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MPC for tracking with optimal closed-loop performance[☆]

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ABSTRACT

In the recent paper [Limon, D., Alvarado, I., Alamo, T., & Camacho, E.F. (2008). MPC for tracking of piecewise constant references for constrained linear systems. *Automatica*, *44*, 2382–2387], a novel predictive control technique for tracking changing target operating points has been proposed. Asymptotic stability of any admissible equilibrium point is achieved by adding an artificial steady state and input as decision variables, specializing the terminal conditions and adding an *offset cost function* to the functional.

In this paper, the closed-loop performance of this controller is studied and it is demonstrated that the *offset cost function* plays an important role in the performance of the model predictive control (MPC) for tracking. Firstly, the controller formulation has been enhanced by considering a convex, positive definite and subdifferential function as the offset cost function. Then it is demonstrated that this formulation ensures convergence to an equilibrium point which minimizes the offset cost function. Thus, in case of target operation points which are not reachable steady states or inputs for the constrained system, the proposed control law steers the system to an admissible steady state (different to the target) which is optimal with relation to the offset cost function. Therefore, the offset cost function plays the role of a steady-state target optimizer which is built into the controller. On the other hand, optimal performance of the MPC for tracking is studied and it is demonstrated that under some conditions on both the offset and the terminal cost functions optimal closed-loop performance is locally achieved.

1. Introduction

Model predictive control (MPC) is one of the most successful techniques of advanced control in the process industry (Camacho & Bordons, 2004). Considering a suitable penalization of the terminal state and an additional terminal constraint, asymptotic stability and constraint satisfaction of the closed-loop system can be proved (Mayne, Rawlings, Rao, & Scokaert, 2000). Moreover, if the terminal cost is the infinite-horizon optimal cost of the unconstrained system, then the MPC control law results in being optimal in a neighborhood of the steady state (Hu & Linnemann, 2002).

These stabilizing conditions' terminal ingredients are suitable for a given operating point, but if the target operating point changes then the feasibility of the controller may be lost and the controller fails to track the reference (Pannocchia & Kerrigan, 2005; Shead & Rossiter, 2007). For such a case, the steady-state target can be determined by solving an optimization problem that determines the steady-state and input targets (Rao & Rawlings, 1999). In the literature some strategies have been proposed for recovering feasibility such as switching strategies (Chisci & Zappa, 2003; Rossiter, Kouvaritakis, & Gossner, 1996), or the command governors approach (Angeli, Casavola, & Mosca, 2000; Bemporad, Casavola, & Mosca, 1997). In Limon, Alvarado, Alamo, and Camacho (2008) a novel MPC for tracking is proposed, which is able to lead the system to any admissible set point in an admissible way. The main characteristics of this controller are: an artificial steady state considered as a decision variable, a cost that penalizes the error with the artificial steady state, an additional term that penalizes the deviation between the artificial steady state and the target steady state (the so-called *offset cost function*) and an extended terminal constraint, the invariant set for tracking. This controller ensures that under any change of the steady-state target, the closed-loop system maintains the feasibility of the controller, converging to the target if admissible. The additional ingredients of the controller have been demonstrated to affect the closed-loop performance of the controlled system (Alvarado, 2007). The objective of this paper is to study this effect and to show that the *offset cost function* plays an important role in the closed-loop performance.

Firstly, the MPC for tracking has been extended to consider a convex, positive definite and subdifferential function as the offset cost function. This choice ensures convergence to a set-point which minimizes the offset cost function and, moreover, allows the proposed MPC for tracking to deal with targets that are inconsistent with the prediction model or the constraints. In this

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case this control law steers the system to an admissible steady state (different to the target) which minimizes the offset cost function. This property means that the offset cost function plays the role of a steady-state target optimizer built in the proposed MPC. Furthermore, under some mild sufficient assumptions on the offset and the terminal cost function, a local optimality property holds, letting the controller achieve optimal closed-loop performance.

This paper is organized as follows. In the following section the constrained tracking problem is stated. In Section 3 the new MPC for tracking is presented, and in Section 4 the property of local optimality is introduced and proved. Finally some conclusions are drawn.

