Prevalence of exponential stability among nearly-integrable Hamiltonian systems *

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Abstract

In the 70's, Nekhorochev proved that for an analytic nearly integrable Hamiltonian system, the action variables of the unperturbed Hamiltonian remain nearly constant over an exponentially long time with respect to the size of the perturbation, provided that the unperturbed Hamiltonian satisfies some generic transversality condition known as *steepness*. Recently, Guzzo has given examples of exponentially stable integrable Hamiltonians which are non steep but satisfy a weak condition of transversality which involves only the affine subspaces spanned by integer vectors.

We generalize this notion for an arbitrary integrable Hamiltonian and prove the Nekhorochev's estimates in this setting. The point in this refinement lies in the fact that we can exhibit a *generic* class of real analytic integrable Hamiltonians which are exponentially stable with *fixed* exponents.

Genericity is proved in the sense of measure since we exhibit a prevalent set of integrable Hamiltonian which satisfy the latter property. This is obtained by an application of a quantitative Sard theorem given by Yomdin.

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