Journal of Process Control 20 (2010) 506–516

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

MPC for tracking zone regions

A. Ferramosca [a,*], D. Limon [a], A.H. González [b,c], D. Odloak [c], E.F. Camacho [a]

[a] Departamento de Ingeniería de Sistemas y Automática, Universidad de Sevilla, Escuela Superior de Ingenieros, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain
[b] Institute of Technological Development for the Chemical Industry (INTEC), CONICET-Universidad Nacional del Litoral (UNL), Güemes 3450, (3000) Santa Fe, Argentina
[c] Department of Chemical Engineering, University of São Paulo, Av. Prof. Luciano Gaulberto, trv 3 380, 61548 São Paulo, Brazil

ARTICLE INFO

Article history:
Received 16 July 2009
Received in revised form 13 November 2009
Accepted 11 February 2010

Keywords:
Model predictive control
Tracking
Optimality
Zone control

ABSTRACT

This paper deals with the problem of tracking target sets using a model predictive control (MPC) law. Some MPC applications require a control strategy in which some system outputs are controlled within specified ranges or zones (zone control), while some other variables – possibly including input variables - are steered to fixed target or set-point. In real applications, this problem is often overcome by including and excluding an appropriate penalization for the output errors in the control cost function. In this way, throughout the continuous operation of the process, the control system keeps switching from one controller to another, and even if a stabilizing control law is developed for each of the control configurations, switching among stable controllers not necessarily produces a stable closed loop system. From a theoretical point of view, the control objective of this kind of problem can be seen as a target set (in the output space) instead of a target point, since inside the zones there are no preferences between one point or another. In this work, a stable MPC formulation for constrained linear systems, with several practical properties is developed for this scenario. The concept of distance from a point to a set is exploited to propose an additional cost term, which ensures both, recursive feasibility and local optimality. The performance of the proposed strategy is illustrated by simulation of an ill-conditioned distillation column.

1. Introduction

1.1. Set-interval control in processing plants

In modern processing plants, MPC controllers are usually implemented as part of a multilevel hierarchy of control functions [1,2]. At the intermediary levels of this control structure, the process unit optimizer computes an optimal economic steady state and passes this information to the MPC in a lower level for implementation. The role of the MPC is then to drive the plant to the most profitable operating condition, fulfilling the constraints and minimizing the dynamic error along the path. In many cases, however, the optimal economic steady state operating condition is not given by a point in the output space (fixed set-point), but is a region into which the output should lie most of the time. In general, based on operational requirements, process outputs can be classified into two broad categories: (1) set-point controlled, outputs to be controlled at a desired value, and (2) set-interval controlled, outputs to be controlled within a desired range. For instance, production rate and product quality may fall into the first category, whereas process variables, such as level, pressure, and temperature in different units/streams may fall into the second category. The reasons for using set-interval control in real applications may be several, and they are all related to the process degrees of freedom: (1) In some problems some inputs of a square system without degrees of freedom are desired to be steered to a specific steady state values (input set-points), and then to account for the lack of degrees of freedom, the use of output zone control arises naturally (for example, it could be desirable, by economic reasons, to drive feed-rate to its maximum). (2) In another class of problems, there are highly correlated outputs to be controlled, and there are not enough inputs to control them independently. Controlling the correlated outputs within zones or ranges is one solution for this kind of problem (for instance, controlling the dense and dilute phase temperatures on an FCC regenerator). (3) A third important class of zone control problems relates to using the surge capacity of tanks to smooth out the operation of a unit. In this case, it is desirable to let the level of the tank float between limits, as necessary, to buffer disturbances between sections of a plant. Conceptually, the output intervals are not output constraints, since they are steady state desired zones that can be transitorily disregarded, while the (dynamic) constraints must be respected at each time. In addition, the determination of the output intervals is related to the steady state operability of the process, and it is not a trivial problem. A special care should be taken about the compatibility between the available input set (given by the input constraints) and the desired output set (given by the output intervals). In [3,4], for instance, an operability

* Corresponding author. Tel.: +34 954 485379; fax: +34 954 487340.
E-mail address: ferramosca@cartuja.us.es (A. Ferramosca).

0959-1524/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jprocont.2010.02.005

index that quantify how much of the region of the desired outputs can be achieved using the available inputs, taking into account the expected disturbance set, is defined. As a result a methodology to obtain the tightest possible operable set of achievable output steady state is derived. Then, the operating control intervals should be subsets of these tightest intervals. In practice, however, the operators are not usually aware of these maximum zones and may select control zones that are not fully consistent with the maximum zones and the operating control zones may be fully or partly unreachable. The MPC controller has to be robust to this poor selection of the control zones.

1.2. Different formulations of the MPC for tracking

Model predictive control (MPC) is one of the most successful techniques of advanced control in the process industry. This is due to its control problem formulation, the natural usage of the model to predict the expected evolution of the plant, the optimal character of the solution and the explicit consideration of hard constraints in the optimization problem. Thanks to the recent developments of the underlying theoretical framework, MPC has become a mature control technique capable to provide controllers ensuring stability, robustness, constraint satisfaction and tractable computation for linear and for nonlinear systems [5]. The control law is calculated by predicting the evolution of the system and computing the admissible sequence of control inputs which makes the system evolves satisfying the constraints and minimizing the predicted cost. This problem can be posed as an optimization problem. To obtain a feedback policy, the obtained sequence of control inputs is applied in a receding horizon manner, solving the optimization problem at each sample time. Considering a suitable penalization of the terminal state and an additional terminal constraint, asymptotic stability and constraints satisfaction of the closed loop system can be proved [6].

Most of the MPC stability and feasibility results consider the regulation problem, that is steering the system to a fixed steady state (typically the origin). It is clear that for a given non-zero set-point, a suitable choice of the steady state can be chosen and the problem can be posed as a regulation problem translating the state and input of the system [7]. However, since the stabilizing choice of the terminal cost and constraints depends on the desired steady state, when the target operating point changes, the feasibility of the controller may be lost and the controller fails to track the reference [8–11], thus requiring to re-design the MPC at each change of the reference. The computational burden that the design of a stabilizing MPC requires may make this approach not viable. For such case, the steady state target can be determined by solving an optimization problem that determines the steady state and input targets. This target calculation can be formulated as different mathematical programs for the cases of perfect target tracking or non-square systems [12], or by solving a unique problem for both situations [13]. In [14], an MPC for tracking is proposed, which is able to steer the system to any admissible set-point in an admissible way. The main characteristics of this approach are: an artificial steady state is considered as a decision variable, a cost that penalizes the error between the predicted variables and the artificial steady state is minimized, an additional term that penalizes the deviation between the artificial steady state and the steady state target is added to the cost function (the so-called offset cost function), and an invariant set for tracking is defined as extended terminal set. This controller ensures both, recursive feasibility and convergence to the target (if admissible) for any change of the steady state target. Furthermore, if the target is not admissible, the system is steered to the closest admissible steady state. In [15], the MPC for tracking is extended considering a general offset cost function. Under some mild sufficient assumptions, the new offset cost function ensures the local optimality property, letting the controller achieve optimal closed loop performance.

1.3. Review of MPC controllers for set-interval control

From the point of view of the controller, several approaches have been developed to account for the set-interval control. Ref. [16], which represents an excellent survey paper of the existing industrial MPC technology, describes a variety of industrial controller and mentions that they always provide a zone control option. That paper presents two ways to implement zone control: (1) defining upper and lower soft constraints, and (2) using the set-point approximation of soft constraints to implement the upper and lower zone boundaries (the DMC-plus algorithm). One of the main problems of these industrial controllers (as was stated in the same paper) is the lack of nominal stability. A second example of zone control can be found in [17], where the authors exemplify the application of this strategy to a FCC system. Although this strategy has shown to have an acceptable performance, stability cannot be proved, even if an infinite horizon is used, since the control system keeps switching from one controller to another throughout the continuous operation of the process. A third example is the closed loop stable MPC controller presented in [18]. In this approach, the authors develop a controller that considers the zone control of the system outputs and incorporates steady state economic targets in the control cost function. Assuming open-loop stable systems, classical stability proofs are extended to the zone control strategy by considering the output set-points as additional decision variables of the control problem. Furthermore, a set of slack variables is included into the formulation to assure both, recursive feasibility of the on-line optimization problem and convergence of the system inputs to the targets. This controller, however, is formulated for stable open-loop stable systems, and since it considers a null controller as local controller, it does not achieve local optimality. An extension of this strategy to the robust case, considering multi-model uncertainty, was proposed in [19].

From a theoretic point of view, the control objective of the zone control problem can be seen as a target set (in the output space) instead of a target point, since inside the zones there are no preferences between one point and another. In this work, the controller proposed in [14,15], is extended to deal with the zone control, generalizing the conditions of the offset cost function to use a distance to a convex target set. This controller ensures recursive feasibility and convergence to the target set for any stabilizable plant. This property holds for any class of convex target sets and also in the case of time-varying target sets. For the case of polyhedral target sets, several formulations of the controller are proposed that allows to derive the control law from the solution of a single quadratic programming problem. One of these formulations allows also to consider target points and target sets simultaneously in such a way that the controller steers the plant to the target point if reachable while it steers the plant to the target set in the other case. Finally, it is worth to remark that the proposed controller inherits the properties of the controller proposed in [14,15].

This paper is organized as follows: in the following section the constrained tracking problem is stated. In Section 3 the new MPC for tracking is introduced and in Section 5 its implementation is presented. Finally an illustrative example is shown and some conclusions are drawn.