2. Problem description

Let a discrete-time linear system be described by

$$x^+ = Ax + Bu \tag{1}$$
$$y = Cx + Du$$

where $x \in \mathbb{R}^n$ is the current state of the system, $u \in \mathbb{R}^m$ is the current input, $y \in \mathbb{R}^p$ is the controlled output and x^+ is the successor state. Note that no assumption is considered on the dimension of the states, inputs and outputs, and hence non-square systems (namely $p > m$ or $p < m$) might be considered.

The controlled output is the variable used to define the target to be tracked by the controller. Since no assumption is made on matrices C and D, these variables might be (a linear combination of) the states, (a linear combination of) the inputs or (a linear combination of) both.

The state of the system and the control input applied at sampling time k are denoted as $x(k)$ and $u(k)$, respectively. The system is subject to hard constraints on state and control:

$$(x(k), u(k)) \in \mathcal{Z} \tag{2}$$

for all $k \geq 0$. $\mathcal{Z} \subset \mathbb{R}^{n+m}$ is a compact convex polyhedron containing the origin in its interior.

Assumption 1. The pair (A, B) is stabilizable and the state is measured at each sampling time.

Under this assumption, the set of steady states and inputs of the system (1) is an m-dimensional linear subspace of $\mathbb{R}^{n+m}$ Alvarado (2007) given by

$$(x_s, u_s) = M_\theta \theta.$$

Every pair of steady-state and input values $(x_s, u_s) \in \mathbb{R}^{n+m}$ is characterized by a given parameter $\theta \in \mathbb{R}^m$. The steady controlled outputs are given by

$$y_s = N_\theta \theta$$

where $N_\theta = [C\ D]M_\theta$.

The problem we consider is the design of an MPC controller $\kappa_N^O(x, y_t)$ to track a (possibly changing) target steady output y_t. If y_t is an admissible steady output (that is, the corresponding operation point fulfills the constraints), the closed-loop system evolves to this target without offset. If y_t is not consistent with the linear model considered for predictions, namely, it is not a possible steady output of system (1) or this is not admissible, the closed-loop system evolves to an admissible steady state which minimizes a given performance index.

3. Enhanced formulation of the MPC for tracking

In this section, the role of the *offset cost function* in the MPC for tracking (Limon et al., 2008) is studied. As will be demonstrated later on, under mild assumptions, this function provides significant properties to the controlled system.

The proposed cost function of the MPC is given by

$$V_N^O(x, y_t; \mathbf{u}, \bar{\theta}) = \sum_{i=0}^{N-1} \|x(i) - \bar{x}_s\|_Q^2 + \|u(i) - \bar{u}_s\|_R^2$$
$$+ \|x(N) - \bar{x}_s\|_P^2 + V_O(\bar{y}_s - y_t)$$

where $x(i)$ denotes the prediction of the state i-samples ahead, the pair $(\bar{x}_s, \bar{u}_s) = M_\theta \bar{\theta}$ is the artificial steady state and input and $\bar{y}_s = N_\theta \bar{\theta}$ the artificial output, all of them parameterized by $\bar{\theta}$; y_t is the target of the controlled variables. The controller is derived from the solution of the optimization problem $P_N^O(x, y_t)$ given by

$$V_N^{O*}(x, y_t) = \min_{\mathbf{u}, \bar{\theta}} V_N^O(x, y_t; \mathbf{u}, \bar{\theta})$$
$$\text{s.t. } x(0) = x,$$
$$x(j+1) = Ax(j) + Bu(j),$$
$$(x(j), u(j)) \in \mathcal{Z}, \quad j = 0, \ldots, N-1$$
$$(\bar{x}_s, \bar{u}_s) = M_\theta \bar{\theta},$$
$$\bar{y}_s = N_\theta \bar{\theta}$$
$$(x(N), \bar{\theta}) \in \Omega_{t,K}^w.$$

Considering the receding horizon policy, the control law is given by

$$\kappa_N^O(x, y_t) = u^*(0; x, y_t).$$

Since the set of constraints of $P_N^O(x, y_t)$ does not depend on y_t, its feasibility region does not depend on the target operating point y_t. Then there exists a polyhedral region $\mathcal{X}_N \subseteq X$ such that for all $x \in \mathcal{X}_N$, $P_N^O(x, y_t)$ is feasible. This is the set of initial states that can be admissibly steered to the projection of $\Omega_{t,K}^w$ onto x in N steps.

