An exact decomposition framework for the electric location-routing problem with heterogeneous fleet

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Agenda

Electric vehicles in transportation

Current situation

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Targets and international agreements

- Carbon neutrality over the next 30 years
- Low emission zones (LEZ)
- EU green deal $\Rightarrow$ ***locating*** charging stations at
  - highways
  - airports
  - ports

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- Public transport vehicles (buses)
- **Logistics vehicles**

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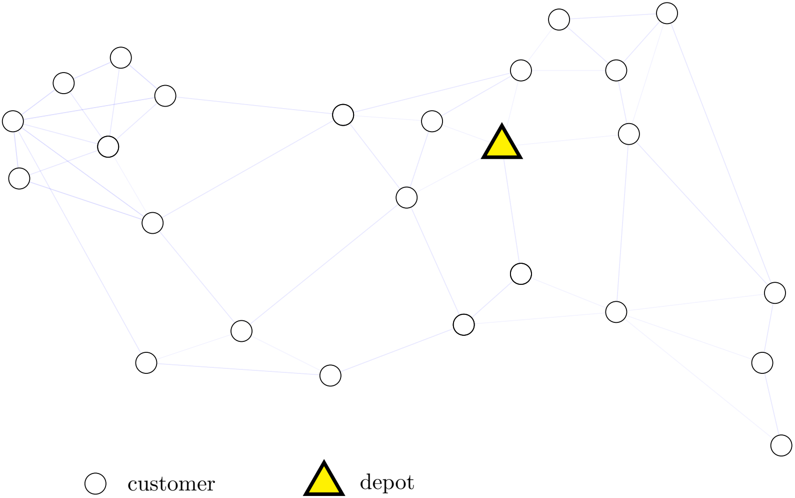
  - lack of charging stations
- Autonomous vehicles

The electric location-routing problem (ELRP)



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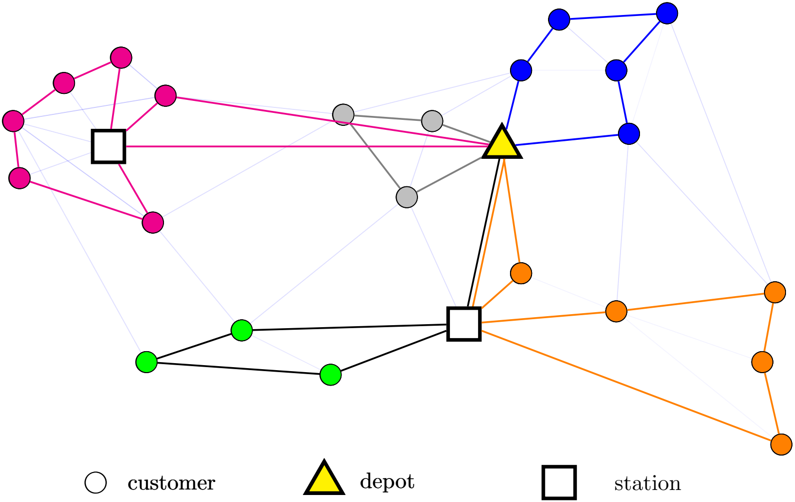

