|k) + \left(M_x + M_v\widehat{K}\right)(x^s(i|k+1)\\
&\qquad\qquad -x^*(i+1|k))\quad i\in\mathbb{I}_{N-2}\\
&u^s(N-1|k+1) = u^{rs}(N-1|k+1)
\end{aligned}$$

we use the notation $(i|k)$ to denote the prediction step i for a trajectory evaluated at the sampling step k. Taking into account equations (7a), (7b) and (7f) it follows that

$$x^{rs}(i|k+1) = x^{r*}(i+1|k) \quad i \in \mathbb{I}_{T-1}$$
$$x^{rs}(T|k+1) = x^{r*}(1|k)$$
$$x^{s}(0|k+1) = Ax(k) + B_u u^*(0|k) + B_d d(k) + \widehat{B}_w w(k)$$
$$x^{s}(i+1|k+1) = Ax^s(i|k+1) + B_u u^s(i|k+1) + B_d d(i|k+1) \quad i \in \mathbb{I}_{N-1}$$

By definition of the matrices M_x and M_v, if $u^*(i+1|k), x^*(i+1|k), d(i+1|k)$ satisfy constraint (5) for a given $v^*(i+1,k)$, then $u^s(i|k+1), x^s(i|k+1), d(i|k+1)$ also satisfy the same constraint for $v^s(i,k+1) = v^*(i+1,k)$ and hence the shifted solution satisfies constraint (7c).

The deviation of the shifted state trajectory from the previous optimal state trajectory is defined as follows

$$x^s(i|k+1) = x^*(i+1|k) + \widehat{A}_K^i \widehat{B}_w w(k)$$

Constraint (7e) holds because $\widehat{A}_K^{N-2} \widehat{B}_w w(k) = 0$ and hence $x^s(N-1|k+1) = x^*(N|k) = x^{rs}(N-1|k+1)$ and $u^s(N-1|k+1) = u^{rs}(N-1|k+1)$.

Taking into account the definition of the sets $\mathcal{Z}_i$ it follows that the shifted solution satisfies constraint (7d) for $i = 1, \ldots, N-1$. For $i = N$ the constraint is satisfies because $u^s(N-1|k+1) = u^{rs}(N-1|k+1)$ and $x^s(N-1|k+1) = x^{rs}(N-1|k+1)$ and the artificial reference is included in $\mathcal{Z}_N$.

Finally, by definition, the shifted artificial reference satisfies constraints (7g)-(7i).

3.3. ISS stability

Stability is proved by demonstrating that the function (10) is an ISS Lyapunov function [16]. Using similar arguments to the stability proof of the nominal case [17], we have that there exists positive constants α_1, α_2 and α_3 such that

$$\alpha_1 \|x(k) - x^\circ(k)\|^2 \leq W(x(k) - x^\circ(k))$$

$$\alpha_2 \|x(k) - x^\circ(k)\|^2 \geq W(x(k) - x^\circ(k))$$

and that $\Delta W = W(x(1|k) - x^\circ(k+1)) - W(x(k) - x^\circ(k))$ holds

$$\Delta W \leq -\alpha_3 \|x(k) - x^\circ(k)\|^2$$

On the other hand, we have that $x(k+1) = x(1|k) + B_w w(k)$. Since the optimal cost function $V_N^*(x, \mathbf{d})$ is a convex function of x defined in a compact set, then it is Lipschitz continuous. This means that there exists a positive constant γ such that

$$\|W(x(k+1) - x^\circ(k+1)) - W(x(1|k) - x^\circ(k+1))\| \leq \gamma \|w(k)\|$$

Therefore, we have that

$$\begin{aligned}\Delta W &= W(x(k+1) - x^\circ(k+1)) - W(x(1|k) - x^\circ(k+1)) \\ &\quad + W(x(1|k) - x^\circ(k+1)) - W(x(k) - x^\circ(k)) \\ &\leq \gamma \|w(k)\| - \alpha_3 \|x(k) - x^\circ(k)\|^2\end{aligned}$$

and then $W(\cdot)$ is an ISS Lyapunov function.

3.4. Robust constraint satisfaction

Taking into account that the optimal control action $u^*(0|k)$ is such that $(x(k), u^*(0|k)) \in \mathcal{Z}_W$ we have that the real control action applied to the system $u(k) = u^*(0|k) + M_w w(k)$ ensures that

$$\begin{aligned}&(x(k), u(k)) \in (x(k), u^*(0|k)) \oplus \mathcal{Z}_W \\ &\in \mathcal{Z} \ominus \mathcal{Z}_W \\ &= \mathcal{Z} \ominus \mathcal{Z}_W \oplus \mathcal{Z}_W \\ &\subseteq \mathcal{Z}\end{aligned}$$

On the other hand, since $E_x x(k) + E_u u^*(0|k) + E_d d(k) = 0$, we have that

$$\begin{aligned}&E_x x(k) + E_u u(k) + E_d d(k) + E_w w(k) \\ &\quad = E_x x(k) + E_u u^*(0|k) + E_d d(k) + E_u M_w w(k) + E_w w(k) \\ &\quad = (E_u M_w + E_w) w(k) = 0\end{aligned}$$

for all $w(k)$.

Remark 1. (Changing economic cost function). A relevant property of the proposed controller is that the recursive feasibility has been proved irrespective of the (possible varying) parameter of the economic cost p, since the set of constraints in the optimization problem does not depend on this function. Therefore this implies that a sudden change of p does not affect the recursive feasibility property of the closed-loop system, and if p remains constant for a sufficient period of time, the system converges to a neighborhood of the optimal reachable trajectory corresponding to the new value of p.

4. Community micro-grid control problem

In this example we consider a community micro-grid which consists of three nano-grids based on the model presented in Refs. [23,29]. Fig. 1 shows a schematic of the topology of the micro-grid.

Each nano-grid is connected to a local load and to the electric utility (EU) and is composed by a renewable energy generator, a cluster of batteries to balance the power of each nano-grid satisfying the short peaks of voltages due to its fast dynamic and a metal hydride based hydrogen storage system, see Fig. 2. The value of the parameters that define the different elements of the nano-grids can be found in Ref. [23].

The energy stored in each nano-grid i is controlled with three manipulable inputs which consists of the power exchanged with the electric utility denoted by P^i_{grid}, the power exchanged with the hydrogen based systems denoted by P^i_{H2} and the contribution of each nano-grid to the local load denoted by P^i_{load}. The state of each nano-grid are the stored energy rate in both storage systems, that is, the stage of charge of the batteries SOC^i and the level of stored hydrogen in the metal hydride deposit MHL^i. The renewable energy generators are modeled using generation profiles of photovoltaic energy sources which can be different. We consider that a prediction with a bounded degree of error of the generation profiles of each PV generator P^i_{PV} and of the local load P_{load} that must be satisfied by the micro-grid are available.

