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Brief paper

Ultimate bounded stability and stabilization of linear systems interconnected with generalized saturated functions[☆]

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ABSTRACT

This paper proposes some ultimate bounded stability analysis and stabilization conditions for systems involving actuators with different nonlinear elements, like for instance both saturation and dead-zone or both saturation and stick–slip. Results are based on the use of a convex differential inclusion approach. Indeed, an adequate property allowing to upper-bound some product terms related to the nonlinearity is provided. Thus, constructive conditions associated to convex optimization schemes are developed to determine suitable regions of the state space in which the closed-loop trajectories can be captured.

1. Introduction

Many industrial processes exhibit non-smooth nonlinearities, generally due to physical, technological or safety constraints, as in the cases of hydraulic servo valves or electric servo motors. The interest for this kind of system mainly comes from the fact that neglecting these nonlinearities, during the stability analysis or the control design, can be a source of undesirable and even catastrophic behaviors (see, for example, Nordin, Ma, & Gutman, 2002; Tarbouriech, Garcia, & Glattfelder, 2007; Taware & Tao, 2003). For all these reasons, the specific case of nonlinear actuators involving saturation elements (position and/or higher dynamics) has been extensively studied in the last ten years Hu and Lin (2001), Kapila and Grigoriadis (2002) and Tarbouriech and Garcia (1997). In particular, several results have been provided in a local context for stability analysis and synthesis purposes, in which the key point is to determine an estimate of the basin of attraction of the closed-loop nonlinear system: see Tarbouriech et al. (2007) for recent advances on this topic. On the other hand, practical actuators often involve more complex nonlinearities, such as friction terms for example, which may generally be represented by hysteresis, backlash, dead-zone or stick–slip elements Gomes, da Rosa, and Albertini (2006), Olsson and Åström (1989) and Shoukat Choudhury, Thornhill, and Shah (2005). However, such nonlinear actuators have been rarely studied, and in many cases only practical solutions without a priori guarantees of stability have been derived. One reason is certainly that such nonlinearities are generally poorly known and that mathematical descriptions are often not very well adapted for stability analysis or synthesis purposes Thiery, Kunze, Karimi, Curnier, and Longchamp (2006). Nevertheless, different solutions can be investigated to guarantee the closed-loop stability requiring some knowledge about the nonlinearities (see for example Corradini, Orlando, & Parlangeli, 2004; Tarbouriech, Prieur, & Queinnec, 2010, and references therein).

The current paper is concerned with the study of nonlinear actuators involving different nonlinear characteristics like both dead-zone and saturation elements. Literature on this subject is very limited. One can however cite preliminary results concerning semi-global stabilization of linear systems interconnected with such nonlinear elements Lin (1997), but with the main drawback that open-loop stability hypothesis has to be satisfied. More recently, attention was paid to bifurcation analysis of such nonlinear systems Ortega, Aracil, Gordillo, and Rubio (2000). The state feedback stabilization problem was addressed in Fong and Hsu (2000), but for the particular case of single input systems. In Gomes da Silva, Robaski, and Reginatto (2002), stability analysis conditions are proposed by considering a hybrid modeling for the closed-loop discrete-time system. Fliegner, Logemann, and Ryan (2003) reported some recent results on integral control, but for single-input single-output open-loop stable systems only.

In this paper, the notion of a generalized saturated function is presented, which is directly related to the notion of convex

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differential inclusion Alamo, Cepeda, Fiacchini, and Camacho (2009) and Fiacchini (2010). Such a tool allows us to address the problem of computing estimates of the domain of attraction for a broad class of actuator nonlinearities. Based on this, an adequate property allowing to upper-bound some pertinent product terms related to the nonlinearity is provided. Then, using quadratic Lyapunov functions, constructive conditions are proposed in a quasi-LMI form, in the sense that the nonlinearity only appears through the product of a matrix by a scalar variable. The proposed approach allows us to characterize both an inner and an outer set. The closed-loop trajectories starting in the outer set are ultimately bounded in the inner set. The outer set is a positively invariant and contractive set for the closed-loop system. The objective of the related optimization problem is then to maximize a measure of the size of the outer set, whereas the inner set is minimized. It is important to point out that the technique proposed does not require the open-loop system to be stable. The main contribution resides in the fact that the results developed encompass those in Alamo, Cepeda, and Limon (2005), Fong and Hsu (2000), Gomes da Silva et al. (2002) and Hsu and Fong (2003). Furthermore, the proposed conditions can be considered as complementary to that ones provided in Dai, Hu, Teel, and Zaccarian (2009) and Hu, Thibodeau, and Teel (2009).

Notation. **1** and **0** denote respectively the identity matrix and the null matrix of appropriate dimensions. Furthermore, 1_m denotes a vector of dimension m with all components equal to 1. The elements of a matrix $A \in \Re^{m\times n}$ are denoted by $A_{(i,j)}$, $i = 1, \dots, m, j = 1, \dots, n$. $A_{(i)}$ and A_i denote the ith row and ith column of matrix A, respectively. A' denotes the transpose of A. He$[A] = A + A'$. $|A|$ is the matrix given by the absolute value of each element of A. For two symmetric matrices, A and B, $A > B$ (resp. $A \geq B$) means that $A - B$ is positive definite (resp. positive semi-definite). For two vectors $x, y \in \Re^n$, the notation $x \geq y$ means that $x_{(i)} - y_{(i)} \geq 0, \forall i = 1, \dots, n$. For any vector $u \in \Re^m$ and $u_0 \in \Re^m$, with $u_0 > 0$, one defines each component of $\text{sat}_{u_0}(u)$ by $\text{sat}_{u_0}(u_{(i)}) = \text{sign}(u_{(i)}) \min(u_{0(i)}, |u_{(i)}|), i = 1, \dots, m$. Given an integer m, the set $\mathcal{V}_m$ is defined as the set of all the subsets of $\mathcal{T}_m = \{1, 2, \dots, m\}$, that is, $\mathcal{V}_m = \{\mathcal{S}; \mathcal{S} \subseteq \mathcal{T}_m\}$. $\mathcal{S}^c$ denotes the complement of $\mathcal{S}$ in $\mathcal{T}_m$ (see Alamo et al., 2005, for more details). Given a set $\mathcal{E}$, $\partial\mathcal{E}$ denotes its boundary. Given a symmetric and positive definite matrix P, an ellipsoid $\mathcal{E}(P, 1)$ is defined by $\mathcal{E}(P, 1) = \{x \in \Re^n; x'Px \leq 1\}$.