Notation and basic definitions: vector (a, b) denotes $[a^T, b^T]^T$; for a given λ and a given set X, $\lambda X \triangleq \{\lambda x : x \in X\}$; $int(X)$ denotes the interior of set X; a matrix T definite positive is denoted as $T > 0$ and $T > P$ denotes that $T - P > 0$. For a given symmetric matrix $P > 0$, $\|x\|_P$ denotes the weighted euclidean norm of x, i.e. $\|x\|_P \triangleq \sqrt{x^T P x}$. Matrix $\mathbf{0}_{n,m} \in \mathbb{R}^{n \times m}$ denotes a matrix of zeros. Matrix $\mathbf{I}_m \in \mathbb{R}^{m \times m}$ denotes the identity matrix. Consider $a \in \mathbb{R}^{n_a}, b \in \mathbb{R}^{n_b}$, and set $\Gamma \subset \mathbb{R}^{n_a+n_b}$, then projection operation is defined as $Proj_a(\Gamma) \triangleq \{a \in \mathbb{R}^{n_a} : \exists b \in \mathbb{R}^{n_b}, (a,b) \in \Gamma\}$. Vector $\mathbf{u}(p)$ denotes the sequence of control action $\mathbf{u}(p) \triangleq \{u(0;p), u(1;p), \ldots\}$, where p is a parameter. Given a subdifferential function $V_O(x)$, notation $\partial V_O(x)$ defines the subdifferential of $V_O(x)$ [20]. Set $\Omega^w_{t,K}$ is the invariant set for tracking, considered in the augmented state (x,θ). $P^Z_N(x,\Gamma_t)$ is the optimization problem for the zone region tracking problem for an horizon of length N, in the set of parameters (x,Γ_t). Γ_t is the target zone.

2. Problem statement

Let a discrete-time linear system be described by:

$$\begin{aligned} x^+ &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the current state of the system, $u \in \mathbb{R}^m$ is the current input, $y \in \mathbb{R}^p$ is the controlled output and x^+ is the successor state. Note that no assumption is considered on the dimension of the states, inputs and outputs and hence non-square systems (namely $p > m$ or $p < m$) might be considered.

The controlled output is the variable used to define the target to be tracked by the controller. Since no assumption is made on matrices C and D, the outputs might be (a linear combination of) the states, (a linear combination of) the inputs or (a linear combination of) both.

The state of the system and the control input applied at sampling time k are denoted as $x(k)$ and $u(k)$ respectively. The system is subject to hard constraints on state and control:

$$(x(k), u(k)) \in \mathcal{Z} \tag{2}$$

for all $k \geqslant 0$. $\mathcal{Z} \subset \mathbb{R}^{n+m}$ is a compact convex polyhedron containing the origin in its interior.

Assumption 1. The pair (A,B) is controllable and the state is measured at each sampling time.

Under this assumption, the set of steady states and inputs of the system (1) is a m-dimensional linear subspace of $\mathbb{R}^{n+m}$ [11] given by

$$(x_s, u_s) = M_\theta \theta$$

where M_θ is a full column rank matrix such that:

$$[A - I \quad B]M_\theta = 0$$

Every pair of steady state and input $(x_s, u_s) \in \mathbb{R}^{n+m}$ is characterized by a given parameter $\theta \in \mathbb{R}^m$. The steady controlled outputs are given by

$$y_s = N_\theta \theta$$

where $N_\theta = [C D]M_\theta$.

The problem we consider is the design of an MPC controller $\kappa^Z_N(x,\Gamma_t)$ such that for a given (possibly time-varying) convex target set (zone region) Γ_t it steers the outputs of system to a steady value contained into the target region satisfying the constraints $(x(k), \kappa^Z_N(x(k),\Gamma_t)) \in \mathcal{Z}$ throughout its evolution.

3. MPC for tracking zone regions

In what follows, an extension of the MPC for tracking [14,15] to the case of target sets is presented. In particular, in [15] the controller is formulated considering a generalized offset cost function. The way this controller handles the tracking problem is characterized by (i) considering an artificial steady state and input as decision variables, (ii) penalizing the deviation of the predicted trajectory with the artificial steady conditions, (iii) adding an offset cost function to penalize the deviation between the artificial and the target equilibrium point and (iv) considering an extended terminal constraint. If the target operating point is an admissible steady state, the closed loop system evolves to this target state without offset. If the target operating point is not consistent with the linear model considered for predictions, namely, it is not an admissible steady state of system (1), the closed loop system evolves to an admissible steady state which minimizes a given performance index.

In this paper, this controller is extended to the case of considering a zone control strategy. To this aim, consider that the target set for the output is a given polyhedron, Γ_t. The cost function of the MPC proposed is, hence, given by:

$$V^Z_N(x,\Gamma_t;\mathbf{u},\bar\theta) \triangleq \sum_{i=0}^{N-1} \|\bar x(i) - \bar x_s\|^2_Q + \|\bar u(i) - \bar u_s\|^2_R + \|\bar x(N) - \bar x_s\|^2_P + V_O(\bar y_s,\Gamma_t) \tag{3}$$

where $\bar x(i)$ denotes the prediction of the state i-samples ahead, the pair $(\bar x_s, \bar u_s) = M_\theta \bar\theta$ is the artificial steady state and input and $\bar y_s = N_\theta \bar\theta$ the artificial output, all of them parameterized by $\bar\theta$; Γ_t is the zone in which the controlled variables have to be steered. The offset cost function $V_O(\bar y_s,\Gamma_t)$ is such that the following assumption is ensured.

Assumption 2.

1. Γ_t is a compact convex set.
2. $V_O(\bar y_s,\Gamma_t)$ is subdifferential and convex w.r.t. $\bar y_s$.
3. If $\bar y_s \in \Gamma_t$, then $V_O(\bar y_s,\Gamma_t) \geqslant 0$. Otherwise, $V_O(\bar y_s,\Gamma_t) > 0$.

Let $P^Z_N(x,\Gamma_t)$ be the optimization problem that defines the controller for tracking of the zone region for the system constrained by Z, with a horizon of length N and whose parameters are the actual state x and the target set Γ_t. This problem is defined as follows:

$$V^{Z*}_N(x,\Gamma_t) = \min_{\mathbf{u},\bar\theta} V^Z_N(x,\Gamma_t;\mathbf{u},\bar\theta) \tag{4a}$$

$$\text{s.t. } x(0) = x, \tag{4b}$$

$$x(j+1) = Ax(j) + Bu(j), \tag{4c}$$

$$(x(j), u(j)) \in \mathcal{Z}, \quad j = 0,\ldots,N-1 \tag{4d}$$

$$(\bar x_s, \bar u_s) = M_\theta \bar\theta, \tag{4e}$$

$$\bar y_s = N_\theta \bar\theta \tag{4f}$$

$$(x(N), \bar\theta) \in \Omega^w_{t,K} \tag{4g}$$

where $\Omega^w_{t,K}$ is the polyhedron that corresponds to the invariant set for tracking, with feedback controller K in the augmented state (x,θ). In what follows, the superscript $*$ will denote the optimal solutions of the optimization problem.

Considering the receding horizon policy, the control law is given by

$$\kappa^Z_N(x,\Gamma_t) \triangleq u^*(0;x,\Gamma_t)$$

where $u^*(0;x,\Gamma_t)$ is the first element of the control sequence $\mathbf{u}^*(x,\Gamma_t)$ which is the optimal solution of problem $P^Z_N(x,\Gamma_t)$. Since the set of constraints of $P^Z_N(x,\Gamma_t)$ does not depend on Γ_t, its feasibility region does not depend on the target region Γ_t. Then there exists a polyhedral region $\mathcal{X}_N \subseteq \mathbb{R}^n$ such that for all $x \in \mathcal{X}_N$, $P^Z_N(x,\Gamma_t)$ is feasible. This is the set of initial states that can be admissibly steered in N steps to the projection of $\Omega^w_{t,K}$ onto x.

Consider the following assumption on the controller parameters:

Assumption 3.

1. Let $R \in \mathbb{R}^{m\times m}$ be a positive semi-definite matrix and $Q \in \mathbb{R}^{n\times n}$ a positive semi-definite matrix such that the pair $(Q^{1/2}, A)$ is observable.

2. Let $K \in \mathbb{R}^{m\times n}$ be a stabilizing control gain such that $(A+BK)$ is Hurwitz.
3. Let $P \in \mathbb{R}^{n\times n}$ be a positive definite matrix such that:

$$(A+BK)^T P(A+BK) - P = -(Q+K^T RK)$$

4. Let $\Omega^w_{t,K} \subseteq \mathbb{R}^{n+m}$ be an admissible polyhedral invariant set for tracking for system (1) subject to (2), for a given gain K. That is, given the extended state $w=(x,\theta)$, for all $w \in \Omega^w_{t,K}$, then $w^+ = A_w w \in \Omega^w_{t,K}$, where A_w is the closed loop matrix given by

$$A_w = \begin{bmatrix} A+BK & BL \\ 0 & I_m \end{bmatrix}$$

and $L = [-K\, I_m]M_\theta$. See [14] for more details.

The set of admissible steady outputs consistent with the invariant set for tracking $\Omega^w_{t,K}$ is given:

$$\mathscr{Y}_s \triangleq \{y_s = N_\theta\theta : (x_s,u_s) = M_\theta\theta,\ \text{and}\ (x_s,\theta)\in\Omega^w_{t,K}\}$$

This set is potentially the set of all admissible outputs for system (1) subject to (2).