In order to design the controller, the dynamics of the SOC and the MHL of each nano-grid are modeled with two integrators taking into account the power balance equations. We consider the following model provided in Ref. [23] for a sampling time of 30 min:

$$\begin{aligned}SOC^1(k+1) &= SOC^1(k) + 7.6285 P^1_{H2}(k) - 5.5847 P^1_{grid}(k) \\ &\quad -5.5847 P^1_{load}(k) + 5.5847 \left(\overline{P}^1_{PV}(k) + \tilde{P}^1_{PV}(k)\right) \\ MHL^1(k+1) &= MHL^1(k) - 3.4495 P^1_{H2}\end{aligned} \tag{11}$$

where $\overline{P}^1_{PV}$, $\tilde{P}^1_{PV}$ are the predicted value and the prediction error

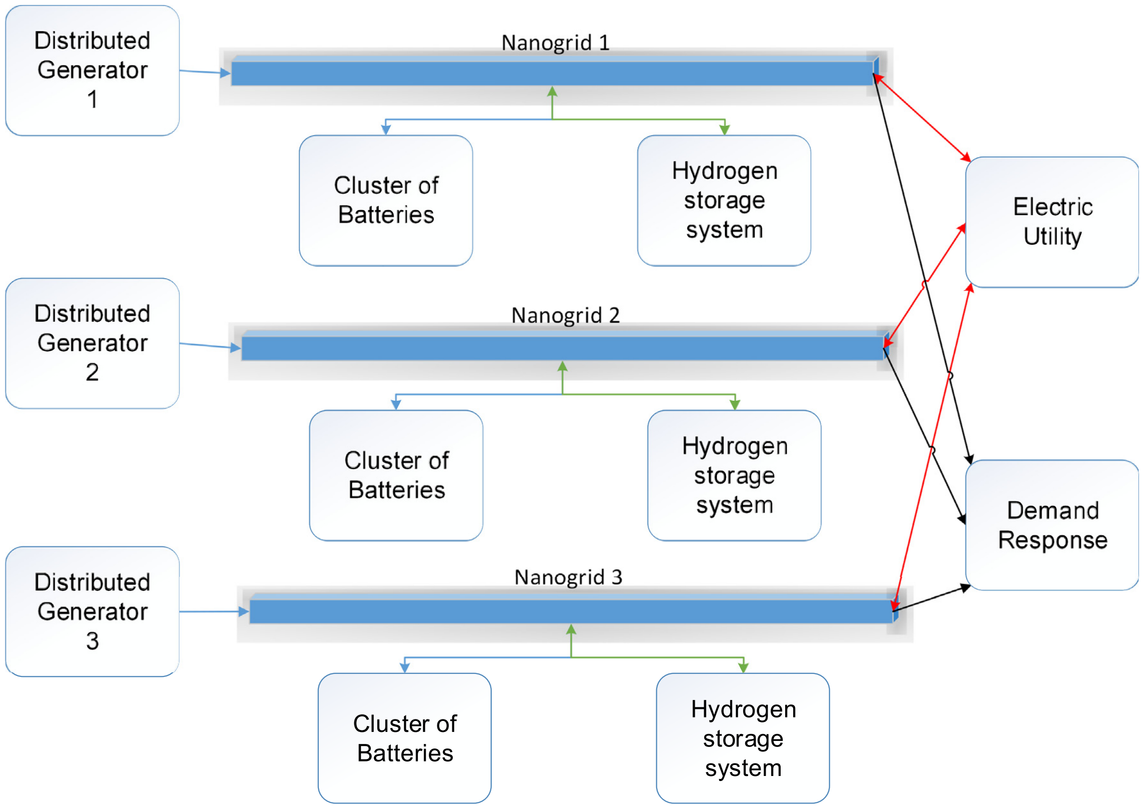

Fig. 1. Community micro-grid scheme.

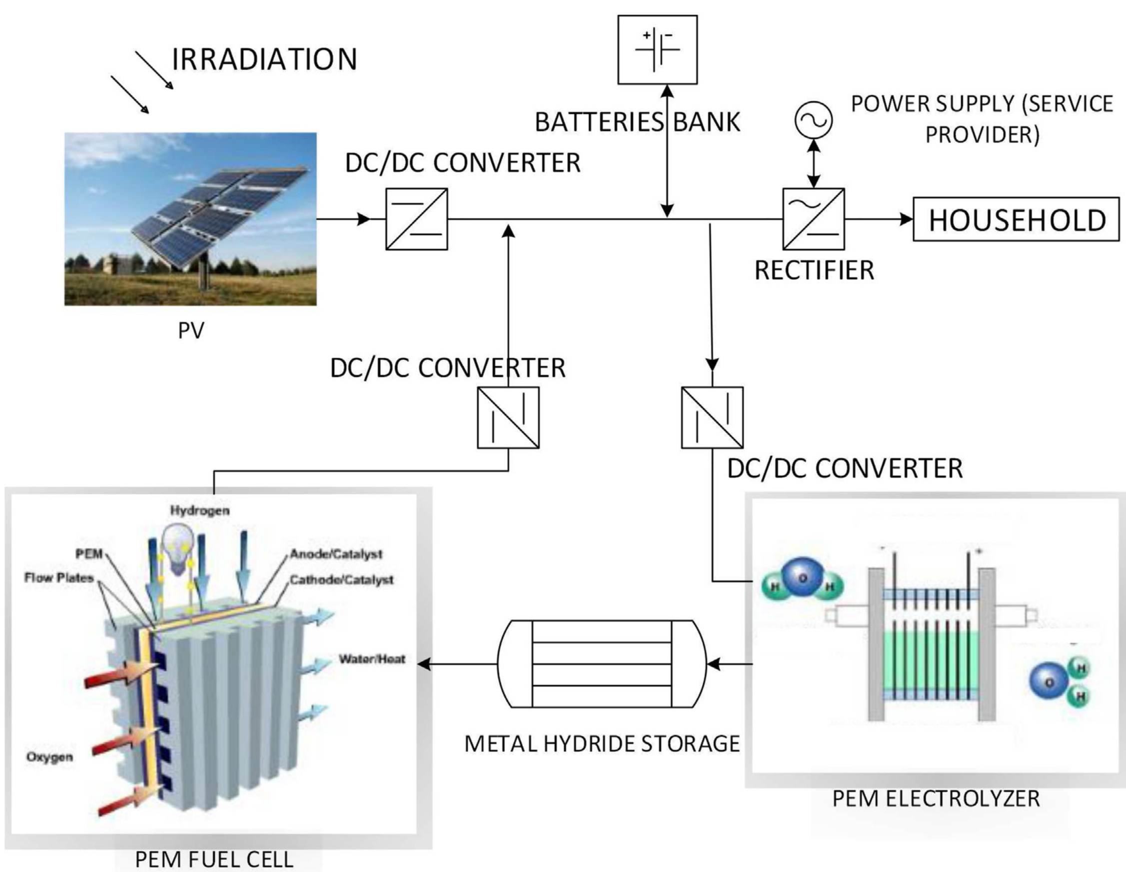

Fig. 2. Nano-grid scheme.

respectively of the power generated in nano-grid 1. In order to present different behavior for every nanogrid we propose that the storage systems of the nanogrid 2 are a 30% lower than in the first nanogrid and in the case of the nanogrid 3, the storage systems are a 20% higher than in nanogrid 1.