2. Generalized saturated functions

In this section, the notion of generalized saturated functions is introduced. As it will be shown, this class of functions encompasses many common nonlinearities which appear in real control processes. The following definition introduces this notion for scalar (possibly time-varying) nonlinearities.

Definition 1 (*Scalar Case*)**.** The scalar function $\varphi : \Re \times \Re \to \Re$ is said to be a generalized saturated function with saturation level $u_0 \in \Re, u_0 > 0$, dead-zone $\sigma \in \Re, \sigma \geq 0$, and linear slope $\mu \in \Re, \mu > 0$, if

$$-\Gamma(-u) \leq \varphi(u, t) \leq \Gamma(u), \quad \forall u, \forall t$$

where $\Gamma(u) = \max\{\mu(u + \sigma), -u_0\}$.

See Fig. 1 for an example of a scalar generalized saturated function. Any nonlinear function involving a combination of dead-zone, stick–slip and saturation elements can be shown to be an element of the class of generalized saturated functions (see Figs. 2 and 3). Moreover, it is also worth mentioning that some time-varying nonlinear phenomena like, for example, hysteresis and friction can be easily modeled by means of this class of functions (see Fig. 4).

The notion of (scalar) generalized saturated function is easily extended to the vector case.

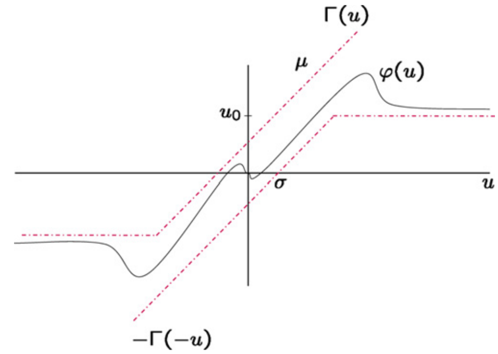

Fig. 1. Generalized saturated function (scalar case).

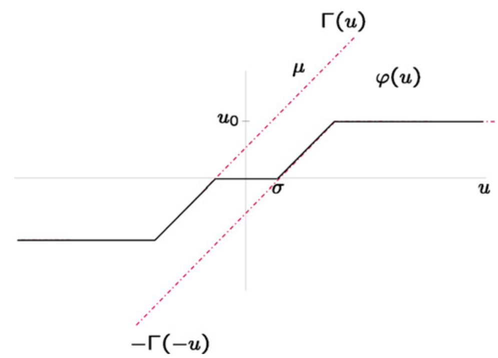

Fig. 2. Input–output characteristics of a nonlinear actuator involving a dead-zone plus a saturation element.

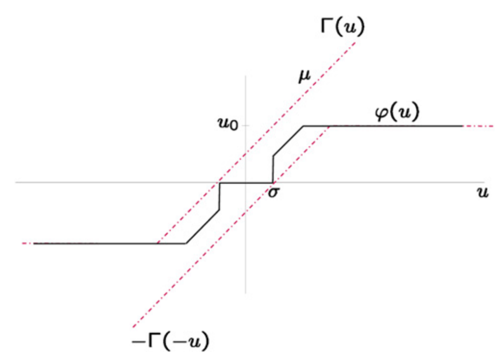

Fig. 3. Input–output characteristics of a nonlinear actuator involving a stick–slip plus a saturation element.

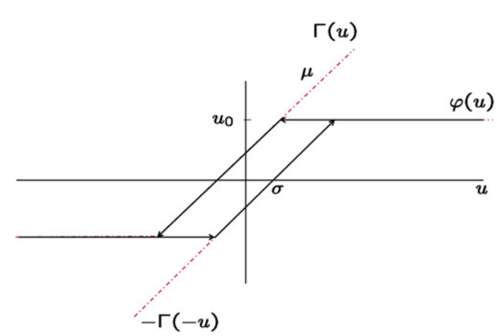

Fig. 4. Input–output characteristics of a nonlinear actuator involving a hysteresis plus a saturation element.

Definition 2 (*Vector Case*)**.** The function $\varphi : \Re^m \times \Re \to \Re^m$ is said to be a generalized saturated function with saturation level $u_0 \in \Re^m, u_0 > 0$, dead-zone $\sigma \in \Re^m, \sigma \geq 0$, and linear slope

$\mu \in \Re^m$, $\mu > 0$, if

$$-\Gamma(-u) \leq \varphi(u,t) \leq \Gamma(u), \quad \forall u, \forall t$$

where the m components of function $\Gamma(u)$ are given by

$$\Gamma_{(i)}(u) = \max\{\mu_{(i)}(u_{(i)} + \sigma_{(i)}), -u_{0(i)}\}, \quad i = 1, \ldots, m.$$

The main results of the paper are based on the following common properties of the (scalar and vector) generalized saturated function (see the Appendix for a proof).

Property 1 (*Scalar Case*). *Suppose that the scalar function $\varphi : \Re \times \Re \to \Re$ is a generalized saturated function with saturation level $u_0 \in \Re, u_0 > 0$, dead-zone $\sigma \in \Re, \sigma \geq 0$, and linear slope $\mu \in \Re, \mu > 0$. Then the following inequality is satisfied for every $z \in \Re$, $u \in \Re$, and $t \in \Re$:*

$$z\varphi(u,t) \leq \max\{z\mu u + |z|\mu\sigma, -|z|u_0\}. \tag{1}$$

By extension, a general property may be stated in the vector case (see the Appendix for a proof).