Taking into account the proposed conditions on the controller parameters, in the following theorem it is proved asymptotic stability and constraints satisfaction of the controlled system.

Theorem 1 (Stability). *Consider that Assumptions 1–3 hold and consider a given target operation zone Γ_t. Then for any feasible initial state $x_0 \in \mathscr{X}_N$, the system controlled by the proposed MPC controller $\kappa^Z_N(x,\Gamma_t)$ is stable, fulfils the constraints throughout the time evolution and, besides*

(i) *If $\Gamma_t \cap \mathscr{Y}_s \neq \emptyset$ then the closed loop system asymptotically converges to a steady output $y(\infty)\in\Gamma_t$.*
(ii) *If $\Gamma_t \cap \mathscr{Y}_s = \emptyset$, the closed loop system asymptotically converges to a steady output $y(\infty) = y_s^*$, such that*

$$y_s^* \triangleq \arg\min_{y_s\in\mathscr{Y}_s} V_O(y_s,\Gamma_t)$$

Proof. The first part of the proof is devoted to prove the feasibility of the controlled system, that is, $x(k+1)\in\mathscr{X}_N$, for all $x(k)\in\mathscr{X}_N$, and Γ_t. Consider the optimal solution of $P^Z_N(x(k),\Gamma_t)$, then the successor state is $x(k+1) = Ax(k) + B\kappa^Z_N(x(k),\Gamma_t)$. Define the following sequences:

$$\begin{aligned}\mathbf{u}(x(k+1),\Gamma_t) &\triangleq [u^*(1;x(k),\Gamma_t),\ldots,u^*(N-1;x(k),\Gamma_t),\\ &\qquad K(x^*(N-1;x(k),\Gamma_t) - \bar{x}_s^*(x(k),\Gamma_t)) + \bar{u}_s^*(x(k),\Gamma_t)]\\ &\qquad \times\bar{\theta}(x(k+1),\Gamma_t) \triangleq \bar{\theta}^*(x(k),\Gamma_t)\end{aligned}$$

Then, following a similar procedure to [14], it is proved that the pair $(\mathbf{u},\bar{\theta})$ is a feasible solution for the optimization problem $P^Z_N(x(k+1),\Gamma_t)$.

Convergence is derived by means of a similar argument to the Lasalle's principle. Consider the proposed feasible solution. Taking into account the properties of the feasible nominal trajectories for $x(k+1)$, the condition *(4)* of Assumption 3 and using standard procedures in MPC [6] it is possible to obtain:

$$\begin{aligned}\Delta V^Z_N(x(k),\Gamma_t) &= V^Z_N(x(k+1),\Gamma_t;\mathbf{u},\bar{\theta}) - V^{Z*}_N(x(k),\Gamma_t)\\ &\leqslant -\|x(k)-\bar{x}_s^*(x(k),\Gamma_t)\|^2_Q - \|u^*(0;x(k),\Gamma_t)\\ &\quad -\bar{u}_s^*(x(k),\Gamma_t)\|^2_R\\ &\leqslant\ -\|x(k)-\bar{x}_s^*(x(k),\Gamma_t)\|^2_Q\end{aligned}$$

By optimality, we have that $V^{Z*}_N(x(k+1),\Gamma_t) \leqslant V^Z_N(x(k+1),\Gamma_t;\mathbf{u},\bar{\theta})$ and then:

$$\begin{aligned}\Delta V^{Z*}_N(x(k),\Gamma_t) &= V^{Z*}_N(x(k+1),\Gamma_t) - V^{Z*}_N(x(k),\Gamma_t)\\ &\leqslant -\|x(k)-\bar{x}_s^*(x(k),\Gamma_t)\|^2_Q\end{aligned}$$

Taking into account that $(Q^{1/2},A)$ is observable, we have that the system evolves to an operating point $(\bar{x}_s^*,\bar{u}_s^*) = M_\theta\bar{\theta}^*$ such that $(\bar{x}_s^*,\bar{\theta}^*)\in\Omega^w_{t,K}$.

The proof will be finished proving that $\bar{y}_s^*$, and hence the couple $(\bar{x}_s^*,\bar{u}_s^*)$, is the minimizer of the offset cost function $V_O(\bar{y}_s,\Gamma_t)$.

This result is proved by contradiction. First, assume that x is such that $x=\bar{x}_s^*$ (i.e. $\|x-\bar{x}_s^*\|_Q = 0$). Then the optimal control sequence is given by $\mathbf{u}^* = [\bar{u}_s^*,\ldots,\bar{u}_s^*]$ and $y(\infty) = \bar{y}_s^*$. Define, then, the convex set of all possible optimal solutions:

$$\Upsilon \triangleq \{y_s : y_s = arg\min_{\bar{y}_s\in\mathscr{Y}_s} V_O(\bar{y}_s,\Gamma_t)\}$$

and consider that $\bar{y}_s^* \notin \Upsilon$. Then, there exists an optimal $\tilde{y}_s\in\Upsilon$, for which the optimal value of the offset cost function is less then the value given by $\bar{y}_s^*$, i.e. $V_O(\tilde{y}_s,\Gamma_t) < V_O(\bar{y}_s^*,\Gamma_t)$. Define $\tilde{\theta}$ as the parameter (contained in the projection of $\Omega^w_{t,K}$ onto θ) such that $\tilde{y}_s = N_\theta\tilde{\theta}$.

It can be proved [11] that there exists a $\hat{\lambda}\in[0,1)$ such that for every $\lambda\in[\hat{\lambda},1)$, the convex combination $\hat{\theta} = \lambda\bar{\theta}^* + (1-\lambda)\tilde{\theta}$ is such that the control law $u = Kx + L\hat{\theta}$ (with $L = [-K, I_m]M_\theta$) steers the system from $\bar{x}_s^*$ to $\hat{x}_s$ fulfilling the constraints.

If $\mathbf{u}$ is defined as the sequence of control actions derived from the control law $u = K(x-\hat{x}_s)+\hat{u}_s$, then $(\mathbf{u},\bar{\theta})$ is a feasible solution of problem $P^Z_N(\bar{x}_s^*,\Gamma_t)$ [14]. From Assumption 3,

$$V^{Z*}_N(\bar{x}_s^*,\Gamma_t) \leqslant V^Z_N(\bar{x}_s^*,\Gamma_t;\mathbf{u},\hat{\theta}) = \|\bar{x}_s^*-\hat{x}_s\|^2_P + V_O(\hat{y}_s,\Gamma_t)$$

Consider now $V^Z_N(\bar{x}_s^*,\Gamma_t;\mathbf{u},\hat{\theta})$. It is clear that

$$\begin{aligned}V^Z_N(\bar{x}_s^*,\Gamma_t;\mathbf{u},\hat{\theta}) &= \|\bar{x}_s^*-\hat{x}_s\|^2_P + V_O(\hat{y}_s,\Gamma_t) = \|\bar{\theta}^*-\hat{\theta}\|^2_H + V_O(\hat{y}_s,\Gamma_t)\\ &= (1-\lambda)^2\|\bar{\theta}^*-\tilde{\theta}\|^2_H + V_O(\hat{y}_s,\Gamma_t)\end{aligned}$$

where $H = M_x^T P M_x$ and $M_x = [I_n, \mathbf{0}_{n,m}]M_\theta$. The partial of V^Z_N about λ is:

$$\frac{\partial V^Z_N}{\partial\lambda} = -2(1-\lambda)\|\bar{\theta}^*-\tilde{\theta}\|^2_H + g^T(\bar{y}_s^*-\tilde{y}_s)$$

where $g^T\in\partial V_O(\hat{y}_s,\Gamma_t)$ and $\partial V_O(\hat{y}_s,\Gamma_t)$ represents the subdifferential of $V_O(\hat{y}_s,\Gamma_t)$, [20]. If $\lambda = 1$,

$$\left.\frac{\partial V^Z_N}{\partial\lambda}\right|_{\lambda=1} = g^{*T}(\bar{y}_s^*-\tilde{y}_s)$$

where $g^{*T}\in\partial V_O(\bar{y}_s^*,\Gamma_t)$. Due to the convexity of V_O [20], we can state that

$$\left.\frac{\partial V^Z_N}{\partial\lambda}\right|_{\lambda=1} = g^{*T}(\bar{y}_s^*-\tilde{y}_s) \geqslant V_O(\bar{y}_s^*,\Gamma_t) - V_O(\tilde{y}_s,\Gamma_t)$$

The fact that $V_O(\bar{y}_s^*,\Gamma_t) - V_O(\tilde{y}_s,\Gamma_t) > 0$ implies that there exists a $\lambda\in[\hat{\lambda},1)$ such that $V^Z_N(\bar{x}_s^*,\Gamma_t;\mathbf{u},\hat{\theta})$ is smaller than the value that $V^Z_N(\bar{x}_s^*,\Gamma_t;\mathbf{u},\hat{\theta})$ assumes for $\lambda = 1$, which on the other hand, is exactly equal to $V^{Z*}_N(\bar{x}_s^*,\Gamma_t)$.

This fact contradicts the optimality of the solution, proving that $\bar{y}_s^*$, and hence the couple $(\bar{x}_s^*,\bar{u}_s^*)$, is the minimizer of the offset cost function $V_O(\bar{y}_s,\Gamma_t)$, proving the second assertion of the theorem. The first one is a direct consequence of the latter. □

4. Properties of the proposed controller

4.1. Steady-state optimization

In practice it is not unusual that the zones chosen as target sets are not fully consistent with the model and, thus, fully or partly unreachable. This may happen when no point in the zone is an admissible operating point for the system.