The power sent to the load by the three nano-grids must satisfy the local demand, which leads to the following algebraic equation

$$\sum_{i=1}^{3} P^i_{load}(k) = \overline{P}_{load}(k) + \tilde{P}_{load}(k) \tag{12}$$

where $\overline{P}_{load}$,$\tilde{P}_{load}$ are the predicted value and the prediction error respectively of the local load.

The corresponding matrices of the whole micro-grid are the following:

$$B_u = \begin{bmatrix} bu & 0 & 0 \\ 0 & 1.3bu & 0 \\ 0 & 0 & 0.8bu \end{bmatrix} \quad bu = \begin{bmatrix} 7.6285 & 5.5847 & 5.5847 \\ -3.4495 & 0 & 0 \end{bmatrix}$$
$$B_d = B_w \begin{bmatrix} bd & 0 & 0 & 0 \\ 0 & 1.3bd & 0 & 0 \\ 0 & 0 & 0.8bd & 0 \end{bmatrix} \quad bd \begin{bmatrix} -5.5847 \\ 0 \end{bmatrix}$$

where

$$x = \left[SOC^1 \;\; MHL^1 \;\; SOC^2 \;\; MHL^2 \;\; SOC^3 \;\; MHL^3\right]^T$$

$$u = \left[P^1_{H2} \;\; P^1_{grid} \;\; P^1_{load} \;\; P^2_{H2} \;\; P^2_{grid} \;\; P^2_{load} \;\; P^3_{H2} \;\; P^3_{grid} \;\; P^3_{load}\right]^T$$

$$d = \left[\overline{P}^1_{PV} \;\; \overline{P}^2_{PV} \;\; \overline{P}^3_{PV} \;\; \overline{P}_{load}\right]^T$$

$$w = \left[\tilde{P}^1_{PV} \;\; \tilde{P}^2_{PV} \;\; \tilde{P}^3_{PV} \;\; \tilde{P}_{load}\right]^T$$

Constraint (12) can be modeled with the following matrices:

$$E_u = [0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 1]$$

$$E_d = E_w = [0 \;\; 0 \;\; 0 \;\; -1]$$

$$E_x = 0$$

We consider the following technological constraints on the power and energy storage systems that define the set $\mathscr{Z}$

$$-0.9kW \le P^i_{H_2} \le 0.9kW$$

$$-2.5kW \le P^i_{grid} \le 2kW$$

$$-0.2kW \le \overline{P}^i_{load} \le 2kW$$

$$45\% \le SOC^i \le 90\%$$

$$20\% \le MHL^i \le 90\%$$

The prediction errors in the load $\tilde{P}^i_{load}$ and energy generation $\tilde{P}^i_{PV}$, with $i = 1, 2, 3$ are assumed to be lower than 0.3094, 0.3279, 0.3752 and 0.34 kW respectively.

4.1. Economic cost function

The main control objectives can be described as follows:

(i) Maximize the profit of the energy exchange between the micro-grid and the EU taking into account the prices of the intraday electricity market, the contract constraints, generation disturbances and distribution of the charge for each nano-grid.
(ii) **Fulfill** a real demand described by a predicted periodic term denoted as $\overline{P}_{load}$ with a prediction error denoted as $\tilde{P}_{load}$.
(iii) **Fulfill** the operational constraint in order to prevent damage the equipments.

Taking into account these objectives, the economic cost function is composed by the following terms:

$$l^1 = c_1(k)\left|P_{of}(k) - \sum_{i=1}^{3} P^i_{grid}(k)\right|^2$$
$$l^2 = c_2 \sum_{i=1}^{3} \left|SOC^{ref} - SOC^i(k)\right|^2 + \left|MHL^{ref} - MHL^i(k)\right|^2$$
$$l^3 = c_3 \sum_{i=1}^{3} \left|P^i_{H2}(k)\right|^2 + \left|P^i_{grid}(k)\right|^2 + \left|P^i_{load}(k)\right|^2$$

The first term l^1 tries to maximize the profit of the energy exchange between the micro-grid and the EU. To this end, it penalizes the square of the deviation from the agreed power reference $P_{of}(k)$. The parameter $c_1(k)$ varies periodically with time, depending on the prices of the intraday electricity market, to take into account that in general is more profitable to satisfy the agreed power when the prices are higher. The second term l^2 tries to extend the life time of the storage systems of the nano-grids by penalizing the deviation from a desired operation point. The third term l^3 tries to reduce the amount of power exchange by each nano-grid by minimizing the powers. The weights c_1, c_2 and c_3 can be tuned to change the relative relevance of each term. In the simulations, we consider two different set of daily values, defined in Table 1, for the parameters of the cost function to demonstrate that a sudden change in the economic cost function does not affect the stability and constraint satisfaction properties of the proposed controller.

4.2. Controller design

To design the proposed controller, a solution defined by matrices $M_v \in \mathrm{IR}^{9\times 4}$ and $M_w = \in \mathrm{IR}^{9\times 8}$ for the algebraic equation is obtained. Matrix M_v is an orthonormal basis for the null space of E_u obtained from the singular value decomposition. Matrix M_w provides a particular solution to the equation $E_w w(k) = 0$. There are infinite solutions to this equation. An inappropriate selection of matrix M_w may lead to optimization problems with a reduced

Table 1
Values of the parameters for the two different cost functions considered.