Property 2 (*Vector Case*). *Suppose that $\varphi : \Re^m \times \Re \to \Re^m$ is a generalized saturated function with saturation level $u_0 \in \Re^m, u_0 > 0$, dead-zone $\sigma \in \Re^m, \sigma \geq 0$, and linear slope $\mu \in \Re^m, \mu > 0$. Then the following inequality is satisfied for every $z \in \Re^m$, $u \in \Re^m$, and $t \in \Re$:*

$$z'\varphi(u,t) \leq \max_{\mathcal{S} \in \mathcal{V}_m} \left\{ \sum_{i \in \mathcal{S}^c} z_{(i)}\mu_{(i)}u_{(i)} + |z_{(i)}|\mu_{(i)}\sigma_{(i)} - \sum_{i \in \mathcal{S}} |z_{(i)}|u_{0(i)} \right\}. \tag{2}$$

Relation (2) is enclosed in the convex differential inclusion framework. Notice that the component-wise nonlinear inclusion $-\Gamma(-u) \leq \varphi(u,t) \leq \Gamma(u)$, and the inequality (2) are valid for every $u \in \Re^m$. This is clearly an advantage with respect to most of the inclusions (and/or sector conditions) that can be found in the literature, which are valid only on a bounded region of $\Re^m$ (see Alamo, Cepeda, Limon, & Camacho, 2006; Hu & Lin, 2001; Pittet, Tarbouriech, & Burgat, 1997; Tarbouriech et al., 2007).

3. Problem statement

Consider the following continuous-time system:

$$\dot{x}(t) = Ax(t) + B\varphi(Kx(t), t) \tag{3}$$

where $x(t) \in \Re^n$ is the state. Matrices A, B and K are constant matrices of appropriate dimensions. According to Section 2, the function $\varphi : \Re^m \times \Re \to \Re^m$ is assumed to be a generalized saturated function with saturation level $u_0 \in \Re^m, u_0 > 0$, dead-zone $\sigma \in \Re^m, \sigma \geq 0$, and linear slope $\mu \in \Re^m, \mu > 0$.

When studying the behavior of such a system (3), any nonlinear function $\varphi(.,.)$ bounded by $\Gamma(.)$ and $-\Gamma(-.)$ is suitable. This means that the closed-loop trajectories do not necessarily converge to the origin. However, under some conditions (see Theorem 1), the ultimate boundedness of the trajectories Khalil (2002) can be obtained. The first problem under consideration is then to evaluate a domain, as small as possible, where it is guaranteed that the trajectories will be ultimately bounded. Such a domain is called the inner set.

Remark 1. The case of presence of dead-zone is a particular case, which induces the system to behave in open-loop inside the inner ellipsoid.

On the other hand, according to the saturation level u_0, we face the classical problem of determining admissible initial state sets. In other words, we want to estimate an outer set of safe operation, such that the associated closed-loop trajectories are contractive, as far as they are captured in the inner set. Hence, the problem we intend to solve by exploiting Property 2 can be summarized as follows:

Problem 1 (*Analysis Problem*). Given the state feedback K,

- compute an outer ellipsoidal invariant and contractive set Ω_1, as large as possible, for the closed-loop system (3);
- compute an inner set Ω_2, as small as possible, in which the closed-loop trajectories, initiated in the outer ellipsoid Ω_1, are ultimately bounded.

Throughout the paper, this problem is addressed by considering quadratic Lyapunov functions. Related to the computation of both ellipsoidal sets Ω_1 and Ω_2, optimization issues are discussed. Moreover, some remarks regarding the control design are also provided.

4. Stability analysis and stabilization conditions

4.1. Main result

The following theorem uses Property 2 (and Properties 3 and 4 given in the Appendix) in order to solve Problem 1.

Theorem 1. *If there exist three symmetric positive definite matrices $W \in \Re^{n\times n}$, $Q \in \Re^{n\times n}$, $R \in \Re^{n\times n}$, matrix $Y^S \in \Re^{m\times n}$ for every $\mathcal{S} \in \mathcal{V}_m$, positive vector β and positive scalar θ satisfying:*

$$\begin{bmatrix} \mathrm{He}\left[AW + \sum_{i\in\mathcal{S}^c} B_i\mu_{(i)}K_{(i)}W + \sum_{i\in\mathcal{S}} B_iY^S_{(i)}\right] + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)}R + \frac{B_iB_i'}{\beta_{(i)}}\right) - \theta W & \theta W \\ \theta W & -\theta Q \end{bmatrix} < \mathbf{0}, \quad \forall i \in \mathcal{V}_m \tag{4}$$

$$W - Q \geq \mathbf{0} \tag{5}$$

$$\begin{bmatrix} R & W \\ W & Q \end{bmatrix} \geq \mathbf{0} \tag{6}$$

$$\begin{bmatrix} u^2_{0(i)} & Y^S_{(i)} \\ \left(Y^S_{(i)}\right)' & W \end{bmatrix} > \mathbf{0}, \quad \forall i \in \mathcal{S} \tag{7}$$

then the closed-loop trajectories of the nonlinear system (3) initiated in the outer ellipsoid $\mathcal{E}(W^{-1},1)$ are ultimately bounded in the set $\mathcal{E}(Q^{-1},1)$.

Proof. Consider the quadratic Lyapunov function $V(x) = x'Px$, with $P = P' > \mathbf{0}$ and $P = W^{-1}$. We want to prove that $\dot{V}(x) < 0$ along the trajectories of the nonlinear system (3) for any x such that $x'Px \leq 1$ and $x'Ux \geq 1$ with $U = U' > \mathbf{0}$ and $U = Q^{-1}$.