From the latter theorem it can be clearly seen that in this case, the proposed controller steers the system to the optimal operating point according to the offset cost function $V_O(\bar{y}_s,\Gamma_t)$. Then it can be considered that the proposed controller has a steady state optimizer built in and $V_O(\bar{y}_s, \Gamma_t)$ defines the function to optimize.

4.2. Feasibility for any reachable target zone

The controller is able to guarantee feasibility for any Γ_t and for any prediction horizon N. Then, it can be derived that the proposed controller is able to track any admissible target zone (i.e. $\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$) even for $N = 1$, if the system starts from an admissible equilibrium point. Nevertheless, a prediction horizon $N > 1$ is always a better choice, because, if on one hand a small prediction horizon reduces the computational effort, on the other hand the performances of the controller improve with N increasing.

4.3. Changing target zones

Taking into account Theorem 1, stability is proved for any offset cost function satisfying Assumption 2. Since the set of constraints of $P_N^Z(x, \Gamma_t)$ does not depend on Γ_t, its feasibility region does not depend on the target operating point Γ_t. Therefore, if Γ_t varies with the time, the results of the theorem still hold. This property will be shown in the example.

4.4. Input target

The zone control problem can be formulated considering input targets u_t that must satisfy some constraint (i.e. $u_{min} \leqslant u_t \leqslant u_{max}$) to allow the outputs to be inside of a certain zone [21]. These input targets are basically specific values for the inputs that are desirable to achieve for economic reasons. The proposed controller can be formulated considering input targets by defining an offset cost function $V_O(\bar{u}_s, \Gamma_{u,t})$ subdifferential and convex w.r.t. $\bar{u}_s$, where $\Gamma_{u,t}$ is a convex polyhedron.

Moreover, all the results and properties of the proposed controller remain valid because this case is equivalent to considering $C = 0$ and $D = I$.

4.5. Enlargement of the domain of attraction

The domain of attraction of the MPC is the set of states that can be admissible steered to $\Omega \triangleq Proj_x \Omega_{t,K}^w$ in N steps. The fact that this set is an invariant set for any equilibrium points makes this set (potentially) larger than the one calculated for regulation to a fixed equilibrium point. Consequently, the domain of attraction of the proposed controller is (potentially) larger than the domain of the standard MPC. This property is particularly interesting for small values of the control horizon.

4.6. Terminal constraint

The optimization problem $P_N^Z(x, \Gamma_t)$ can also be formulated by posing the terminal contraint as a terminal equality constraint, by considering $P = 0$ and $\Omega_{t,K}^w$ such that:

$$\Omega_{t,K}^w \triangleq \{(x, \theta) : M_\theta \theta \in \mathcal{Z}, \quad x = M_x \theta\}$$

4.7. Robustness and output feedback

It has been demonstrated that asymptotically stabilizing predictive control laws may exhibit zero-robustness, that is, any disturbance may make the controller to be unfeasible or the asymptotic stability property may not hold [22]. In this case, taking into account that the control law is derived from a multiparametric convex problem, the closed loop system is input-to-state stable for sufficiently small uncertainties [23].

This property is very interesting for an output feedback formulation [24], since it allows to ensure asymptotic stability for the control law based on the estimated state using an asymptotically stable observer. A robust formulation of the proposed controller can be obtained by extending the formulation presented in [25] for state feedback and [26] for output feedback. In this case, offset free control can be achieved by means of disturbances models [10] or adding an outer loop which manages the targets. [11].

4.8. Convexity of the optimization problem

Since all the ingredients (functions and sets) of the optimization problem $P_N^Z(x, \Gamma_t)$ are convex, then it derives that $P_N^Z(x, \Gamma_t)$ is a convex mathematical programming problem that can be efficiently solved in polynomial time by specialized algorithms [20].

5. Formulations of the MPC for tracking target sets leading to QP problems

Consider the target set Γ_t and define as y_t a specific point that belongs to the zone region, typically the center of the zone. As it has been stated in Theorem 1, in the problem of tracking a target set, three situations can be addressed.

(a) There not exists an admissible steady output in the zone, i.e. $\Gamma_t \cap \mathcal{Y}_s = \emptyset$.
(b) There exists an admissible steady state in the zone, but the desired output is not admissible, i.e. $\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$ and $y_t \notin \mathcal{Y}_s$.
(c) There exists an admissible steady state in the zone and the desired output is admissible, i.e. $\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$ and $y_t \in \mathcal{Y}_s$.

These three situations are shown in Fig. 1 where the double integrator system presented in [14] has been considered. This system is given by

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 & 0.5 \\ 1.0 & 0.5 \end{bmatrix}, \text{ and } \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

which is constrained to $\|x\|_\infty \leqslant 5$ and $\|u\|_\infty \leqslant 0.3$. In the picture, the domain of attraction $\mathcal{X}_N$ for $N = 3$, the invariant set for tracking $\Omega_{t,K} = Proj_x(\Omega_{t,K}^w)$, and the region of admissible steady state $\mathcal{X}_s$ are depicted respectively in blue, green and pink line. Notice that $\mathcal{X}_s \equiv \mathcal{Y}_s$, since $C = \mathbf{I}_2$. The three target set situations previous mentioned are represented by the three boxes labeled as (a), (b) and (c).

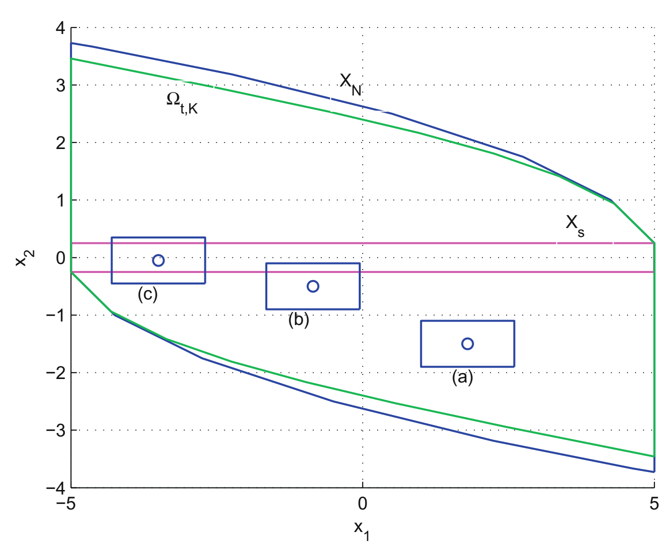


Fig. 1. Target sets for the double integrator system.

The center of each box, depicted as a circle, is the desirable target point into the zone region, y_t. In particular the zone region and the desirable target point for each case are:

(a) $\Gamma_t = \{1 \leqslant y_1 \leqslant 2.6, -1.9 \leqslant y_2 \leqslant -1.1\}$ and $y_t = (1.8, -1.5)$.
(b) $\Gamma_t = \{-1.65 \leqslant y_1 \leqslant -0.05, -0.9 \leqslant y_2 \leqslant -0.1\}$ and $y_t = (-0.85, -0.5)$.
(c) $\Gamma_t = \{-4.3 \leqslant y_1 \leqslant -2.7, -0.45 \leqslant y_2 \leqslant -0.35\}$ and $y_t = (-3.5, -0.05)$.

The controller presented in the paper will steer the system to that point which minimizes the offset cost function. This point can be a point belonging to $\mathcal{Y}_s$ (case (a)) or a point belonging to the intersection of $\mathcal{Y}_s$ with Γ_t (cases (b) and (c)). The controller implementation presented in Section 5.3, in the case (c), will steer the system exactly to the desired set-point y_t.

The optimization problem $P_N^Z(x, \Gamma_t)$ is a convex mathematical programming problem that can be efficiently solved by specialized algorithms [20]; fortunately this can be re-casted as a standard quadratic programming problem for a certain selection of the ingredients. To this aim, in this section, three different implementations of the MPC for tracking with target sets are presented, which ensures that the optimization problem can be formulated as a QP problem.

5.1. Distance from a set: ∞-norm

Consider that Γ_t is a set-interval zone defined as

$$\Gamma_t \triangleq \{y : y_{min} \leqslant y \leqslant y_{max}\}$$

Define as y_t the desirable target point into the zone region, typically the center of the zone.

In this implementation, the offset cost function is chosen as the distance from the y_s to the target region Γ_t, measured by a ∞-norm. Hence, the offset cost function $V_O(\bar{y}_s, \Gamma_t)$ is given by:

$$V_O(\bar{y}_s, \Gamma_t) \triangleq \min_{y \in \Gamma_t} \|\bar{y}_s - y\|_\infty$$

Consider the following lemma:

Lemma 1 [27]. *The set $\Xi \triangleq \{\bar{y}_s : \min_{y \in \Gamma_t} \|\bar{y}_s - y\|_\infty \leqslant \lambda\}$ is given by*

$$\begin{aligned} y + \lambda \mathbf{1} &\leqslant y_{max} \\ -y - \lambda \mathbf{1} &\leqslant -y_{min} \\ \lambda &\geqslant 0 \end{aligned}$$

where $\mathbf{1} \in \mathbb{R}^p$ is a vector of all unitary elements.