Parameters	Cost function 1			Cost function 2		
	0 h–8h	8 h–16 h	17 h–24 h	0 h–8h	8 h–16 h	17 h–24 h
c_1	10	100	10	10	100	10
c_2	0.05	0.05	0.05	0.05	0.05	0.05
c_3	40	40	40	200	200	200
SOC^{ref}	50	50	50	60	60	60
MHL^{ref}	50	50	50	60	60	60
P_{of}	0 W	1000 W	0 W	0 W	0 W	0 W

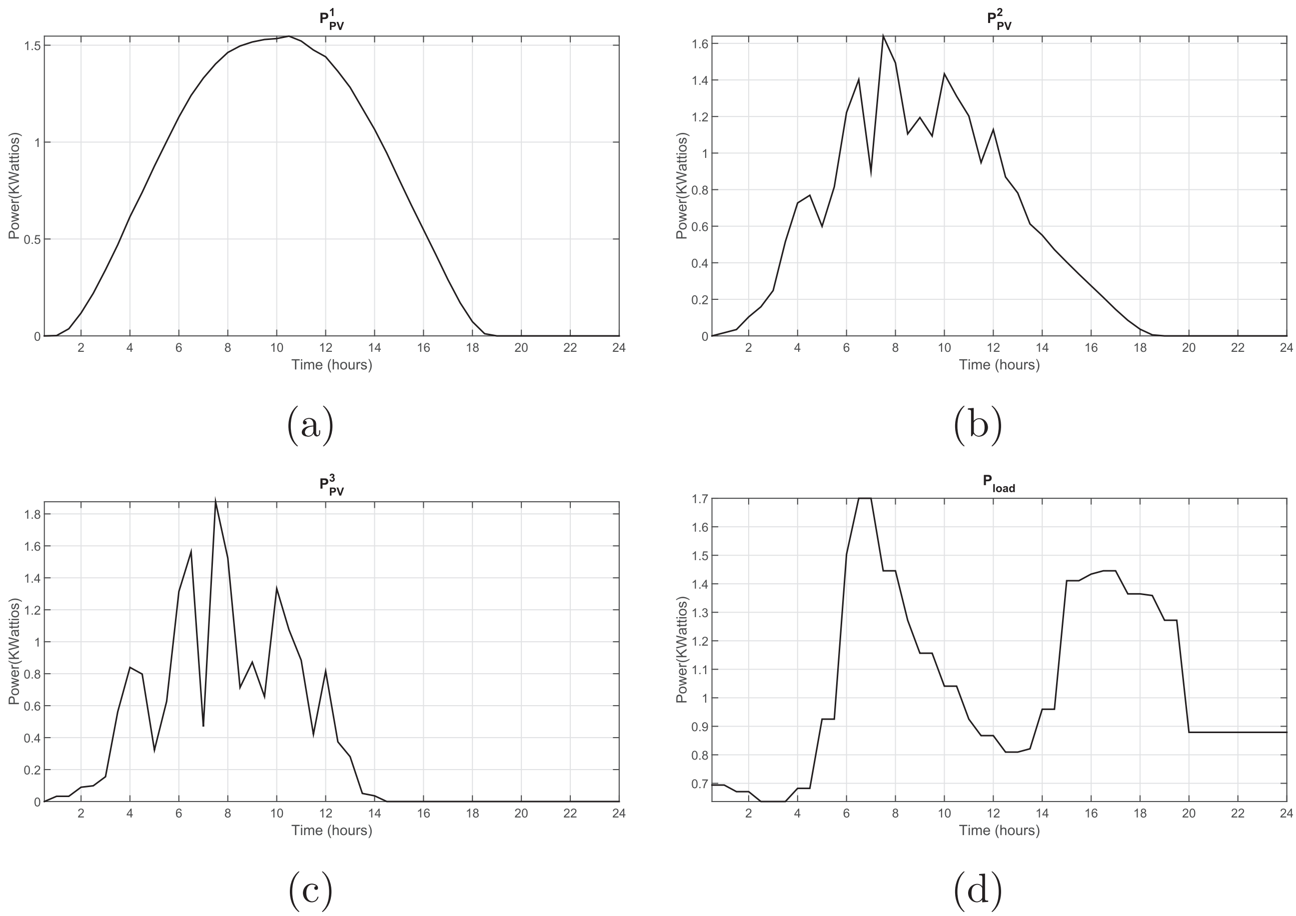

Fig. 3. (a) Predicted renewable generation profile 1; (b) Predicted renewable generation profile 2; (c) Predicted renewable generation profile 3; (d) Predicted local load profile.

feasibility region. In these simulations M_w is obtained from the solution to the equation $E_u u(k) + E_w w(k) = 0$ using the Moore-Penrose pseudo-inverse. We assume that the effect in the system produced by the predicted disturbances and its corresponding prediction error are the same and $E_d = E_w$. Therefore, the matrices M_w and M_d are equal.

To define the local controller $\widehat{K}$, a LQR control law has been designed based on the following weighting matrices Q_K and R_K that penalizes each state and input inversely to the difference between its maximum and minimum values.

$$\begin{aligned} Q_K &= diag\left(1 \Big/ \left(x_i^{max} - x_i^{min}\right)\right) \\ R_K &= 0.1 M_v^T diag\left(1 \Big/ \left(u_i^{max} - u_i^{min}\right)\right) M_v \end{aligned} \tag{13}$$

The control gain obtained using these weights satisfies the uncertainty rejection assumption and yields a MPC optimization problem with a nonempty feasibility region. In particular, the maximum eigenvalue of the matrix $(A + \widehat{B}K)^{47}$ is $0.1252 \cdot 10^{-23}$ thus σ is lower than the precision of the simulations carried out with Matlab.

4.3. Simulations results

To demonstrate the different properties of the proposed controller a ten day simulation in which three different prediction errors behaviors and two different economic cost functions have been taken into account has been carried out. The simulations have been made with Matlab 2014b using the solver *cplex* to solve the resulting QP optimization problem. The number of decision variables is 2208 because a simultaneous formulation was used in which the level of all storage systems, powers of each nano-grid and auxiliary control input for both the predicted and the artificial trajectories were included as decision variables. For this simulation, the initial level of energy in batteries and the metal hydride deposit of each nano-grid is $[74.9823\ 73.0028\ 74.8227\ 73.6751\ 75.4939\ 73.8573]^T$.

The renewable generation power profiles has been obtained using combinations of sunny/cloudy day profiles[1]. The expected values of the local load of the community micro-grid, $\overline{P}_{load}^i$, shown in Fig. 3(d), correspond to the standard daily demand of a house. These profiles are assumed to be periodic with a period of 24 h. All the profiles have been discretized using a sampling time of thirty minutes. Fig. 3(a,b,c) shows the profiles used in the simulations for expected values $\overline{P}_{PV}^i$.

The prediction errors of each signal are assumed to be lower than 20% of the maximal value of its daily evolution which are 0.3094, 0.3279, 0.3752, 0.34 KW respectively.

For the first 80 h the real prediction errors of load and the PV generation are constant and take the following value and sign

[1] These profiles are obtained from Ref. [23]. Special thanks to Valverde, L. and the energy engineering department of the University of Seville.