If relation (4) is satisfied then one gets, by applying the Schur complement, that

$$\mathcal{L} = \mathrm{He}\left[AW + \sum_{i\in\mathcal{S}^c} B_i\mu_{(i)}K_{(i)}W + \sum_{i\in\mathcal{S}} B_iY^S_{(i)}\right] + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)}R + \frac{B_iB_i'}{\beta_{(i)}}\right) - \theta W + \theta WQ^{-1}W < 0.$$

Hence, there exists an $\epsilon > 0$ small enough such that

$$\mathcal{L} < -\epsilon\mathbf{1}. \tag{8}$$

From (7) and Property 3 (see Appendix), one gets

$$B_iY^S_{(i)} + (B_iY^S_{(i)})' \geq -\alpha_{(i)}u_{0(i)}W - \frac{B_iB_i'}{\alpha_{(i)}}u_{0(i)}, \quad \forall \alpha_{(i)} > 0, \forall i \in \mathcal{S}.$$

Hence, it follows from relation (8)

$$\begin{aligned}&\text{He}\left[AW + \sum_{i\in\mathcal{S}^c} B_i\mu_{(i)}K_{(i)}W\right] \\ &\quad + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)}R + \frac{B_iB_i'}{\beta_{(i)}}\right) - \sum_{i\in\mathcal{S}}\left(\alpha_{(i)}W + \frac{B_iB_i'}{\alpha_{(i)}}\right)u_{0(i)} \\ &\quad - \theta W + \theta W Q^{-1}W \leq \mathcal{L} < -\epsilon\mathbf{1}.\end{aligned}$$

Thus, by pre- and post-multiplying both sides of the previous inequality by $x'P$ and Px, respectively, one obtains

$$\begin{aligned}&x'\left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)} + \left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)}\right)'\right. \\ &\quad + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)}PRP + \frac{PB_iB_i'P}{\beta_{(i)}}\right) \\ &\quad \left. - \sum_{i\in\mathcal{S}}\left(\alpha_{(i)}P + \frac{PB_iB_i'P}{\alpha_{(i)}}\right)u_{0(i)} - \theta P + \theta U\right)x \\ &\leq x'P\mathcal{L}Px < -\epsilon x'P^2x. \qquad (9)\end{aligned}$$

Clearly one gets $\epsilon x'P^2x \geq \sum_{i\in\mathcal{S}}\epsilon\frac{x'P^2x}{m}$. Thus, by denoting $\bar{\epsilon} = \frac{\epsilon x'P^2x}{m}$, it follows from inequality (9)

$$\begin{aligned}&x'\left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)} + \left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)}\right)'\right. \\ &\quad \left. + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)}PRP + \frac{PB_iB_i'P}{\beta_{(i)}}\right) - \theta P + \theta U\right)x \\ &\quad - \sum_{i\in\mathcal{S}}\left(\alpha_{(i)}x'Px + \frac{x'PB_iB_i'Px}{\alpha_{(i)}} - \frac{\bar{\epsilon}}{u_{0(i)}}\right)u_{0(i)} \\ &\leq x'P\mathcal{L}Px + \epsilon x'P^2x < 0. \qquad (10)\end{aligned}$$

Since $x'Px \leq 1$ for all $x \in \mathcal{E}(W^{-1}, 1)$, by definition, it follows that

$$\begin{aligned}&x'\left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)} + \left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)}\right)'\right. \\ &\quad \left. + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)}PRP + \frac{PB_iB_i'P}{\beta_{(i)}}\right) - \theta P + \theta U\right)x \\ &\quad - \sum_{i\in\mathcal{S}}\left(\alpha_{(i)} + \frac{x'PB_iB_i'Px}{\alpha_{(i)}} - \frac{\bar{\epsilon}}{u_{0(i)}}\right)u_{0(i)} \qquad (11)\end{aligned}$$

is a lower bound of the left-hand term of (10). Furthermore, from relation (6) it follows that $x'PRPx \geq x'Ux$. Thus, since we are considering the decreasing of V for x such that $x'Ux \geq 1$, it follows that

$$\begin{aligned}&x'\left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)} + \left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)}\right)'\right. \\ &\quad \left. - \theta P + \theta U\right)x - \sum_{i\in\mathcal{S}}\left(\alpha_{(i)} + \frac{x'PB_iB_i'Px}{\alpha_{(i)}} - \frac{\bar{\epsilon}}{u_{0(i)}}\right)u_{0(i)} \\ &\quad + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)} + \frac{x'PB_iB_i'Px}{\beta_{(i)}}\right) \qquad (12)\end{aligned}$$

is a lower bound of (11) and therefore of the left-hand term of (10). From this, for a proper positive value of $\alpha_{(i)}$ (note that this value has not to be expressly obtained) and using Property 4 the satisfaction of

$$\begin{aligned}&x'\left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)} + \left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)}\right)'\right. \\ &\quad \left. - \theta P + \theta U\right)x - 2\sum_{i\in\mathcal{S}}|x'PB_i|u_{0(i)} \\ &\quad + \sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}\left(\beta_{(i)} + \frac{x'PB_iB_i'Px}{\beta_{(i)}}\right) < 0 \qquad (13)\end{aligned}$$

implies the satisfaction of inequality (10). By using the fact that $2|a| \leq b + \frac{a^2}{b}$, for all $b > 0$ and for all a, it follows that

$$\begin{aligned}&x'\left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)} + \left(PA + \sum_{i\in\mathcal{S}^c}PB_i\mu_{(i)}K_{(i)}\right)'\right. \\ &\quad \left. - \theta P + \theta U\right)x + 2\sum_{i\in\mathcal{S}^c}\mu_{(i)}\sigma_{(i)}|x'PB_i| \\ &\quad - 2\sum_{i\in\mathcal{S}}|x'PB_i|u_{0(i)} < 0. \qquad (14)\end{aligned}$$

The inequality is satisfied for every $S \in \mathcal{V}_m$, and therefore one gets

$$\begin{aligned}&x'(A'P + PA - \theta P + \theta U)x \\ &\quad + 2\max_{\mathcal{S}\in\mathcal{V}_m}\left\{\sum_{i\in\mathcal{S}^c}\left(x'PB_i\mu_{(i)}K_{(i)}x + |x'PB_i|\mu_{(i)}\sigma_{(i)}\right)\right. \\ &\quad \left. - \sum_{i\in\mathcal{S}}|x'PB_i|u_{0(i)}\right\} < 0. \qquad (15)\end{aligned}$$

