First steps in the study of weakly dissipative two-dimensional maps*

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Key words: weakly dissipative dynamics, twist map approximations, splitting of separatrices

Abstract

The effect of tiny dissipative perturbations in area-preserving maps (APM) is considered. In an equivalent way this can be applied to model the behaviour of Hamiltonian systems with $1\frac{1}{2}$ or 2 degrees of freedom under the effect of a small friction.

The study is done about elliptic fixed points of APM and the dissipation is radial. That is, if $(0,0)$ is such a fixed point and the APM map is $(x,y) \to T(x,y)$, maps of the form $(1-\varepsilon)\circ T$ are considered for very small $\varepsilon > 0$.

For sufficiently small dissipation, in a neighbourhood of an elliptic fixed point, which becomes an attracting focus, some periodic points related to different resonances survive. The invariant manifolds of the hyperbolic ones determine the basin of attraction of the elliptic ones.

Increasing the dissipation parameter produces the destruction of some resonances. Several figures describing the process of destruction will be also shown.

Formulas for the measure of the points captured into any of the resonances which survive are obtained for general maps around an elliptic fixed point. The range of the dissipation parameter where these formulas give a good approximation is discussed.

The illustrations correspond to a dissipative perturbation of the classical Hénon map, but the same patterns are proved to exist for generic maps.

*oral communication.