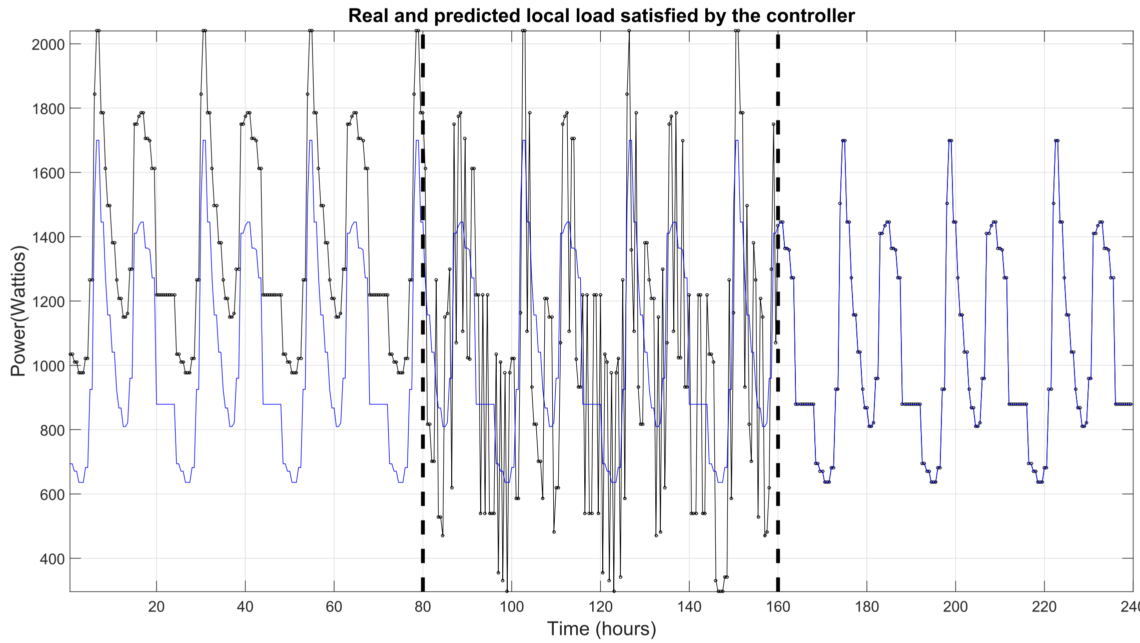

Fig. 4. Trajectory of predicted local demand (blue) and real local demand (black with circle). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$w^{s1}(k) = [-0.3094 \quad -0.3279 \quad -0.3752 \quad 0.34]^T, \forall k$$

This prediction error is the worst disturbance that can be applied to the control-loop system because in this case the real generation is decreased and the load is increased, thus the storage systems are emptied. The objective of the first stage is to demonstrate robust constraint satisfaction for the worst possible case. Note that we assume that the real generation power is always greater than or equal to zero. From the hour 80 to 160, the prediction errors switch randomly between the extreme values of the set $\mathscr{W}$, that is between $w^{s1}(k)$ and $-w^{s1}(k)$. The objective of this stage is to show a more realistic behavior of the controller. During this stage the parameters of the cost function change abruptly after 120 h. This transition shows that the constraints are satisfied even when the cost suddenly changes in a way shown in Table 1 where

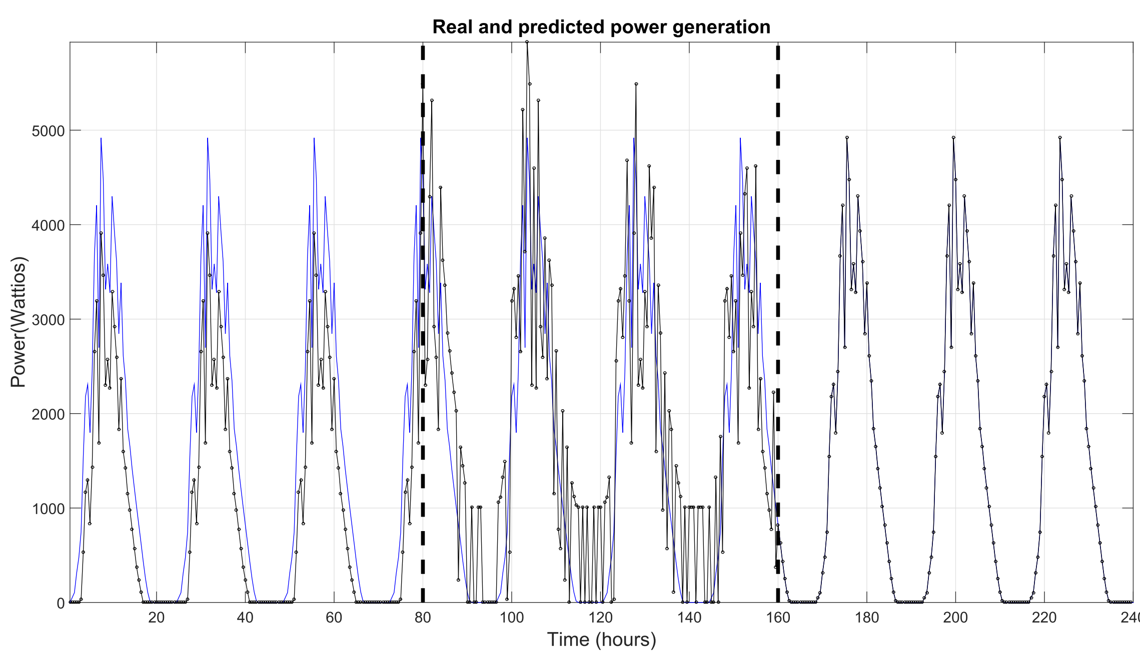

Fig. 5. Trajectory of predicted total generation (blue) and real total generation (black with circle). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