Then, by using Property 2 with $z = B'Px$, one has

$$\begin{aligned}&2x'PAx + 2x'PB\varphi(Kx, t) - \theta x'Px + \theta x'Ux \\ &\quad \leq x'(A'P + PA - \theta P + \theta U)x \\ &\quad + 2\max_{\mathcal{S}\in\mathcal{V}_m}\left\{\sum_{i\in\mathcal{S}^c}\left(x'PB_i\mu_{(i)}K_{(i)}x + |x'PB_i|\mu_{(i)}\sigma_{(i)}\right)\right. \\ &\quad \left. - \sum_{i\in\mathcal{S}}|x'PB_i|u_{0(i)}\right\} < 0\end{aligned}$$

along the trajectories of the closed-loop system, for all $x \in \mathcal{E}(W^{-1}, 1)$ and $x \notin \mathcal{E}(Q^{-1}, 1)$. Hence, the satisfaction of relations (4), (6) and (7) guarantees, by referring to the S-procedure Boyd, El Ghaoui, Feron, and Balakrishnan (1994), that $\dot{V}(x) < 0$ for any x such that $x'Px \leq 1$ and $x'Ux \geq 1$. Furthermore, from the definition of the ellipsoids $\mathcal{E}(W^{-1}, 1)$ and $\mathcal{E}(Q^{-1}, 1)$, the condition (5) means that $\mathcal{E}(Q^{-1}, 1)$ is included in $\mathcal{E}(W^{-1}, 1)$. Finally, the satisfaction of conditions (4)–(7) means that the ellipsoid $\mathcal{E}(W^{-1}, 1)$ is contractive with respect to the trajectories of the nonlinear system (3) until the state enters the set $\mathcal{E}(Q^{-1}, 1)$. In other words, the closed-loop trajectories initiated in the outer ellipsoid $\mathcal{E}(W^{-1}, 1)$ are ultimately bounded in the inner ellipsoid $\mathcal{E}(Q^{-1}, 1)$. □

Remark 2. Note that the conditions stated in Theorem 1 do not impose the asymptotic stability of the origin of the state space, and then, do not require any stability condition on the open-loop matrix A.

Remark 3. Theorem 1 gives a solution to the analysis Problem 1. The conditions can be directly extended to consider the control design case by considering matrix Y instead of terms KW, as further optimization variables.

Remark 4. In Theorem 1, the outer ellipsoid $\mathcal{E}(W^{-1}, 1)$ is contractive and invariant contrarily to the inner ellipsoid $\mathcal{E}(Q^{-1}, 1)$. Nevertheless, we can determine the smallest invariant ellipsoid whose shape is determined by W and containing $\mathcal{E}(Q^{-1}, 1)$. For this, it suffices to compute the greatest scalar ρ such that the resulting ellipsoid $\mathcal{E}(W^{-1}, \rho^{-1})$ is invariant and contains $\mathcal{E}(Q^{-1}, 1)$; in other words one has to compute the maximal scalar $\rho \geq 1$ such that $W - \rho Q \geq \mathbf{0}$.

Remark 5. It can be of interest to impose a rate of convergence $\gamma, \gamma > 0$, for the closed-loop system trajectories by adding in the (1, 1) block of matrix in relation (4) the term $+\gamma W$. Moreover, from the satisfaction of (4) and since according to Remark 4 there exists a scalar $\rho \geq 1$ such that $\mathcal{E}(Q^{-1}, 1) \subset \mathcal{E}(W^{-1}, \rho^{-1})$, it follows $\dot{V}(x) < \theta x'Px - \theta x'Q^{-1}x \leq \theta(1-\rho)x'Px$. The positive scalar $-\theta(1-\rho)$ then represents a rate of convergence for the closed-loop system trajectories.

Remark 6. Consider a (normalized) saturation which is equivalent to the generalized saturated function with $\mu_{(i)} = 1, \sigma_{(i)} = 0, u_{0(i)} = 1, \forall i = 1, \ldots, m$. By adapting to this case the conditions of Theorem 1, it follows that the variable β does not appear and R is no more affecting (4) but is involved only in (6). This relation admits infinite solutions R provided that $Q > 0$, as assumed in the statement of the theorem, then (6) does not affect the problem. Moreover, by choosing $Q = W$, (5) holds and hence, in this case, only conditions (4) and (7) of Theorem 1 have to be tackled. With $Q = W$, relation (4) reads: $\mathrm{He}\left[AW + \sum_{i\in S^c} B_i K_{(i)} W + \sum_{i\in S} B_i Y^S_{(i)}\right] < 0$ and therefore the current Theorem 1 is equivalent to Theorem 1 in Alamo et al. (2005). Then the current result can be viewed as an extension of the results proposed in Alamo et al. (2005), and thus also of those in Hu and Lin (2001) and Hu, Lin, and Chen (2002).

4.2. Optimization aspects

Theorem 1 provides a condition on ellipsoids $\mathcal{E}(W^{-1}, 1)$ and $\mathcal{E}(Q^{-1}, 1)$ with $\mathcal{E}(Q^{-1}, 1) \subseteq \mathcal{E}(W^{-1}, 1)$, such that $\mathcal{E}(W^{-1}, 1)$ is contractive and $\mathcal{E}(Q^{-1}, 1)$ captures the ultimate bounded closed-loop trajectories. Finally, an optimization problem may be stated, for the analysis or the synthesis case, to simultaneously evaluate the smallest inner ellipsoid and the largest outer ellipsoid[1]:

$$\begin{aligned} &\min \eta_1 \mathrm{Trace}(M) + \eta_2 \mathrm{Trace}(Q) \\ &\text{subject to (4)–(7) and } \begin{bmatrix} M & \mathbf{1} \\ \mathbf{1} & W \end{bmatrix} > \mathbf{0} \end{aligned} \tag{16}$$