Consider the following assumption on the controller parameters:

Assumption 2. (1) Let $R \in \mathbb{R}^{m \times m}$ be a positive definite matrix and $Q \in \mathbb{R}^{n \times n}$ a positive semi-definite matrix such that the pair $(Q^{1/2}, A)$ is observable.
(2) Let the offset cost function $V_O : \mathbb{R}^p \to \mathbb{R}$ be a convex, positive definite and subdifferentiable function such that $V_O(0) = 0$.
(3) Let $K \in \mathbb{R}^{m \times n}$ be a stabilizing control gain such that $(A + BK)$ is Hurwitz.
(4) Let $P \in \mathbb{R}^{n \times n}$ be a positive definite matrix such that

$$(A + BK)^{\mathrm{T}} P (A + BK) - P = -(Q + K^{\mathrm{T}} R K).$$

(5) Let $\Omega_{t,K}^w \subseteq \mathbb{R}^{n+m}$ be an admissible polyhedral invariant set for tracking for system (1) subject to (2), for a given gain K. That is, for all $(x, \theta) \in \Omega_{t,K}^w$, then $((A + BK)x + BL\theta, \theta) \in \Omega_{t,K}^w$ where $L = [-K\ I_m]M_\theta$. See Limon et al. (2008) for more details.

The set of admissible steady outputs consistent with the invariant set for tracking $\Omega_{t,K}^w$ is given by

$$\mathcal{Y}_s = \{y_s = N_\theta \theta : (x_t, u_t) = M_\theta \theta, \text{ and } (x_t, \theta) \in \Omega_{t,K}^w\}.$$

This set is potentially the set of all admissible outputs for system (1) subject to (2), (Limon et al., 2008).

Taking into account the proposed conditions on the controller parameters, the following theorem proves the asymptotic stability and constraint satisfaction of the controlled system.

Theorem 1 (*Stability*). *Consider that Assumptions 1 and 2 hold and consider a given target operation point y_t. Then for any feasible initial state $x_0 \in \mathcal{X}_N$, the system controlled by the proposed MPC controller $\kappa_N^O(x, y_t)$ is stable, fulfills the constraints throughout the time and, if $y_t \in \mathcal{Y}_s$, converges to an equilibrium point y_t such that $\lim_{k\to\infty} \|y(k) - y_t\| = 0$. If $y_t \notin \mathcal{Y}_s$, the closed-loop system asymptotically converges to a steady state and input (x_s^*, u_s^*) and $y_s^* = Cx_s^* + Du_s^*$, where*

$$y_s^* = \arg\min_{y_s \in \mathcal{Y}_s} V_O(y_s - y_t).$$

Proof. Feasibility and convergence can be proved by following a similar procedure to Limon et al. (2008).

The proof will be finished by demonstrating that $(\bar{x}_s^*, \bar{u}_s^*)$ is the minimizer of the offset cost function $V_O(\bar{y}_s - y_t)$, proving the second assertion of the theorem. The first assertion is a direct consequence of the latter.

This result is obtained by contradiction. Consider the following set of the optimal solutions:

$$\Gamma = \{\bar{y}_s : \bar{y}_s = \arg\min_{\bar{y}_s \in \mathcal{Y}_s} V_O(\bar{y}_s - y_t)\}.$$

Consider that $\bar{y}_s^* \notin \Gamma$. Then there exists a $\tilde{y}_s \in \Gamma$, such that $V_O(\tilde{y}_s - y_t) < V_O(\bar{y}_s^* - y_t)$. Define $\tilde{\theta}$ as a parameter (contained in the projection of $\Omega_{t,K}^w$ onto θ) such that $\tilde{y}_s = N_\theta \tilde{\theta}$.

In can be proved (Alvarado, 2007) that there exists a $\hat{\lambda} \in [0, 1)$ such that for every $\lambda \in [\hat{\lambda}, 1)$, the parameter $\hat{\theta} = \lambda\bar{\theta}^* + (1 - \lambda)\tilde{\theta}$ is such that the control law $u = Kx + L\hat{\theta}$ (with $L = [-K, I_m]M_\theta$) steers the system from $\bar{x}_s^*$ to $\hat{x}_s$ fulfilling the constraints.

