Discussion on: "GPC Robust Design Using Linear and/or Bilinear Matrix Inequalities"

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The paper by J.V. Salcedo and M. Martínez addresses the problem of obtaining a robust generalized predictive control (GPC) for the control of uncertain systems. The uncertainty structure of the plant has been modelled to be linear fractional. The proposed synthesis strategy is based on the formulation of the design problems by means of linear and/or bilinear matrix inequalities (LMIs and/or BMIs). The selection of the parameters of the GPC becomes a non-convex problem. The authors use a multi-objective genetic algorithm to address the non-convex optimization issue.

In many situations, the exact and deterministic solution of a robust design problem is intractable from a computational point of view (see e.g. [11]). Therefore, it is not surprising that the authors finally resort to the use of a genetic algorithm to obtain the design parameters of the proposed robust controller. In this discussion paper we briefly recall different strategies to circumvent the unaffordable computational requirements often associated to robust design problems.

If Θ represents the set of possible design parameters, a worst-case approach to the robust synthesis of controllers consists in obtaining $\theta \in \Theta$ such that the corresponding closed-loop system satisfies a set of specifications for every possible realization of the uncertainty. Generally, the uncertainty is assumed to belong to a bounded set Δ. If the set Δ exhibits an infinite cardinality, the corresponding worst-case design problem can be often formulated as a semi infinite programming problem. There exists, however, a good number of vertex results that, when applicable, allow one to affirm that in order to guarantee robust satisfaction of the constraints of the problem, only a specially chosen subset of finite cardinality of Δ has to be considered. For example, the celebrated Karitonov's theorem is only one of the vertex results that can be used to study stability when interval uncertainty affects the coefficients of a given characteristic polynomial (see [7] for details). Other vertex results are applicable in presence of interval matrix uncertainty [3,17,18,23,31]. However, the number of extreme realizations of the uncertainty required to guarantee robust satisfaction of the constraints grows exponentially with the dimension of the uncertain interval matrix.

In the case when no vertex result is applicable, or when the required number of extreme realizations is too large, other strategies have to be taken into account. As it is mentioned in the paper by J.V. Salcedo and M. Martínez, it is possible to bound the effect of the uncertainty by means of scaling techniques. We cite here the classical scaling techniques from μ-theory [32], and those used in the realm of robust optimization [9,10]. See also the review paper on LMI relaxations [24]. All these techniques provide sufficient conditions for robustness.

In addition to the fact that worst-case control design problems are often intractable from a computational point of view, they have proved to yield conservative results. An alternative to worst-case design is the randomized approach to robust control [25,26,29]. The use of randomization is based on the notion of ε-level probabilistic solution [8]. Given a probabilistic measure Pr_{Δ} on the uncertainty set Δ, we say that the design parameter θ is an ε-level probabilistic solution if the probability that the closed-loop system corresponding to θ violates the constraints of the problem is not greater than ε. This randomized approach is of practical relevance because in many industrial processes and applications, allowing a (small) probability of failure allows one to considerably reduce the production costs. For example, consider a process in which the cost of rejecting a reduced number of products that do not satisfy the specifications is much smaller than the cost of running a process in which the probability of failure is minimized.

Resorting to the notion of ε-level probabilistic solution not only provides less conservative results

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than the worst-case approach, but it also reduces the complexity of the design problem. Under the assumption that the robust constraints of the problem depend in a convex way on the design parameter θ, it is possible to obtain an ε-level probabilistic solution in polynomial time (provided that the original robust problem is strictly feasible) [25].

There exists a good number of efficient strategies to obtain a probabilistic solution to a convex robust design problem. For example, we mention here the sequential algorithms based on stochastic gradient [15,16,21] or ellipsoid iterations [19]; see also [4,14] for other classes of sequential algorithms. A non-sequential strategy, denoted as the *scenario approach*, has been introduced in [12,13]. In this approach, the original robust control problem is reformulated in terms of a single convex optimization problem with sampled constraints which are randomly generated. See also [1] and [22].

A classic approach for not necessarily convex uncertain problems is based upon statistical learning theory, see [27] and [28] for further details. In particular, the use of this theory for feedback design of uncertain systems has been initiated in [29]; subsequent work along this direction include [30] and [20]. Statistical learning theory states that it is possible to obtain an ε-level probabilistic solution to a robust non-convex optimization problem by means of a single optimization problem with sampled constraints randomly generated. An a priori upper bound of the number of samples required to guarantee that the obtained solution is an ε-level probabilistic solution with probability no smaller than $1-\delta$ can be obtained using the results of this theory. See [2] for recent results on this topic.

We conclude that the use of randomization not only provides a way to circumvent the possible non-finite cardinality of the uncertain set Δ, but it also serves to reduce the conservativeness of the results obtained with a worst-case robust design. Extensions of the paper by J.V. Salcedo and M. Martínez could be easily obtained by the use of statistical learning theory. The GPC design parameters could be obtained by means of the solution of a non-convex optimization problem subject to constraints randomly generated. It would be also interesting to consider the possibility of formulating the closed-loop performance specifications without using Lyapunov functions. As it is shown in [6] and references therein, this allows one to greatly reduce the number of decision variables. The resulting non-convex optimization problem stemming from the use of randomization and the formulation of the constraints without using Lyapunov functions could be solved by means of non-smooth optimization [5,6].

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Final Comments by the Authors J.V. Salcedo and M. Martínez

The authors think that some comments of the discussion by VanAntwerp and Braatz must be clarified. First, the satisfaction of constraints on the manipulated and state variables is not taken into account in the paper, and also any comment about them is not present. However, constraints can be easily incorporated following similar ideas of work [1] as matrix inequalities, which is treated in detail in the authors work [2].

Second, the mass-spring example requires a computation time of $300 \times 2\,\text{hr}$. However, the problem solved is not a standard problem of minimization subject to bilinear and linear matrix inequalities (BMIs and LMIs) regarding multiple objectives. Here a multiobjective optimization problem is solved [see Eq. (33) in the paper] with two objectives:

- Maximization of uncertainty range of K.
- Minimization of ∞-norm of reference/output channel.

The solution to this problem is not a single controller, it is an approximation to the optimal Pareto front (see Fig. 4 in the paper) composed by a big number of controllers. Each controller in the Pareto front is optimal in the sense that the uncertain range is maximized given a certain level of ∞-norm, or conversely, the ∞-norm is minimized given a certain uncertainty range.

In each generation (300 in total) for a population of 2000 generalized predictive control (GPC) controllers, the set of matrix inequalities is checked for every GPC. This explains the computation time of one generation. The big number of generations is required to reach an approximation of Pareto front.

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