Shadowing orbits for chains of invariant tori*

Marian Gidea   Clark Robinson

Key words: Shadowing orbits, correctly aligned windows, Arnold diffusion.

Abstract

Let $F : M \to M$ be a C^2-diffeomorphism of a smooth d-dimensional manifold M, and A be an annulus embedded in M. Assume that A is normally hyperbolic, and the restriction f of F to A is a twist map. Consider a sequence of invariant one-dimensional C^0-Lipschitz tori $\{T_i\}_{i\in\mathbb{Z}}$ (essential invariant circles) in the annulus. Assume that the restriction of f to each torus is topologically transitive. Each torus has an unstable manifold $W^u(T_i)$ and a stable manifold $W^s(T_i)$. Assume that for each $i \in \mathbb{N}$, one of the following two conditions hold: either the unstable manifold $W^u(T_i)$ of the torus T_i is topologically crossing the stable manifold $W^s(T_{i+1})$ of the torus T_{i+1} at a point away from the annulus, or there is no invariant torus contained in the region bounded by T_i and T_{i+1}. Then there exists an orbit of F that shadows the sequence of tori. The proof is based on the method of correctly aligned windows. This problem is related to the Arnold diffusion problem.

*oral communication