where $\eta_i, i = 1, 2$, are weighting parameters. The search for the largest outer ellipsoidal set mainly corresponds to evaluate the influence of the saturation u_0 on the stability properties of the nonlinear closed-loop system Alamo et al. (2005), Gomes da Silva and Tarbouriech (2005) and Hu et al. (2002). The determination of the smallest inner ellipsoidal set corresponds to the evaluation of the smallest domain in which the closed-loop trajectories are ultimately bounded. The conditions stated in Theorem 1 are not LMI conditions, due to vector β, and due to the products θW and θQ implying that the optimization problem proposed above is not convex. Note however, that, as pointed out in new Remark 5, θ is a sort of measure of how high the decreasing rate is required to be ensured. Posing θ very close to zero, means that inside $\mathcal{E}(W^{-1}, 1)$ and outside $\mathcal{E}(Q^{-1}, 1)$ the convergence rate is required to be close to null, that is, we are trying to approximate the basin of attraction (outer set) and the ultimately bounded stability region (inner set). Vice versa, high values of θ would entail the requirement of a high convergence rate, convergence rate that could be not possible to be guaranteed inside any region (in fact, infeasibility can occur). Then, in the following, θ is viewed as a design parameter and fixed as a small value since we are interested in the set optimization problem. On the other hand, the influence of the choice of vector β cannot be neglected. Either an iterative search on the components of this vector or an encapsulation of the LMI optimization step (16) in some overall nonlinear optimization procedure (such as *fminsearch* Matlab function) may be considered. In the single-input case ($m = 1$), β is a scalar and the optimal solution of (16) can be obtained from a search over a mere one-dimensional grid. For systems with $m \geq 2$, a preliminary hypothesis (which has been clearly sufficient in all tested examples) may be to consider a vector $\beta = a_1 1_m$ with scalar a_1, that is, one yet considers a mere one-dimensional parameter of the problem. Hence, in all these cases an optimal or sub-optimal solution for (16) can be easily obtained from the solution of LMI-based problems. Such a point is illustrated in the numerical examples.

5. Numerical examples

5.1. Single input example

Let us consider the following single-input unstable example borrowed from Fong and Hsu (2000):

$$A = \begin{bmatrix} 0.5 & -1 \\ 1 & 0.5 \end{bmatrix}; \qquad B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}.$$

The nonlinear element $\varphi(Kx, t)$ is the one described in Fig. 2 (dead-zone + saturation) with $u_0 = 4, \mu = 1$ and $\sigma = 0.4$. The S-procedure parameter is set to: $\theta = 10^{-6}$. Let us first consider the controller gain provided in Fong and Hsu (2000): $K = K_a = [0.93\ -3.84]$. The optimization of the ellipsoid sizes with respect to the scalar β has been performed using the Matlab *fminsearch* function, from an initial guess 0.5, and with $\eta_1 = \eta_2 = 1$. The optimal inner and outer ellipsoidal sets solution to (16), plotted in Fig. 5 (dashed lines), have then been obtained for $\beta = 0.0125$.

The synthesis problem is classically a compromise between several objectives, in the present case, a small activity around the origin (small inner ellipsoid), a large domain of admissible initial state (large outer ellipsoid), a not-too-high gain and a convenient pole placement for the linear closed-loop system ($\varphi(Kx, t) = Kx$). In this example, one considers a pole-placement requirement in a disk of ray 5 and centered in -5 Peaucelle, Arzelier, Bachelier, and Bernussou (2000), and the optimization problem is solved with the trade-off objective given by $\eta_1 = \eta_2 = 1$. One then obtains the controller gain $K = K_s = \begin{bmatrix} 0.6750 & -8.1591 \end{bmatrix}$and $\beta = 0.0035$ as the solution to the encapsulation of problem (16) in *fminsearch*. The optimal inner and outer ellipsoids associated to this controller are plotted in Fig. 5 in solid lines. Trajectories starting from several initial states are also plotted in Fig. 5. They illustrate that the outer ellipsoid obtained through Theorem 1 with the optimization procedure (16) remains a good approximation of the domain of attraction.

A zoom of the inner ellipsoids and, by the way, of the trajectories of the system for the controller gains K_a (dashed lines) and K_s (solid

[1] Rigorously speaking, we do not compute the smallest and largest ellipsoids, due to the nonlinearities in the optimization problem. What we denote abusively the smallest and largest ellipsoids are only sub-optimal solutions, which depend also on the measure chosen to evaluate the size of the ellipsoid.

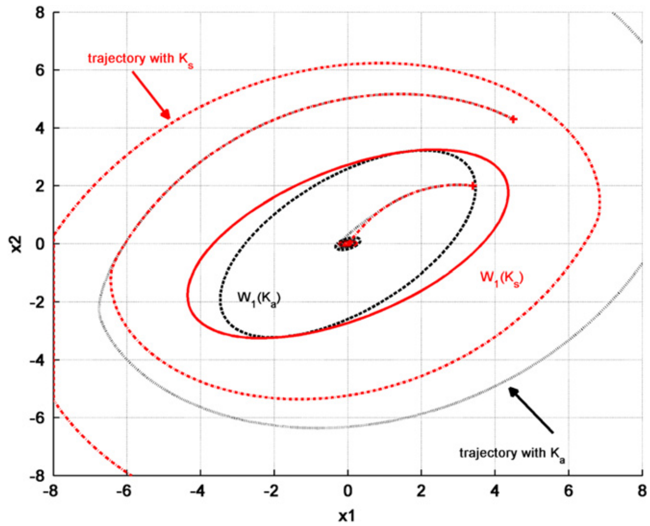

Fig. 5. Example 1—Inner and outer ellipsoids related to K_a (dashed lines) and to K_s (solid lines). State space trajectories initiated from various states, related to the controllers K_a (dotted line) K_s (dashed–dotted line).