Thanks to this lemma, and considering the offset cost function in its epigraph form, the optimization problem $P_N^Z(x, \Gamma_t)$ can be posed as a standard quadratic programming problem, by adding a new decision variable λ, such that

$$V_O(y_s, \Gamma_t) \leqslant \lambda$$

Thanks to the previous statements, the cost function can be written in the form:

$$V_N^Z(x, \Gamma_t; \mathbf{u}, \bar{\theta}, \lambda) \triangleq \sum_{i=0}^{N-1} \|\bar{x}(i) - \bar{x}_s\|_Q^2 + \|\bar{u}(i) - \bar{u}_s\|_R^2 + \|\bar{x}(N) - \bar{x}_s\|_P^2 + \lambda$$

where λ is a new optimization variable, and the optimization problem $P_N^Z(x, \Gamma_t)$ is posed as:

$$\begin{aligned} V_N^{Z*}(x, \Gamma_t) = \min_{\mathbf{u}, \bar{\theta}, \lambda} & V_N^Z(x, \Gamma_t; \mathbf{u}, \bar{\theta}, \lambda) \\ \text{s.t.} \quad & (4b), (4c), (4d), (4e), (4f), (4g) \\ & \bar{y}_s + \lambda \mathbf{1} \leqslant y_{max} \\ & -\bar{y}_s - \lambda \mathbf{1} \leqslant -y_{min} \\ & \lambda \geqslant 0 \end{aligned}$$

which is a formulation of $P_N^Z(x, \Gamma_t)$ as a QP problem.

In Fig. 2 the trajectories for the double integrator system, from the initial state $x_0 = (-3, 2)$, for the three situations above mentioned, using a ∞-norm distances are plotted.

See how the controller steers the system to the point that minimize the ∞-norm distance. In particular, see that in cases (b) and (c) the system converges to a point inside the zone regions. The role of the ∞-norm is important in cases such (a). In this case, in fact, the system converges to one of those points that minimize the ∞-norm distance from the target region.

5.2. Distance from a set: 1-norm

Consider that Γ_t is a set-interval zone defined as

$$\Gamma_t \triangleq \{y : y_{min} \leqslant y \leqslant y_{max}\}$$

Define as y_t the desirable target point into the zone region, typically the center of the zone.

In this implementation, the offset cost function is chosen as the distance from y_s to the target region Γ_t, measured using a 1-norm:

$$V_O(\bar{y}_s, \Gamma_t) \triangleq \min_{y \in \Gamma_t} \|\bar{y}_s - y\|_1$$

As in the previous case, the optimization problem $P_N^Z(x, \Gamma_t)$ can be posed as a standard quadratic programming problem, by considering the offset cost function in its epigraph form $V_O(y_s, \Gamma_t) \leqslant \lambda$ and by resorting the following lemma.

Lemma 2 [27]. *The set $\Xi \triangleq \{\bar{y}_s : \min_{y \in \Gamma_t} \|\bar{y}_s - y\|_1 \leqslant \lambda\}$ is given by*

$$\begin{aligned} \mathbf{1}^T y + \lambda &\leqslant \mathbf{1}^T y_{max} \\ -\mathbf{1}^T y - \lambda &\leqslant -\mathbf{1}^T y_{min} \\ \lambda &\geqslant 0 \end{aligned}$$

The cost function to minimize is given by (5) and the optimization problem $P_N^Z(x, \Gamma_t)$ is given by:

$$\begin{aligned} V_N^{Z*}(x, \Gamma_t) = \min_{\mathbf{u}, \bar{\theta}, \lambda} & V_N^Z(x, \Gamma_t; \mathbf{u}, \bar{\theta}, \lambda) \\ \text{s.t.} \quad & (4b), (4c), (4d), (4e), (4f), (4g) \\ & \mathbf{1}^T \bar{y}_s + \lambda \leqslant \mathbf{1}^T y_{max} \\ & -\mathbf{1}^T \bar{y}_s - \lambda \leqslant -\mathbf{1}^T y_{min} \\ & \lambda \geqslant 0 \end{aligned}$$

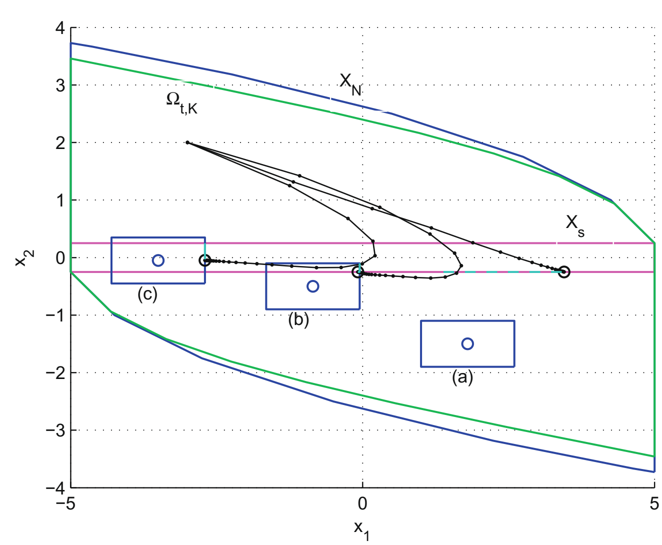


Fig. 2. The double integrator system: ∞-norm distance.

where λ is a new optimization variable, and which is a formulation of $P_N^Z(x, \Gamma_t)$ as a QP problem.

In Fig. 3 the trajectories for the double integrator system, from the initial state $x_0 = (-3, 2)$, for the three situations above mentioned, using a 1-norm distances are plotted.

See how the controller steers the system to the point that minimize the 1-norm distance. In particular, see that in cases (b) and (c) the system converges to a point inside the zone regions. The role of the norm is important in this case (a). In this case, the system converges to one of those points that minimize the 1-norm distance (see also Fig. 2).

5.3. Scaling factor

In this implementation, the target region is defined as

$$\Gamma_t \triangleq y_t \oplus \Sigma_t$$

where y_t is a desired target point and Σ_t is a polyhedron that defines the zone. The offset cost function $V_O(\bar{y}_s, \Gamma_t)$ is chosen as a kind of distance from y_s to the target region Γ_t, given by

$$\begin{aligned} V_O(\bar{y}_s, \Gamma_t) = \min_{\lambda, y} & \lambda \\ \text{s.t.} & \lambda \geqslant 0 \\ & y - y_t \in \lambda \Sigma_t \end{aligned}$$

This measure is such that, if $y \notin \Gamma_t$ then $\lambda > 1$, and if $y \in \Gamma_t$ then $\lambda \in [0, 1]$. In particular, if $y = y_t$, hence $\lambda = 0$. Therefore, λ has the double role of measuring the distance to a set and to a point.

In order to formulate the optimization problem as a QP, the cost function is chosen as in (5) and is minimized considering the following constraint:

$$\bar{y}_s - y_t \in \lambda \Sigma_t$$

with $\lambda \geqslant 0$. This means that $\bar{y}_s$ should remain in a zone that is an homothetic transformation of Γ_t centered in y_t.

Then, the optimization problem $P_N^Z(x, \Gamma_t)$ is given by:

$$\begin{aligned} V_N^{Z*}(x, \Gamma_t) = \min_{\mathbf{u}, \bar{\theta}, \lambda} & V_N^Z(x, \Gamma_t; \mathbf{u}, \bar{\theta}, \lambda) \\ \text{s.t.} \quad & (4b), (4c), (4d), (4e), (4f), (4g) \\ & \bar{y}_s - y_t \in \lambda \Sigma_t \\ & \lambda \geqslant 0 \end{aligned}$$

where λ is an optimization variable, and which is a formulation of $P_N^Z(x, \Gamma_t)$ as a QP problem. Notice that problem $P_N^Z(x, \Gamma_t)$ is a QP problem, for any Σ_t that is a convex polyhedron.

In Fig. 4 the trajectories for the double integrator system, from the initial state $x_0 = (-3, 2)$, for the three situations above mentioned, using the homothetic transformation method are plotted.

The zone regions are depicted in solid line while their homothetic transformation are depicted in dotted line. Notice that, when $y_t \in \Gamma_t \cap \mathcal{Y}_s$, the homothetic transformation of Γ_t is the target point y_t. See how the controller steers the system to the point that minimize the offset cost function w.r.t. the homothetic transformation.

6. Example

In this section, an example to test the performance of the proposed controller, is presented. The system adopted is an ill-conditioned distillation column (see Fig. 5).

This is a typical example in chemical industry in which, instead of output set-points, the system has output zones. The linear model for this plant, presented in [28,21], is given by:

$$\begin{bmatrix} y_D \\ x_B \end{bmatrix} = \frac{1}{75s + 1} \begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix}$$

The manipulated input variables are: L and V, the reflux and boil-up flow rates, respectively. The model is ill-conditioned, which implies that controlling the two outputs y_D and x_B independently is difficult, due to the strong interaction between them.

The objective of the controller is to maintain the system within some specified zones.

The system is constrained to $8 \leqslant L \leqslant 36.5, 7 \leqslant V \leqslant 37.5$. An MPC with $N = 3$ has been considered. The weighting matrices chosen for the set up of the controller are $Q = I_n$ and $R = I_m$.

The objective of the simulation is to show how the proposed controller manages a target set given by a combination of both, output set-points and output zones. To this aim, 4 changes of these target sets have been considered, that are in fact changes of the zones into which the outputs should be steered. In particular, in the first change of reference, we considered the case in which both target set and desirable set-point are not admissible ($\Gamma_t \cap \mathcal{Y}_s = \emptyset$ and $y_t \notin \mathcal{Y}_s$), while the case in which both target set and desirable set-point are admissible is considered in the other 3 changes ($\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$ and $y_t \in \mathcal{Y}_s$).