Most relevant studies to the ELRP

ELRP with time windows and partial recharging (ELRP-TWPR):
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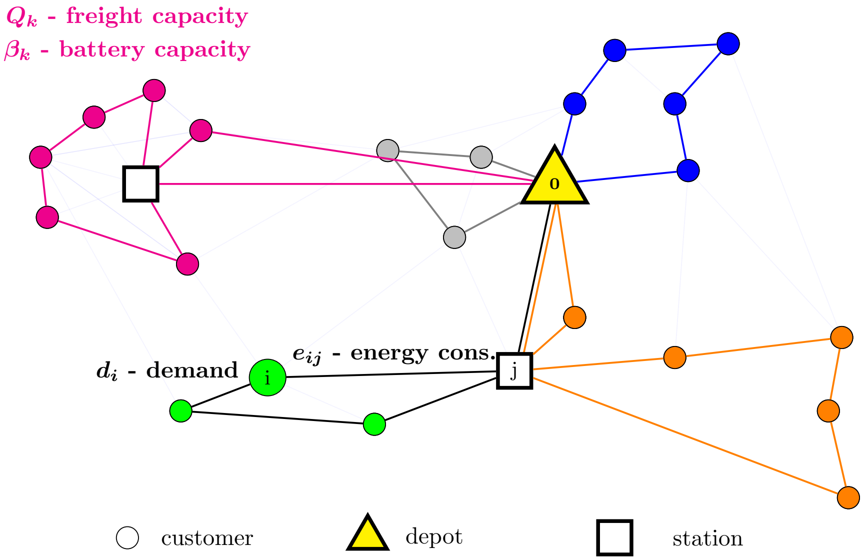
Our problem: ELRP with partial recharging for **heterogeneous fleet** (no time windows)

Notation - The physical network

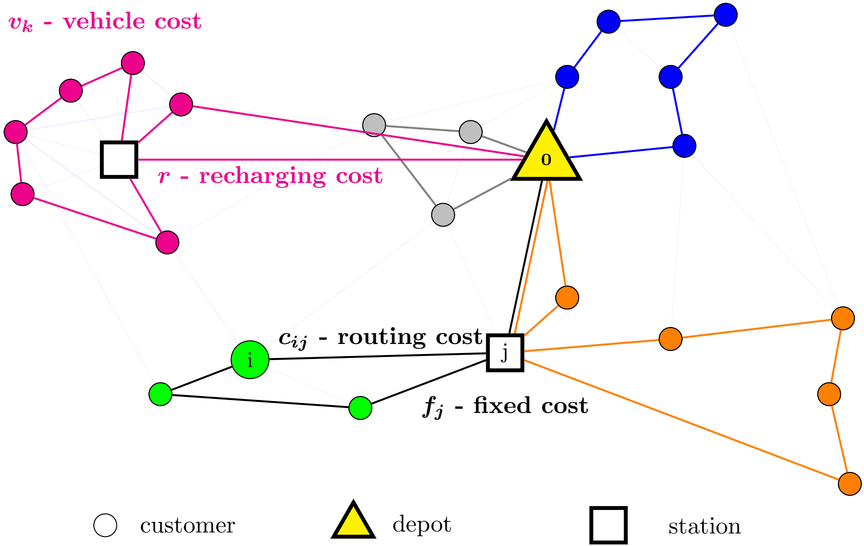
- $G = (N, A)$ with node set $N = I \cup J \cup \{0\}$ and arc set A
  - $'0'$ is the depot node.
  - $I = \{1, \ldots, n\}$ is the set of customer locations.
  - J is the set of potential locations for charging stations (not necessarily identical).
  - I and J are **not necessarily disjoint** (we assume $I \subset J$).
- K is the (finite) set of vehicles (**non-identical**).

(Çalık et al., 2021)

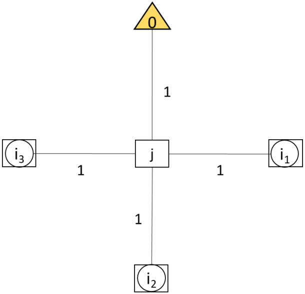
Notation - Capacity and range parameters



Notation - Cost parameters



Notation - Modeling challenges: an extreme case ($\beta = 1$)



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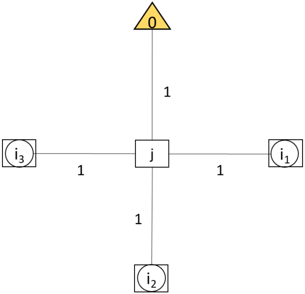