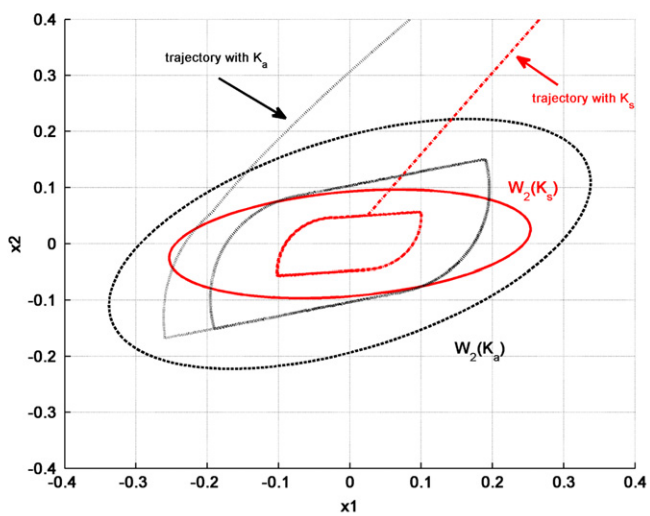

Fig. 6. Example 1—Zoom on the inner ellipsoids related to K_a (dashed lines) and to K_s (solid lines). State space trajectories related to the controllers K_a (dotted lines) K_s (dashed–dotted lines).

lines), is shown in Fig. 6. It illustrates that around the origin, the system is not controlled and does not converge to the origin but to a limit cycle. This limit cycle remains however confined inside the inner ellipsoid.

According to Remark 6 focusing on the case of saturation only, the ellipsoidal approximation of the basin of attraction with the controller K_a is plotted in Fig. 7 (in black). Note that when the optimization (in Theorem 1 framework) is oriented mainly on the maximal outer set solution (in blue, with $\eta_1 = 1, \eta_2 = 0.01$), it approaches the case with saturation only. However, this is to the detriment of the size of the inner ellipsoid. A compromise is given by the solution issued from Theorem 1 with $\eta_1 = \eta_2 = 1$ (in red).

5.2. Multivariable example

The second example is a multi-input example, with three states and two inputs derived from Amato, Cosentino, and Merola (2007), which intends to illustrate the computational burden associated to the nonlinear influence of β. System (3) is defined by the following

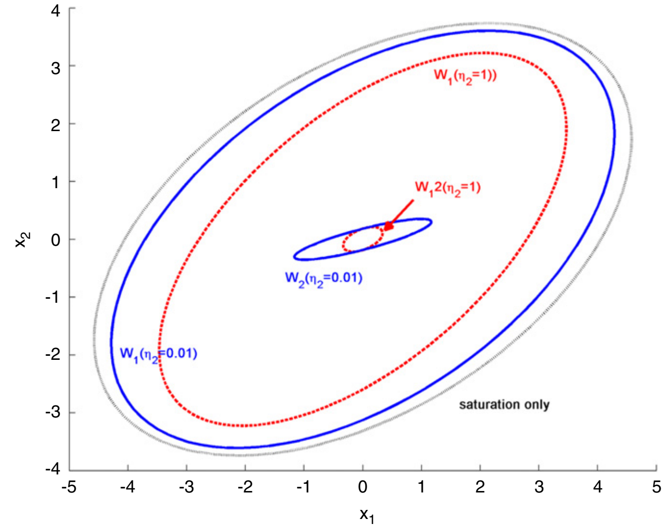

Fig. 7. Example 1—Ellipsoidal approximations of the basin of attraction. The dotted ellipsoid refers to the case with saturation only Alamo et al. (2005). The solid line outer (W_1) and inner (W_2) ellipsoids refer to the case with $\eta_2 = 0.01$. The dashed line outer (W_1) and inner (W_2) ellipsoids refer to the case with $\eta_2 = 1$.

Table 1
Illustration of the influence of β in the calculus of the optimal inner and outer ellipsoids.

	Case 1	Case 2
β	$\begin{bmatrix}0.7250\\0.7250\end{bmatrix}$	$\begin{bmatrix}0.1621\\0.8142\end{bmatrix}$
$\sqrt{\det(W)}$	0.0576	0.0283
$\sqrt{\det(Q)}$	1.3219	1.5099
nb iteration in *fminsearch*	8	26
nb execution pb (16)	16	52

data:

$$A = \begin{bmatrix} -0.5 & 1.5 & 4 \\ 4.3 & 6.0 & 5.0 \\ 3.2 & 6.8 & 7.2 \end{bmatrix}; \qquad B = \begin{bmatrix} -0.7 & -1.3 \\ 0 & -4.3 \\ 0.8 & -1.5 \end{bmatrix} \tag{17}$$

with nonlinear elements given by:

$$u_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \qquad \mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \sigma = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}.$$

Note that the open-loop system is unstable, with spectrum given by $\{13.9600; -0.6300 \pm 0.7368i\}$. One considers the following control gain:

$$K = \begin{bmatrix} -1.5120 & -2.5839 & -5.2308 \\ 2.0067 & 3.1215 & 4.3454 \end{bmatrix}$$

for which the linear closed-loop spectrum ($\varphi(Kx, t) = Kx$) is $\{-8.1394; -2.4180 \pm 1.6404i\}$.

Two cases are considered to determine the smallest and largest inner and outer ellipsoids, using *fminsearch* and problem (16): the first case is with $\beta = a1_m$, and the second case considers that all components of β are independent. The results, presented in Table 1 provide a comparison in terms of computation time and volume[2] of ellipsoids. They illustrate that considering only one parameter for β is sufficient to give a good approximation of the ellipsoids with a reasonable computation burden.

Finally, Figs. 8 and 9 show the outer and inner ellipsoids, respectively, for the two evaluation cases. State space trajectories

[2] The expression $\sqrt{\det(W)}$ is proportional to the volume of the ellipsoid $\mathcal{E}(W^{-1}, 1)$.

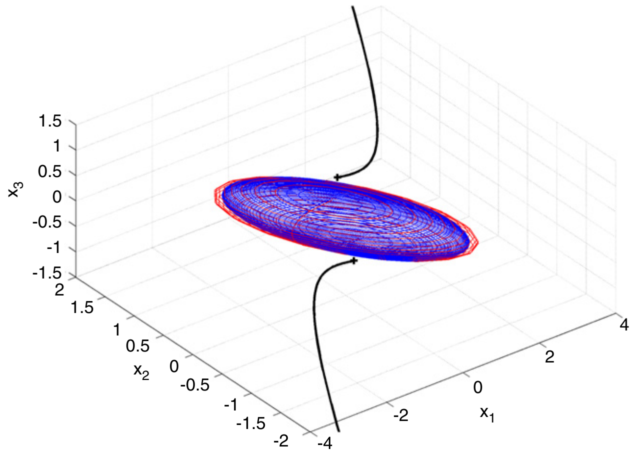

Fig. 8. Example 2—3D outer ellipsoids related to case 1 (internal blue ellipsoid) and case 2 (external red ellipsoid) and unstable state space trajectories. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

illustrate both the correct approximation of the overall invariant domain for the system (unstable trajectories for initial states taken outside the outer ellipsoids) and the confinement of the stable trajectories in the inner ellipsoids after some transient time.