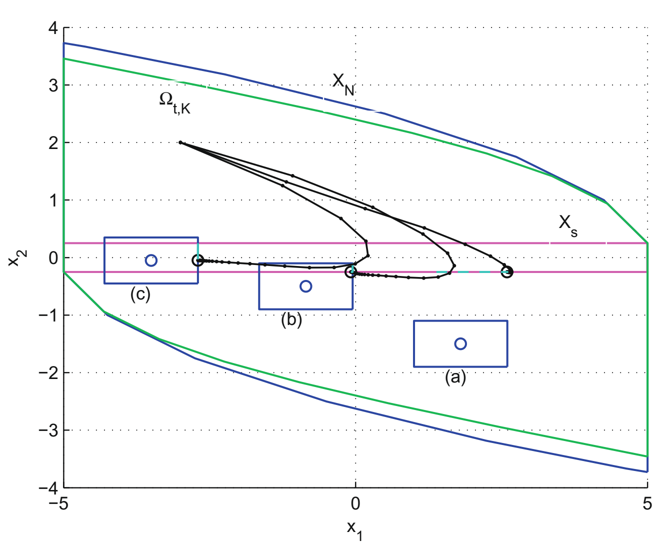


Fig. 3. The double integrator system: 1-norm distance.

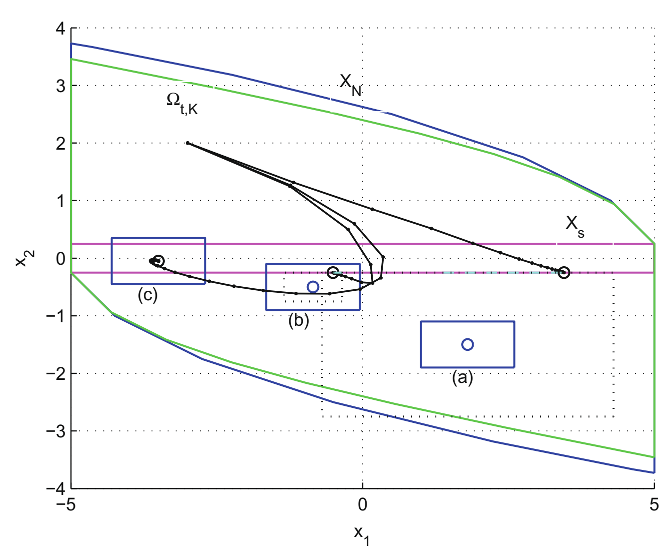


Fig. 4. The double integrator system: homothetic transformation.

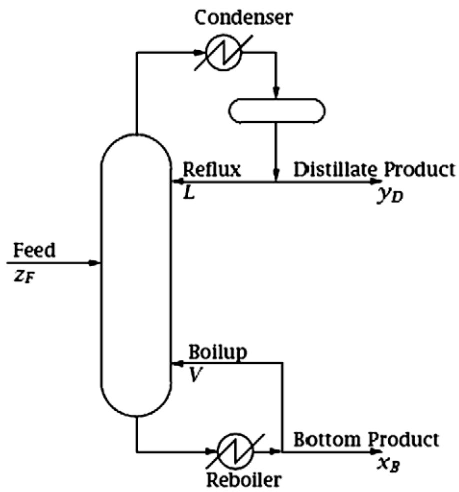


Fig. 5. Ill-conditioned distillation column.

It is convenient to remark that, inside the zones, there are no preferences between one point and another. The starting point is $y_0 = (0.1; 0.9)$. Moreover, an other objective of the example is to show the 3 different implementations of the controller, presented in Section 5.

6.1. Distance from a set: ∞-norm

In this section, the results of the simulations for the ∞-norm distance from a set implementation of the offset cost function are presented. Figs. 6 and 7 show the state-space evolution of the system. The domain of attraction $\mathcal{X}_N$ for $N = 3$, the invariant set for tracking $\Omega_{t,K} = Proj_x(\Omega^w_{t,K})$, and the region of admissible steady state $\mathcal{X}_s$ are depicted respectively in dashed–dotted, solid and dotted line. The zone regions are represented as boxes, and the desirable target points y_t, are represented as circles and considered as the center of the target zones. Fig. 7 is a zoom of Fig. 6. The 4 situations described in the introduction of this section, are labeled as (1)–(4). In Fig. 8 the time evolution of the outputs is depicted. The evolutions of the outputs and the artificial references are drawn respectively in solid and dashed line. The zones are drawn in thick-solid lines. See how the controller steers (whenever possible) the system into the output zone, even if the initial condition

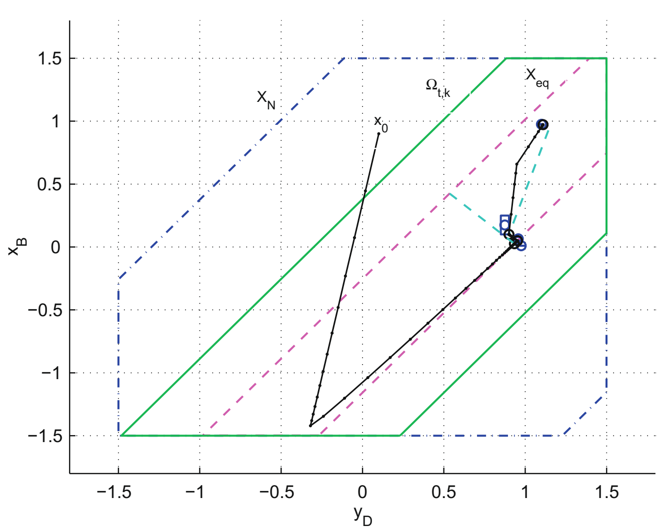


Fig. 6. Outputs evolution for the ∞-norm.

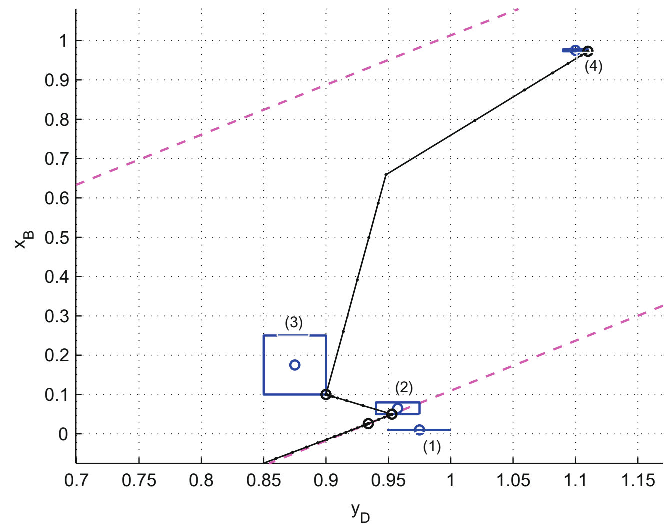


Fig. 7. Outputs evolution for the ∞-norm: zoom.

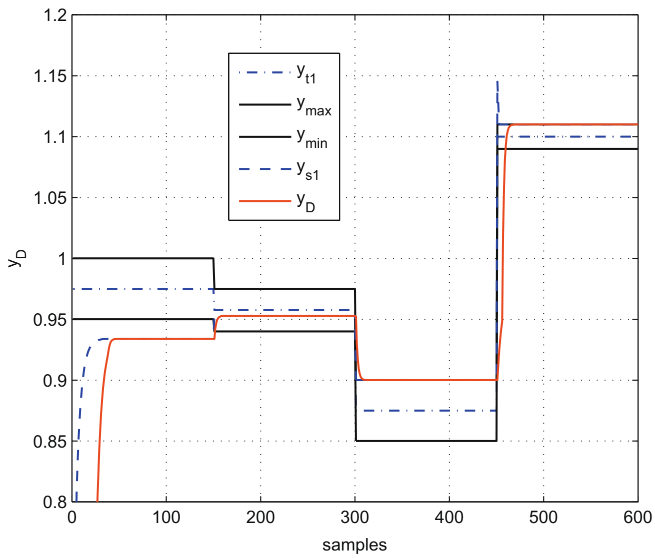
stays out of the zone. Furthermore, if the output zone is not admissible, that is $\Gamma_t \cap \mathcal{Y}_s = \emptyset$, the controller steers the system to the admissible point that minimizes the offset cost function. This can be seen in the first output zone change (from sample 0 to 150), in which the outputs y_D and x_B are steered to stationary value out of the corresponding zones. In the other cases, it can be seen that the controller steers y_D into the zone. This happens because $\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$ and $y_t \in \mathcal{Y}_s$. Furthermore, and despite it was not simulated, the proposed algorithm also allows the possibility to include input target, i.e., specific values for the inputs that are desirable to achieve for economic reasons.