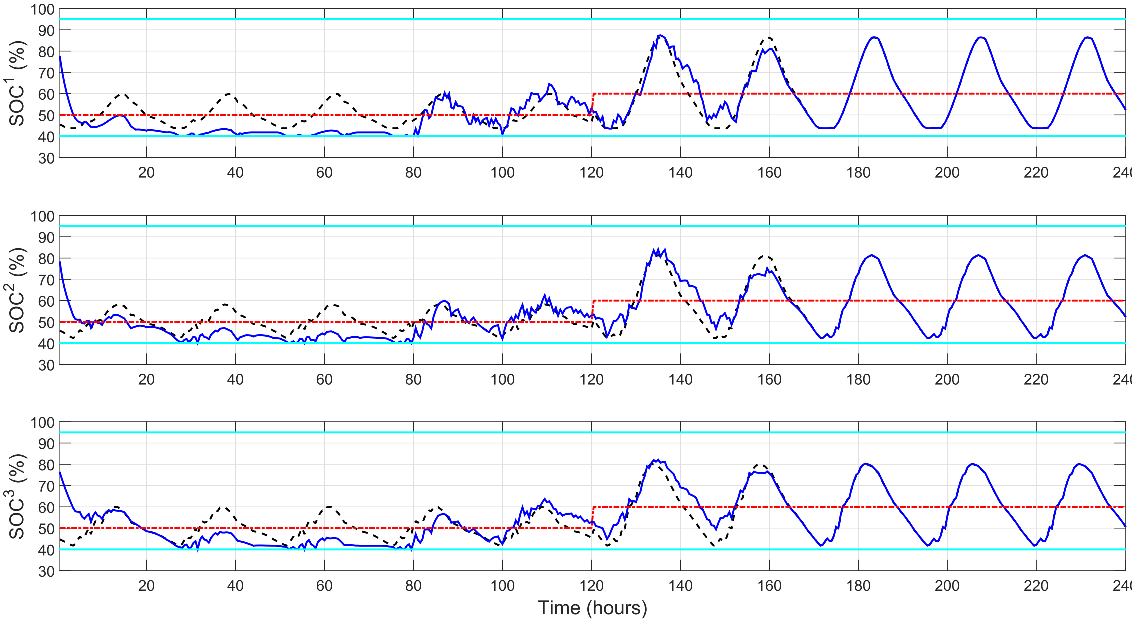

Fig. 6. Trajectory of the *SOC* of all nano-grids: robust controller with disturbances (blue), robust planner (black discontinuous), level constraints (cyan), desired operation point (red discontinuous). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the previous and new values of the cost function parameters are presented. From the hour 160 to the end of the simulation, the prediction error is assumed equal to zero showing the convergence of the robust controller closed-loop trajectories to the new optimal trajectory provided by the robust planner. The real and predicted total generation for each stage is shown in Fig. 5 and the real and predicted local load is shown in Fig. 4.

Figs. 6 and 7 show the trajectories of the energy stored in the batteries and the hydrogen systems of both the closed loop system (blue) and the robust planner (black with circle). In the first stage, the robust controller drives the closed-loop system as close as possible to limits (cyan) without violating the constraints but without reaching the trajectory provided by the robust planner. In the second stage, the robust controller drives the closed-loop system to a neighborhood of the robust planner trajectory. In addition, in this stage the economic cost function changes showing how the closed-loop system maintains recursive feasibility and stability even in this extreme situation. In the

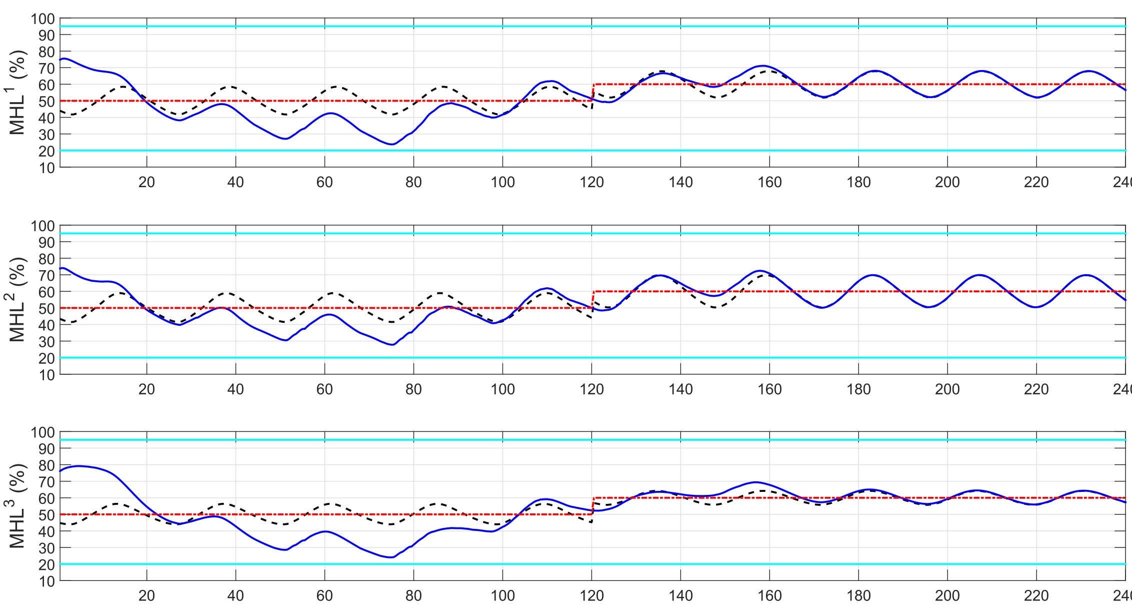

Fig. 7. Trajectory of the *MHL* of all nano-grids: robust controller with disturbances (blue), robust planner (black discontinuous), level constraints (cyan), desired operation point (red discontinuous). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

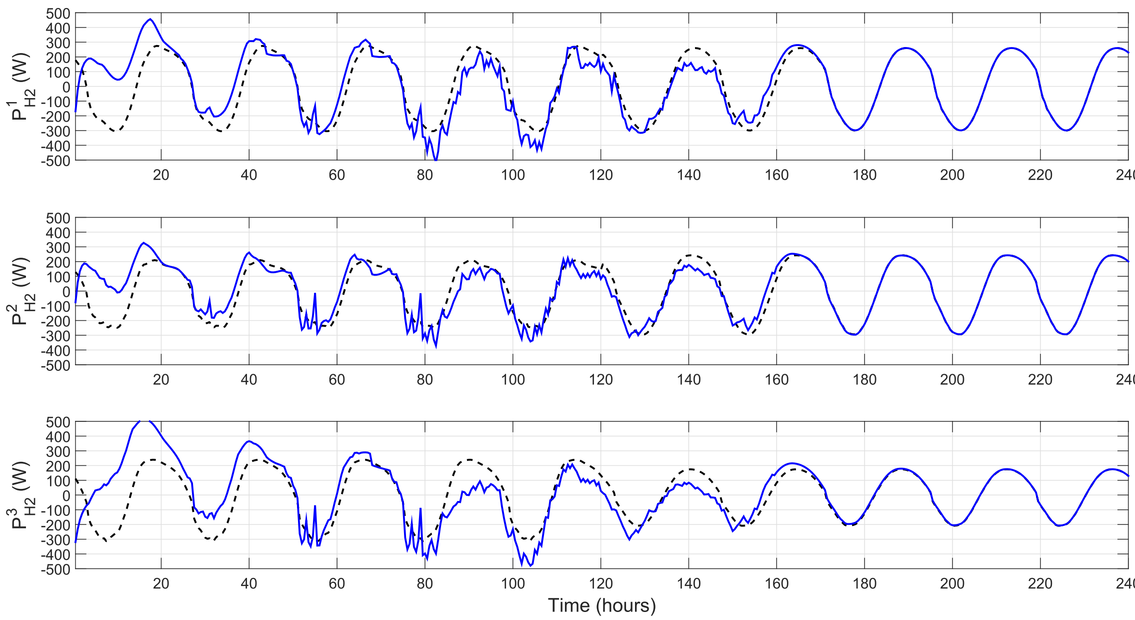

Fig. 8. Trajectories of Power P_{H2} of all nano-grids: robust planner (black discontinuous), robust controller (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