Defining as $\mathbf{u}$ the sequence of control actions derived from the control law $u = K(x - \hat{x}_s) + \hat{u}_s$, it is inferred that $(\mathbf{u}, \bar{x}_s^*, \hat{\theta})$ is a feasible solution for $P_N^O(\bar{x}_s^*, y_t)$ (Limon et al., 2008). From Assumption 2,

$$\begin{aligned} V_N^{O*}(\bar{x}_s^*, y_t) &\le V_N^O(\bar{x}_s^*, y_t; \mathbf{u}, \hat{y}_s) \\ &= \|\bar{x}_s^* - \hat{x}_s\|_P^2 + V_O(\hat{y}_s - y_t). \end{aligned}$$

Then, defining $H = M_x^{\mathrm{T}} P M_x$ and considering the previous statements,

$$\begin{aligned} V_N^O(\bar{x}_s^*, y_t; \mathbf{u}, \hat{y}_s) &= \|\bar{x}_s^* - \hat{x}_s\|_P^2 + V_O(\hat{y}_s - y_t) \\ &= \|\bar{\theta}_s^* - \hat{\theta}_s\|_H^2 + V_O(\hat{y}_s - y_t) \\ &= (1 - \lambda)^2 \|\bar{\theta}_s^* - \tilde{\theta}_s\|_H^2 + V_O(\hat{y}_s - y_t). \end{aligned}$$

The partial of V_N^O about λ is

$$\frac{\partial V_N^O}{\partial \lambda} = -2(1 - \lambda)\|\bar{\theta}_s^* - \tilde{\theta}_s\|_H^2 + g^{\mathrm{T}}(\bar{y}_s^* - \tilde{y}_s)$$

where $g^{\mathrm{T}} \in \partial V_O(\hat{y}_s - y_t)$, defining $\partial V_O(\hat{y}_s - y_t)$ as the subdifferential of $V_O(\hat{y}_s - y_t)$, (Boyd & Vandenberghe, 2006). Evaluating this partial for $\lambda = 1$ we obtain that

$$\left.\frac{\partial V_N^O}{\partial \lambda}\right|_{\lambda=1} = g^{*T}(\bar{y}_s^* - \tilde{y}_s)$$

where $g^{*T} \in \partial V_O(\bar{y}_s^* - y_t)$, defining $\partial V_O(\bar{y}_s^* - y_t)$ as the subdifferential of $V_O(\bar{y}_s^* - y_t)$. Taking into account that V_O is a subdifferentiable function, we can state that

$$\left.\frac{\partial V_N^O}{\partial \lambda}\right|_{\lambda=1} = g^{*T}(\bar{y}_s^* - \tilde{y}_s) \ge V_O(\bar{y}_s^* - y_t) - V_O(\tilde{y}_s - y_t).$$

Considering that $V_O(\bar{y}_s^* - y_t) - V_O(\tilde{y}_s - y_t) > 0$, it can be derived that there exists a $\lambda \in [\hat{\lambda}, 1)$ such that $V_N^O(\bar{x}_s^*, y_t; \mathbf{u}, \hat{y}_s)$ is smaller than the value of $V_N^O(\bar{x}_s^*, y_t; \mathbf{u}, \hat{y}_s)$ for $\lambda = 1$, which is equal to $V_N^{O*}(\bar{x}_s^*, y_t)$.

This contradicts the optimality of the solution and hence the result is proved, finishing the proof. □

Remark 1 (*Steady-State Optimization*). It is not unusual that the output target y_t is not contained in $\mathcal{Y}_s$. This may happen when there does not exist an admissible operating point whose steady output is equal to the target or when the target is not a possible steady output of the system (that is, this is not in the subspace spanned by the columns of matrix N_θ). To deal with this situation in predictive controllers, the standard solution is to add an upper level steady-state optimizer to decide the best reachable target of the controller (Rao & Rawlings, 1999; Tatjewski, 2008).

From the latter theorem it can be clearly seen that, in this case, the proposed controller steers the system to the optimal operating point according to the offset cost function $V_O(\cdot)$. Then it can be considered that the proposed controller has a steady-state optimizer built in and $V_O(\cdot)$ defines the function to optimize. Notice that the only mild assumptions on this function are to be convex, positive definite, subdifferentiable and zero when the entry is null (to ensure offset-free control if $y_t \in \mathcal{Y}_s$).