6. Conclusion

In this paper, constructive conditions to deal with ultimate bounded stability analysis or stabilization have been proposed for systems interconnected with actuators involving different nonlinear elements, like for instance both saturation and dead-zone or both saturation and stick–slip. Appropriate properties allow to upper-bound the nonlinearity and then to derive quasi-linear matrix inequality conditions. Optimization schemes have been derived in order to evaluate two ellipsoids such that the trajectories initiated inside the largest outer one will be ultimately bounded in the smallest inner one, without any restriction on the open-loop stability. The main limitation of the approach is related to the optimization scheme which involves several tuning parameters. Some possible approaches are however proposed to select an adequate set of parameters.

In the context of actuators involving different nonlinear elements, there are still some possible interesting extensions and open problems. The technique developed should be extended to handle a larger class of nonlinear elements, like piecewise affine nonlinearities or dynamic nonlinearities, leading to the construction of more adequate Lyapunov functions (more complex than the quadratic ones).

Appendix

A.1. Proof of Property 1

Two cases are considered.

- If $z \geq 0$, one obtains from the inequality: $\varphi(u,t) \leq \Gamma(u)$, $z\varphi(u,t) \leq z\Gamma(u) = z\max\{\mu(u+\sigma), -u_0\} = \max\{z\mu(u+\sigma), -zu_0\} = \max\{z\mu u + |z|\mu\sigma, -|z|u_0\}$.
- If $z < 0$ then one can multiply the inequality $-\Gamma(-u) \leq \varphi(u,t)$ by the negative scalar z to obtain: $z\varphi(u,t) \leq -z\Gamma(-u) = |z|\max\{\mu(-u+\sigma), -u_0\} = \max\{|z|\mu(-u+\sigma), -|z|u_0\} = \max\{z\mu u + |z|\mu\sigma, -|z|u_0\}$.

A.2. Proof of Property 2

Following Property 1, one directly writes

$$z'\varphi(u,t) = \sum_{i=1}^{m} z_{(i)}\varphi_{(i)}(u,t) \leq \sum_{i=1}^{m} \max\{z_{(i)}\mu_{(i)}u_{(i)} + |z_{(i)}|\mu_{(i)}\sigma_{(i)}, -|z_{(i)}|u_{0(i)}\}.$$

Moreover one can verify that

$$\sum_{i=1}^{m} \max\{z_{(i)}\mu_{(i)}u_{(i)} + |z_{(i)}|\mu_{(i)}\sigma_{(i)}, -|z_{(i)}|u_{0(i)}\} = \max_{\mathcal{S}\in\mathcal{V}_m}\left\{\sum_{i\in\mathcal{S}^c} z_{(i)}\mu_{(i)}u_{(i)} + |z_{(i)}|\mu_{(i)}\sigma_{(i)} - \sum_{i\in\mathcal{S}} |z_{(i)}|u_{0(i)}\right\}.$$

A.3. Additional properties

Property 3. *Let a symmetric matrix $W \in \Re^{n\times n}$ and a matrix $Y \in \Re^{1\times n}$ being such that*

$$\begin{bmatrix} u_0^2 & Y \\ Y' & W \end{bmatrix} > \mathbf{0}$$

then, $\forall \overline{B} \in \Re^n$, one has

$$\overline{B}Y + Y'\overline{B}' \geq -\alpha u_0 W - u_0\frac{\overline{B}\,\overline{B}'}{\alpha}, \quad \forall\alpha > 0.$$

The proof is directly extended from Alamo et al. (2005), by considering that: $\mathbf{0} \leq u_0\left(\frac{\sqrt{\alpha}}{u_0}Y' + \frac{\overline{B}}{\sqrt{\alpha}}\right)\left(\frac{\sqrt{\alpha}}{u_0}Y' + \frac{\overline{B}}{\sqrt{\alpha}}\right)'$, $\forall\alpha > 0$.

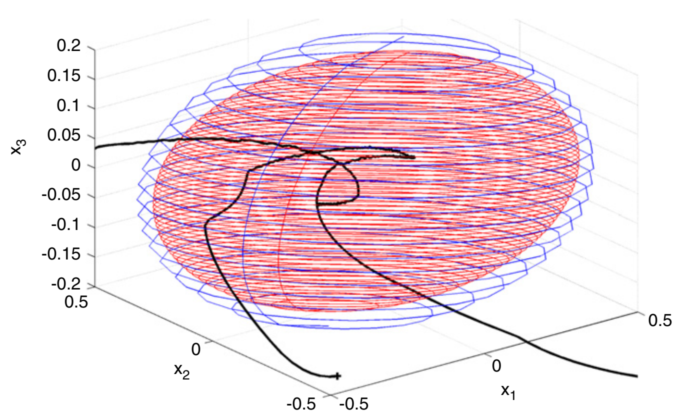

Fig. 9. Example 2—3D inner ellipsoids related to case 1 (external blue ellipsoid) and case 2 (internal red ellipsoid) and stable state space trajectories. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Property 4 (Alamo et al. 2005). *Suppose that $\overline{\epsilon} > 0$. Then, for every $a \in \Re$*

$$-2|a| < \sup_{\overline{\alpha}>0} -\overline{\alpha} - \frac{a^2}{\overline{\alpha}} + \overline{\epsilon}.$$

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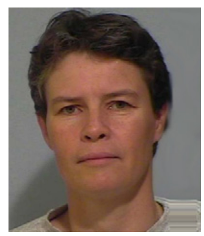

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