third stage, the robust controller converges to the trajectory provided by the robust planner (black discontinuous), which is the best trajectory that the disturbed system can follow when the closed loop system is minimizing the proposed economic cost function without violating the constraints. The trajectory provided by the robust planner does not reach the constraint limits in order to guarantee robust constraint satisfaction in the presence of disturbances.

Figs. 8–10 shows the power trajectories of each nano-grid presenting similar results. Fig. 11 shows the evolution of the cost of the closed-loop system (blue) with the robust controller and how it changes after five days from the previous cost value (green discontinuous) to the new cost value (red discontinuous). Due to the sudden change in the economic cost function in the second stage, the trajectory of the robust planner is different for both economic cost function. Their corresponding costs are $3.3824 \cdot 10^3$ and $5.6825 \cdot 10^4$ respectively. The simulation shows that the robust MPC optimization cost converges to a

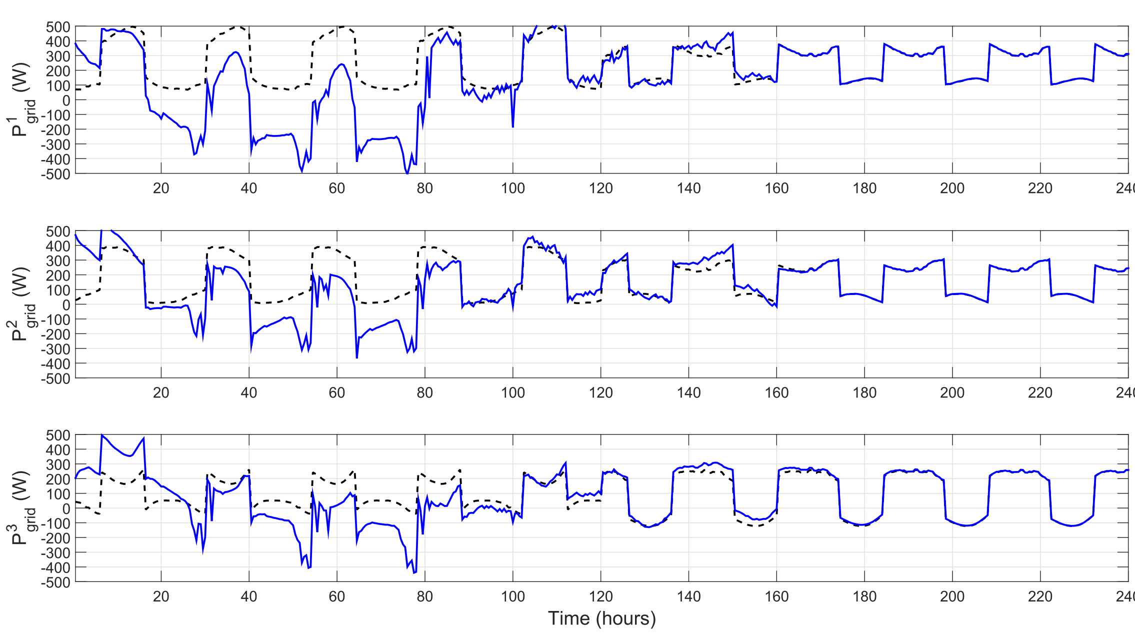

Fig. 9. Trajectories of Power P_{grid} of all nano-grids: robust planner (black discontinuous), robust controller (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

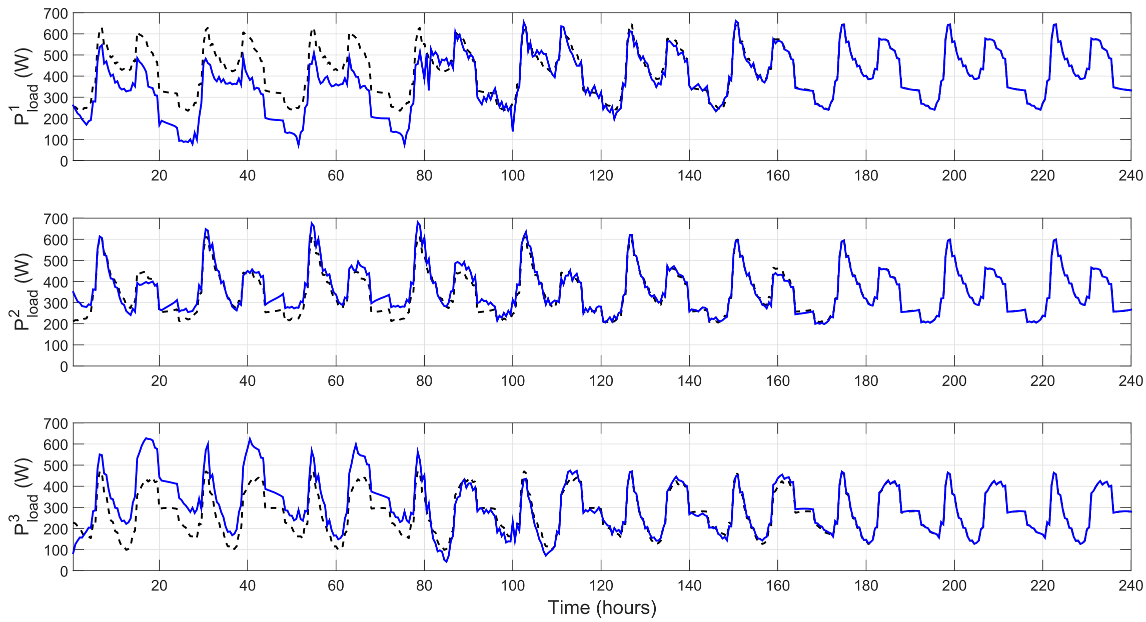

Fig. 10. Trajectories of Power P_{load} of all nano-grids: robust planner (black discontinuous), robust controller (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

neighborhood of the cost of the original trajectory, and then changes suddenly to the new optimal cost without loosing feasibility or violating any constraints and without the necessity to calculate any robust invariant set.