Remark 2 (*Offset Cost Function and Stability*). Taking into account Theorem 1, stability is proved for any offset cost function satisfying Assumption 2. Therefore, if this cost function varies with the time, the results of the theorem still hold.

This property allows us to tune the cost function along the time maintaining the stabilizing properties of the controller. Besides, this property can be exploited to consider an offset cost function which depends on the target, namely $V_O(y_t; \bar{y}_s - y_t)$ defining different optimal criterion for the operating point selection depending on the chosen target.

Remark 3 (*QP Formulation*). The optimization problem $P_N^O(x, y_t)$ is a convex mathematical programming problem that can be efficiently solved. In the case that the offset cost function $V_O(y_t; \bar{y}_s - y_t)$ is such that the region $\{\bar{y}_s : V_O(y_t; \bar{y}_s - y_t) \le 0\}$ is polyhedral, then $P_N^O(x, y_t)$ can be posed as a quadratic programming by means of an epigraph formulation.

Remark 4 (*Robustness*). Taking into account that the control law is derived from a parametric convex problem, the closed-loop system is input-to-state stable for small uncertainties (Limon et al., 2008). In Alvarado, Limon, Alamo, Fiacchini, and Camacho (2007) a robust formulation of this controller has been proposed. In this case, offset free control can be achieved by means of disturbances models (Pannocchia & Kerrigan, 2005) or adding an external loop (Alvarado, 2007).

Remark 5 (*Terminal Equality Constraint*). Following the same arguments, it can be proved that the results of Theorem 1 still hold when posing the terminal constraint as an equality constraint, by considering $(\bar{x}_s, \bar{u}_s) \in \mathcal{Z}$, $x(N) = \bar{x}_s$ and $P = 0$.

4. Local optimal control

Assume that the standard MPC control law to regulate the system to the target y_t, $\kappa_N^r(x, y_t)$ is derived from the solution of $P_N^O(x, y_t)$ subject to $\bar{y}_s = y_t$. The resulting optimization problem, denoted as $P_N^r(x, y_t)$, is feasible for any x in a polyhedral region denoted as $\mathcal{X}_N^r(y_t)$. Under certain assumptions (Mayne et al., 2000), for any feasible initial state $x \in \mathcal{X}_N^r(y_t)$, the control law $\kappa_N^r(x, y_t)$ steers the system to the target fulfilling the constraints. However, this control law is suboptimal since the cost function of the MPC is only minimized for a finite prediction horizon, and hence the MPC does not ensure the best closed-loop performance. Fortunately, as stated in the following lemma, if the terminal cost function is the optimal cost of the unconstrained LQR, then the resulting finite horizon MPC is equal to the constrained LQR in a neighborhood of the terminal region (Bemporad, Morari, Dua, & Pistikopoulos, 2002; Hu & Linnemann, 2002).

Lemma 6. *Consider that Assumptions 1 and 2 hold. Consider that the terminal control gain K is the one of the unconstrained linear quadratic regulator. Let θ_t be the parameter such that $y_t = N_\theta \theta_t$. Define the set $\Upsilon_N(y_t) \subset \mathbb{R}^n$ as $\Upsilon_N(y_t) = \{\bar{x} \in \mathbb{R}^n : \phi(N; \bar{x}, \kappa_\infty(\cdot, y_t), \theta_t) \in \Omega_{t,K}^w\}$. Then for all $x \in \Upsilon_N(y_t)$, $V_N^{r,*}(x, y_t) = V_\infty^*(x, y_t)$ and $\kappa_N^r(x, y_t) = \kappa_\infty(x, y_t)$.*

The proposed MPC for tracking might not ensure this local optimality property under the assumptions of Lemma 6 due to the artificial steady state and input and the functional cost to minimize (Alvarado, 2007). However, as is demonstrated in the following property, under some conditions on the offset cost function $V_O(\cdot)$, this property holds.