6.2. Distance from a set: 1-norm

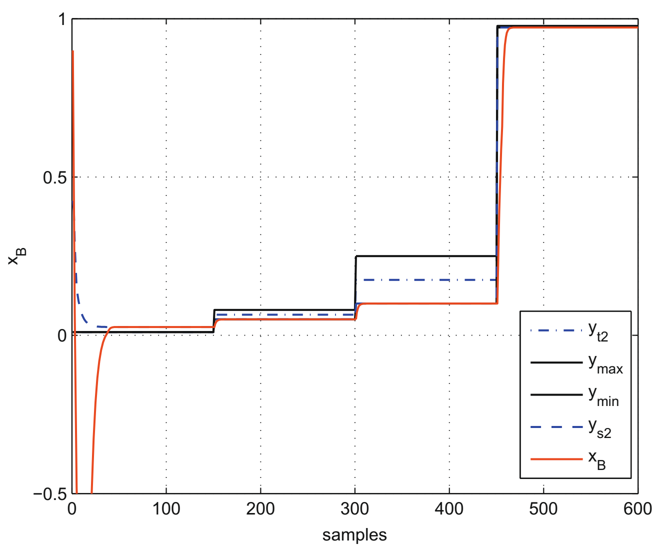
In this example, the controller is set-up for considering a 1-norm as offset cost function. The results of the simulations are presented in Figs. 9 and 10, which show the state-space evolution of the system, and in Fig. 11, which shows the time evolution of the outputs. In Figs. 9 and 10 the domain of attraction $\mathcal{X}_N$ for $N = 3$, the invariant set for tracking $\Omega_{t,K} = Proj_x(\Omega^w_{t,K})$, and the region of admissible steady state $\mathcal{X}_s$ are depicted respectively in dashed–dotted, solid and dotted line. The zone regions are represented as boxes, and the desirable target points y_t, are represented as circles and considered as the center of the target zones. Fig. 10 is a zoom of Fig. 9. The 4 situations described in the introduction of this section, are labeled as (1)–(4). In Fig. 11, the evolutions of the outputs and the artificial references are drawn respectively in solid and dashed line. The zones are drawn in thick-solid lines. As in the previous case, the controller steers (whenever possible) the system into the output zone, even if the initial condition lies out of the zone. In the first output zone change (from sample 0 to 150), it can be seen how output x_B is steered to a stationary value out of the corresponding zones, which is the one that minimizes the offset cost function. This happens because the target zone is not admissible ($\Gamma_t \cap \mathcal{Y}_s = \emptyset$). In the other cases, the controller steers y_D and x_B into the zone. This happens because $\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$ and $y_t \in \mathcal{Y}_s$. This formulation, as the previous one, can also cope with input targets.

6.3. Scaling factor

The last controller implementation proposed in Section 5, the scaling factor, is presented in this section. Figs. 12–14 show the re-



(a) Evolution of y_D.



(b) Evolution of x_B.

Fig. 8. Time evolutions of the outputs: ∞-norm.

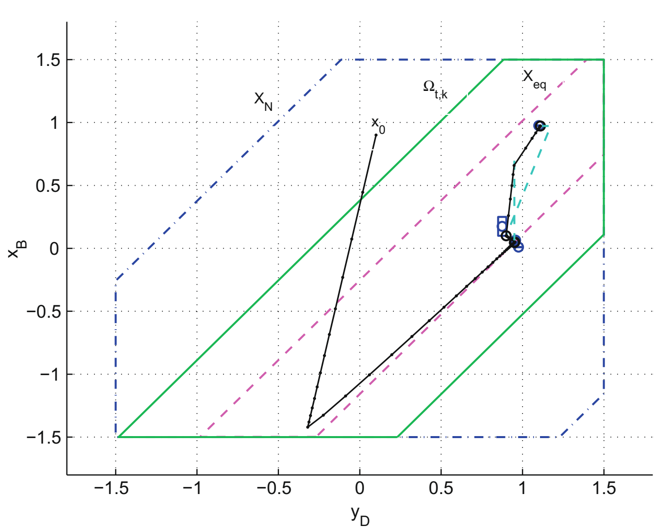


Fig. 9. Outputs evolution for the 1-norm.

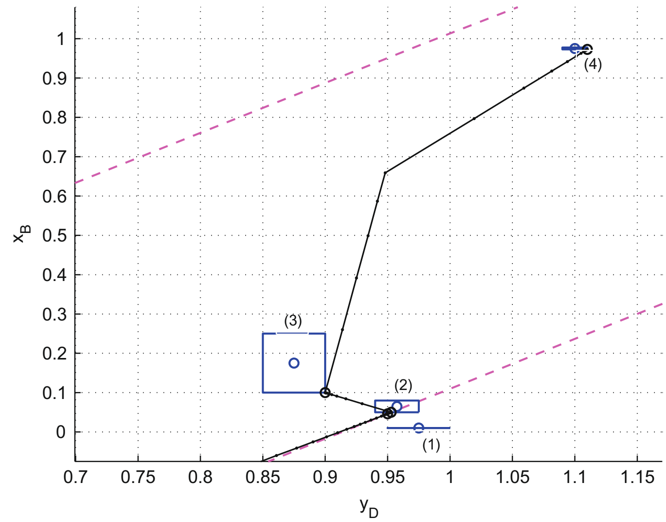
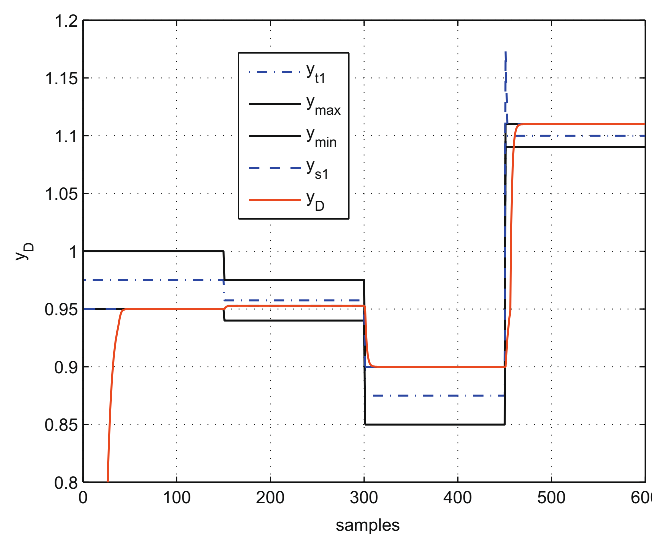
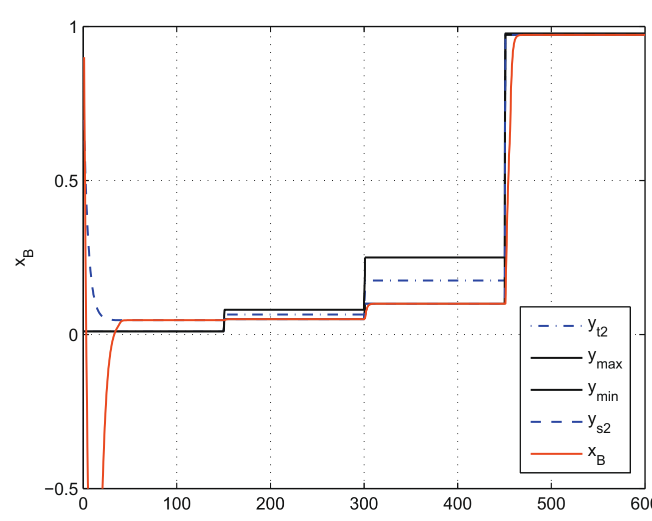


Fig. 10. Outputs evolution for the 1-norm: zoom.



(a) Evolution of y_D.



(b) Evolution of x_B.

Fig. 11. Time evolutions of the outputs: 1-norm.

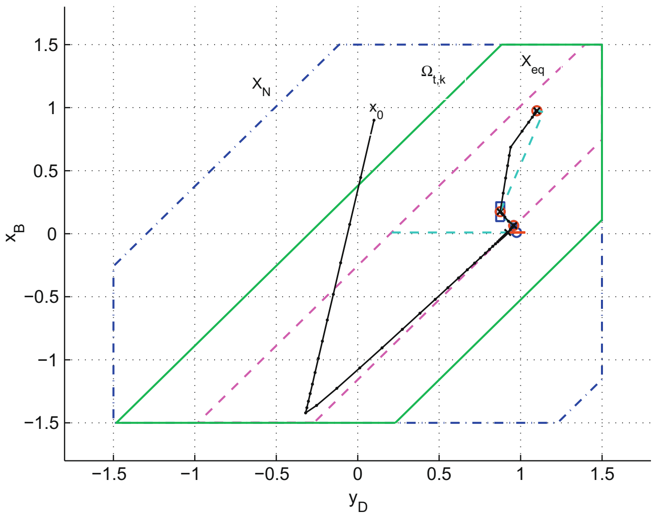
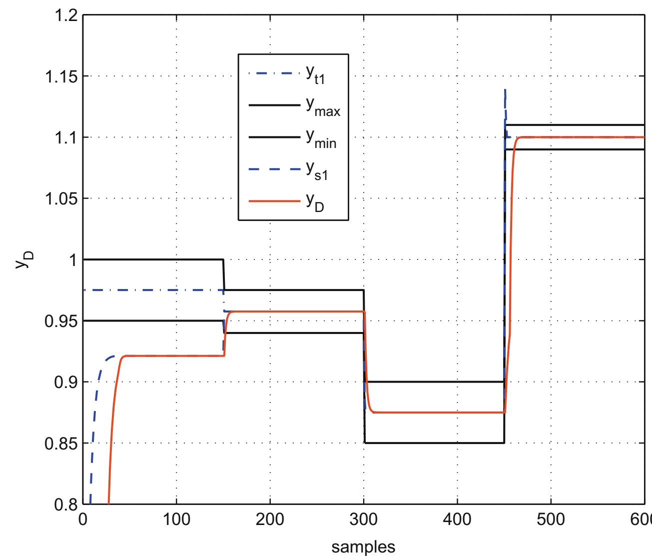
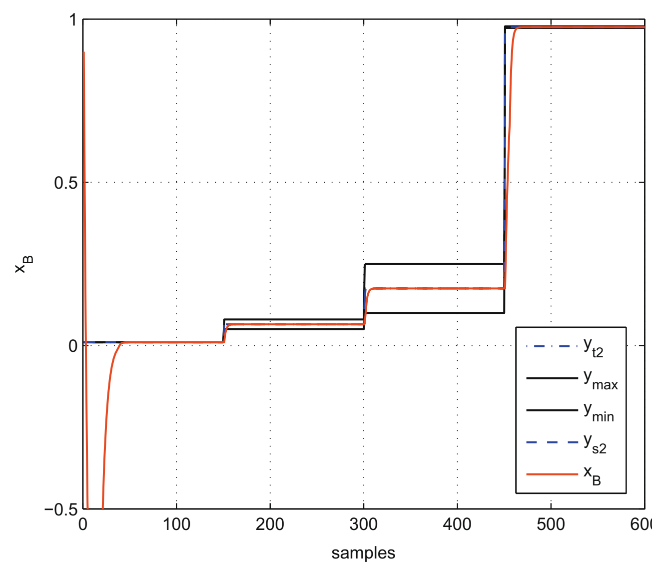


Fig. 12. Outputs evolution for the scaling factor case.