Fig. 12 shows the total P_{grid}, the desired P_{of} and the parameter $c_1(k)$ trajectories. This figure shows how the total P_{grid} tries to follow the desired offered power P_{of} taking into account that the penalty for not fulfilling the contract varies with time as shown by the cost parameter $c_1(k)$. The change in this cost shows how the controller adapts its operation cycle to the economic cost.

5. Conclusions

In this paper, a novel robust economic MPC for periodic signals is applied to uncertain discrete time algebraic-differential linear model. The control objective is to optimize the periodic behavior of

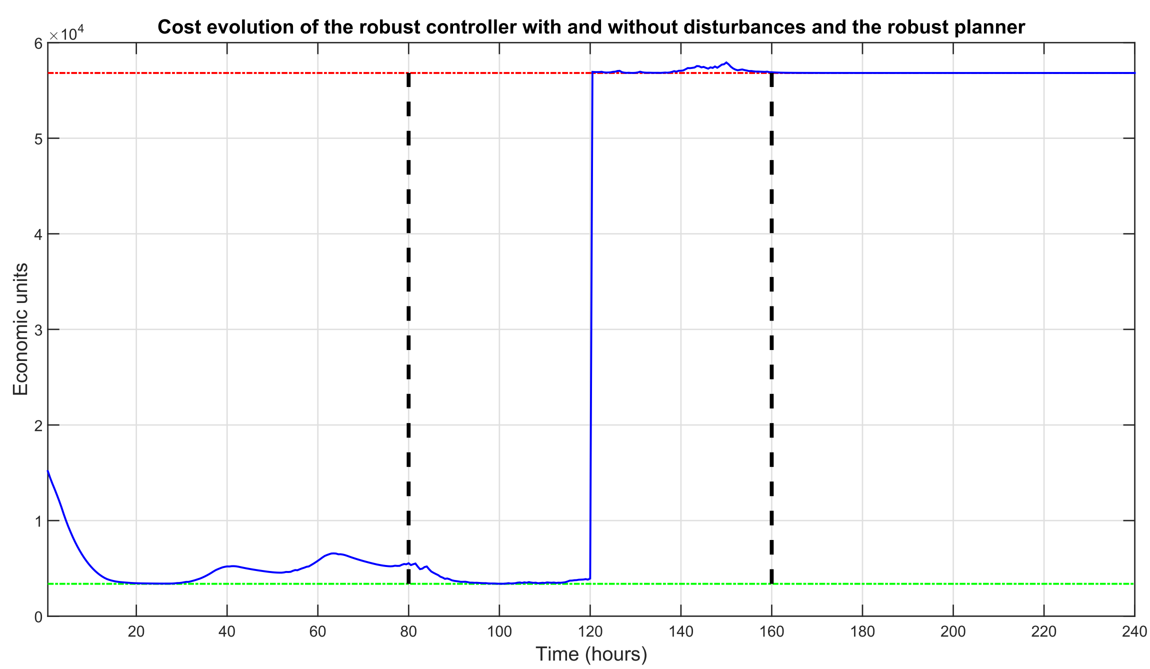

Fig. 11. Trajectory of the optimal cost of the robust controller without disturbances (blue discontinuous), robust controller (blue) and the robust planners 1 and 2 (green/red discontinuous). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

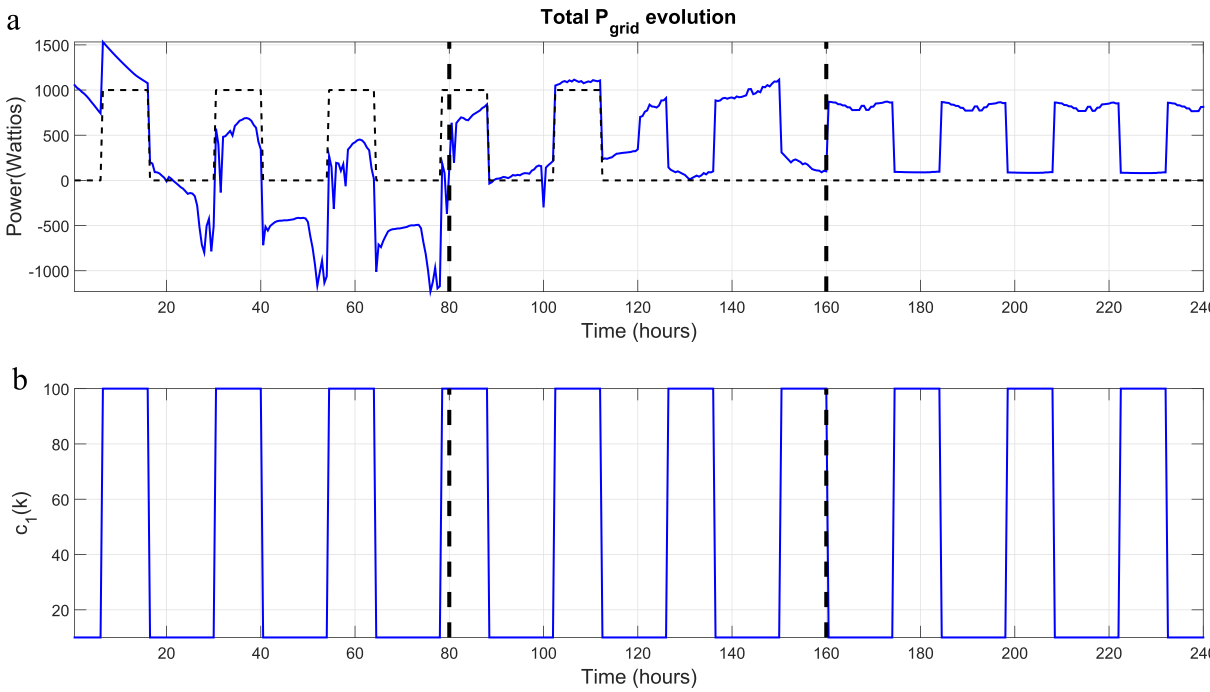

Fig. 12. (a) P_{grid} (blue continuous) and P_{of} (black discontinuous) evolution; (b) Trajectory of parameter $c_1(k)$ (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

this type of process while guaranteeing robust constraint satisfaction. The proposed controller provides closed-loop robust constraint satisfaction even in the presence of sudden changes in the economic cost function and asymptotic convergence to the corresponding new optimal trajectory. The proposed controller is based on the solution of a single quadratic programming optimization problem. The controller is defined without the necessity of the computation of a robust positive invariant set. These features are very important in practical applications and make this controller an appropriate approach to control large scale systems.

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