Assumption 3. Let the offset cost function $V_O(\cdot)$ fulfill Assumption 2.2 and be such that

$$\alpha_1\|y\| \leq V_O(y) \leq \alpha_2\|y\|, \quad \forall y \in \mathcal{Y}_s$$

where α_1 and α_2 are positive real constants.

Property 1 (*Local Optimality*). *Consider that Assumptions 1–3 hold. Then, for all $x \in \mathcal{X}_N^r(y_t)$ there exists an $\alpha^* > 0$ such that, for all $\alpha_1 \geq \alpha^*$:*

- *The proposed MPC for tracking is equal to the MPC for regulation; that is, $\kappa_N^O(x, y_t) = \kappa_N^r(x, y_t)$ and $V_N^{O*}(x, y_t) = V_N^{r*}(x, y_t)$ for all $x \in \mathcal{X}_N^r(y_t)$.*
- *If the terminal control gain K is the one of the unconstrained linear quadratic regulator, then the MPC for tracking control law $\kappa_N^O(x, y_t)$ is equal to the optimal control law $\kappa_\infty(x, y_t)$ for all $x \in \Upsilon(y_t)$.*

Proof. In virtue of the well-known result on the exact penalty functions (Luenberger, 1984), the constant α^* can be chosen as the Lagrange multiplier of the equality constraint $\|\bar{y}_s - y_t\|_1 = 0$ of the optimization problem $P_N^r(x, y_t)$. Since the optimization problem depends on the parameters (x, y_t), the value of this Lagrange multiplier also depends on (x, y_t).

Define the optimization problem $P_{N,\alpha}^m(x, y_t)$ as a particular case of $P_N^O(x, y_t)$ with $V_O(\bar{y}_s - y_t) \triangleq \alpha\|\bar{y}_s - y_t\|_1$. This optimization problem $P_{N,\alpha}^m(x, y_t)$ results from the optimization problem $P_N^r(x, y_t)$ with the last constraint posed as an exact penalty function. Therefore, there exists a finite constant $\alpha^* > 0$ such that for all $\alpha \geq \alpha^*$, $V_{N,\alpha}^{m*}(x, y_t) = V_N^{r*}(x, y_t)$ for all $x \in \mathcal{X}_N^r(y_t)$ (Boyd & Vandenberghe, 2006; Luenberger, 1984).

Consider that $V_O(y) \leq \alpha_2\|y\|$. Then

$$V_{N,\alpha_1}^{m*}(x, y_t) \leq V_N^{O*}(x, y_t) \leq V_{N,\alpha_2}^{m*}(x, y_t).$$

Since $\alpha_2 \geq \alpha_1 \geq \alpha^*$, we have that for all $x \in \mathcal{X}_N^r(y_t)$

$$V_N^{r*}(x, y_t) \leq V_N^{O*}(x, y_t) \leq V_N^{r*}(x, y_t)$$

and hence $V_N^{O*}(x, y_t) = V_N^{r*}(x, y_t)$.

The second claim is derived from Lemma 6, observing that $\Upsilon_N(y_t) \subseteq \mathcal{X}_N^r(y_t)$. □

In order to ensure the local optimality property, the constant α^* should be chosen as the maximum of the Lagrange multiplier in the set of the parameters $(x, y_t) \in \mathcal{X}_N \times \mathcal{Z}_s$. In Rao and Rawlings (1999), the authors state that, in theory, a conservative state-dependent bound for the Lagrange multipliers may be obtained by the use of the Lipschitz continuity of the quadratic programming. In Kerrigan and Maciejowski (2000) the authors propose a way to solve this problem, based on the multi-parametric quadratic programming proposed in Bemporad et al. (2002).

The authors are currently studying the problem of calculating α^* and characterizing the region in which the property of local optimality holds. In Ferramosca, Limon, Alvarado, Alamo, and Camacho (2008) it is proposed a method for calculating a value of α^* by means of a single LP, for which the local optimality region is the invariant set for tracking.

5. Conclusions

In this paper, the role of the *offset cost function* has been studied. In particular a convex, positive definite and subdifferential function is considered.

Under some assumptions, it is proved that the proposed controller steers the system to a point which minimizes the offset cost function. This point is the target if it is admissible. If not, the controller converges to an admissible steady-state optimum according to the offset cost function. Besides, the closed-loop evolution is also optimal in the sense that provides the best possible performance index.

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