Exponentially small splitting of heteroclinic orbits in a family in $\mathbb{R}^3$ *

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Key words: exponentially small splitting of separatrices, Hopf-zero singularity.

Abstract

The phenomenon known as splitting of separatrices has been widely studied by several authors. This phenomenon arises, for instance, when we consider a differential equation in $\mathbb{R}^2$ with a fixed point having coincident branches of stable and unstable manifolds and we perturb it by a periodic or quasi periodic function on time.

In this paper we deal with a slightly different setting. We give an asymptotic expression of the splitting of a heteroclinic orbit in a near integrable vector field of $\mathbb{R}^3$. Such asymptotic will be exponentially small in our perturbative parameter δ.

The fields under consideration in this work are of the form:

$$\begin{aligned}
\frac{dx}{d\tau} &= -\delta xz - y\,(\alpha + c\delta z) + \delta^{p+1} f(\delta x, \delta y, \delta z, \delta) \\
\frac{dy}{d\tau} &= -\delta yz + x\,(\alpha + c\delta z) + \delta^{p+1} g(\delta x, \delta y, \delta z, \delta) \\
\frac{dz}{d\tau} &= \delta \;\; -1 + b(x^2+y^2) + z^2 \;\; + \delta^{p+1} h(\delta x, \delta y, \delta z, \delta),
\end{aligned} \tag{1}$$

where $p > -2$, and f, g, h are real analytic functions in all their variables, whose Taylor series begin at least with terms of degree three.

The motivation to study this problem is that systems of the form (1) with $p = -2$ come from generic unfoldings of the Hopf-zero singularity after normal form procedure up to order two and suitable scalings of the variables and parameters. The authors hope that this work will be a first step towards to prove the existence of Silnikov bifurcations for generic analytic unfoldings of the Hopf-zero singularity.

*oral communication.