(a) Evolution of y_D.



(b) Evolution of x_B.

Fig. 14. Time evolutions of the outputs: scaling factor.

sult of this case's simulation, which are respectively the state-space evolution of the system and the time evolution of the outputs. In particular, in Figs. 12 and 13 the domain of attraction $\mathcal{X}_N$ for $N = 3$, the invariant set for tracking $\Omega_{t,K} = Proj_x(\Omega^w_{t,K})$, and the region of admissible steady state $\mathcal{X}_s$ are depicted respectively in dashed–dotted, solid and dotted line. The zone regions are represented as boxes, and the desirable target points y_t, are represented as circles and considered as the center of the target zones. Fig. 13 is a zoom of Fig. 12. The 4 situations described in the introduction of this section, are labeled as (1)–(4). In Fig. 14, the evolutions of the outputs and the artificial references are drawn respectively in solid and dashed–dotted line. The zones are drawn in thick-solid lines. The main difference between this implementation and the previous is that when $\Gamma_t \cap \mathcal{Y}_s \neq \emptyset$ and $y_t \in \mathcal{Y}_s$ the controller steers the system to exactly y_t, while if $\Gamma_t \cap \mathcal{Y}_s = \emptyset$ (first reference), the controller steers the system to the admissible point that minimizes the offset cost function. This happens because in the last three changes of reference, when $y_t \in \mathcal{Y}_s$, $\lambda = 0$ and $\Gamma_t \equiv y_t$. This last simulation shows that the controller account for the frequent practical case in which a combination of output set-point and zones is given. This last implementation allows the possibility to include input target, as well.

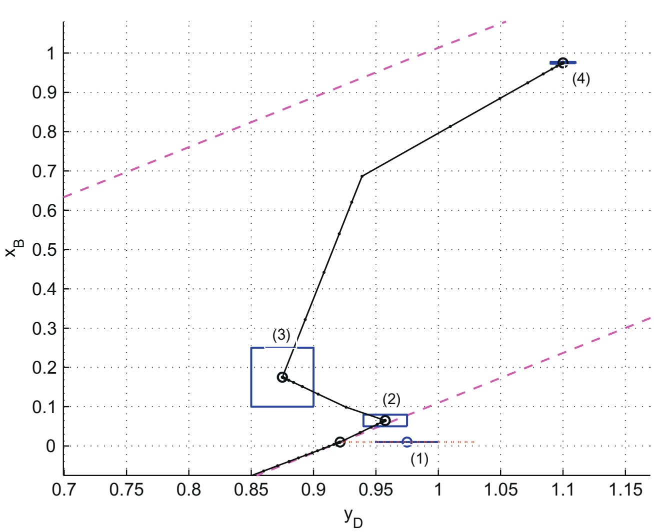


Fig. 13. Outputs evolution for the scaling factor case: zoom.

7. Conclusions

The zone control strategy is implemented in applications where the exact values of the controlled outputs are not important, as long as they remain inside a range with specified limits. In this work, an extension of the MPC for tracking for zone control has been presented, in which the controller considers a set, instead of a point, as target. The concept of deviation between two points used in the offset cost function has been generalized to the concept of distance from a point to a set. A characterization of the offset cost function has been given as the minimal distance between the output and some point inside the target set.

The properties of the presented controller have been tested on an ill-conditioned distillation column. The results have shown how the controller always steers the system into the output zone, even if the initial condition stays out of the zone. If the output zone is not admissible, the controller steers the system to the admissible point that minimizes the offset cost function.

Acknowledgements

The authors acknowledge MEC-Spain for funding this work (contracts DPI2007-66718-C04-01 and DPI2008-05818).

References

[1] D.E. Kassmann, T.A. Badgwell, R.B. Hawkings, Robust target calculation for model predictive control, AIChE Journal 45 (2000) 1007–1023.
[2] P. Tatjewski, Advanced control and on-line process optimization in multilayer structures, Annual Reviews in Control 32 (2008) 71–85.
[3] D.R. Vinson, C. Georgakis, A new measure of process output controllability, Journal of Process Control 10 (2000) 185–194.
[4] F.V. Lima, C. Georgakis, Design of output constraints for model-based non-square controllers using interval operability, Journal of Process Control 18 (2008) 610–620.
[5] E.F. Camacho, C. Bordons, Model Predictive Control, 2nd ed., Springer-Verlag, 2004.
[6] D.Q. Mayne, J.B. Rawlings, C.V. Rao, P.O.M. Scokaert, Constrained model predictive control: stability and optimality, Automatica 36 (2000) 789–814.
[7] K. Muske, J. Rawlings, Model predictive control with linear models, AIChE Journal 39 (1993) 262–287.
[8] J. Rossiter, B. Kouvaritakis, J. Gossner, Guaranteeing feasibility in constrained stable generalized predictive control, IEEE Proceedings on Control theory and Appliactions 143 (1996) 463–469.
[9] A. Bemporad, A. Casavola, E. Mosca, Nonlinear control of constrained linear systems via predictive reference management, IEEE Transactions on Automatic Control 42 (1997) 340–349.
[10] G. Pannocchia, E. Kerrigan, Offset-free reciding horizon control of constrained linear systems, AIChE Journal 51 (2005) 3134–3146.
[11] I. Alvarado, Model predictive control for tracking constrained linear systems, Ph.D. thesis, University de Sevilla, 2007.
[12] K. Muske, Steady-state target optimization in linear model predictive control, in: Proceedings of the ACC, 1997.
[13] C. Rao, J. Rawlings, Steady-states and constraints in model predictive control 45 (1999) 1266–1278.
[14] D. Limon, I. Alvarado, T. Alamo, E. Camacho, MPC for tracking of piece-wise constant references for constrained linear systems, Automatica 44 (9) (2008) 2382–2387.
[15] A. Ferramosca, D. Limon, I. Alvarado, T. Alamo, E.F. Camacho, MPC for tracking with optimal closed-loop performance, Automatica 45 (8) (2009) 1975–1978.
[16] S.J. Qin, T.A. Badgwell, A survey of industrial model predictive control technology, Control Engineering Practice 11 (2003) 733–764.
[17] A.C. Zanin, M.T. de GouvOa, D. Odloak, Integrating real time optimization into the model predictive controller of the fcc system, Control Engineering Practice 10 (2002) 819–831.
[18] A.H. González, D. Odloak, A stable MPC with zone control, Journal of Process Control 19 (2009) 110–122.
[19] A.H. González, J.L. Marchetti, D. Odloak, Robust model predictive control with zone control, IET Control Theory and Application (2009). doi:10.249/iet–cta:20070211.
[20] S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2006.
[21] Y. Wang, Robust model predictive control, Ph.D. thesis, University of Wisconsin–Madison, 2002.
[22] G. Grimm, M.J. Messina, S.Z. Tuna, A.R. Teel, Examples when nonlinear model predictive control is nonrobust, Automatica 40 (2004) 1729–1738.
[23] D. Limon, T. Alamo, D.M. Raimondo, D.M. de la Peña, J.M. Bravo, A. Ferramosca, E.F. Camacho, Input-to-state stability: an unifying framework for robust model predictive control, in: L. Magni, D.M. Raimondo, F. Allgöwer (Eds.), International Workshop on Assessment and Future Direction of Nonlinear Model Predictive Control, Springer, 2009, pp. 1–26.
[24] M.J. Messina, S.E. Tuna, A.R. Teel, Discrete-time certainty equivalence output feedback: allowing discontinuous control laws including those from model predictive control, Automatica 41 (2005) 617–628.
[25] I. Alvarado, D. Limon, T. Alamo, M. Fiacchini, E. Camacho, Robust tube-based MPC for tracking of piece-wise constant references, in: Proceedings of 46th IEEE Conference on Decision and Control, CDC 2007, New Orleans, LA, USA, 2007.
[26] I. Alvarado, D. Limon, T. Alamo, E. Camacho, Output feedback robust tube-based MPC for tracking of piece-wise constant references, in: Proceedings of 46th IEEE Conference on Decision and Control, CDC 2007, New Orleans, LA, USA, 2007.
[27] R.T. Rockafellar, Convex Analysis, Princeton University Press, 1970.
[28] S. Skogestad, M. Morari, J.C. Doyle, Robust control of ill-conditioned plants: High-purity distillation, IEEE Transactions on Automatic Control 33 (12) (1988) 1092–1105.