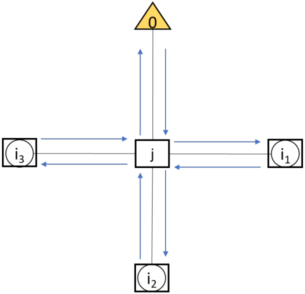

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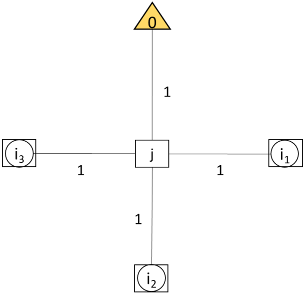

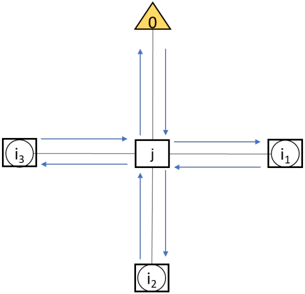

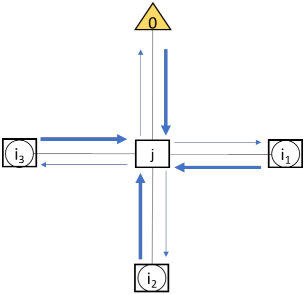

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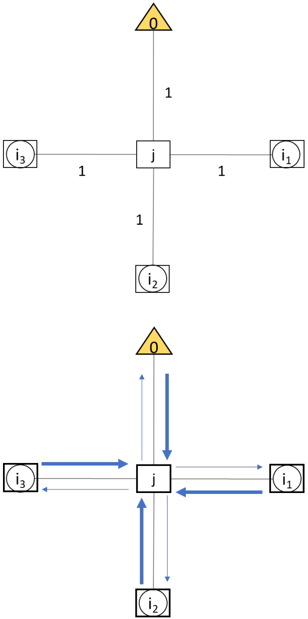
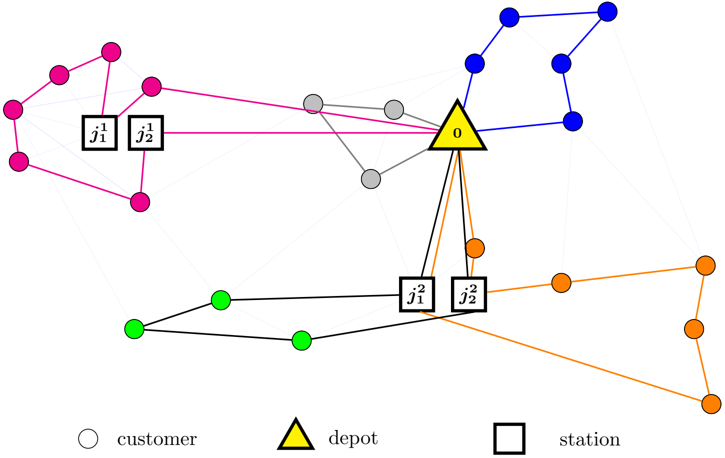


Notation - Multi-visit network $G' = (N', A')$

Multiple copies of potential stations for multiple visits by the same vehicle



- No arcs between copies of the same station
- $f_j = d_j = 0$ for additional copies

How difficult is the problem?

Small EVRP-TW instances (Schneider et al. (2014))

$\|I\|$	5	10	15
$\|J\|$	7 - 8	12 - 14	17 - 22

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The state-of-the-art exact LRP method solves instances with less than 100 nodes (Contardo et al., 2014).

Pre-processing

- Eliminate infeasible arcs
  - violating freight capacities
  - violating battery restrictions
- Find a lower bound on the number of vehicles (*nVMin*)
  $\Rightarrow$ solve a bin packing problem with $\boldsymbol{Q}, \boldsymbol{d}$
- Find a lower bound on the number of stations (*nSMin*)
  $\Rightarrow$ solve a minimum (set) covering problem with $\boldsymbol{\beta}, \boldsymbol{e}$
- Find an upper bound on the number of copies needed
  $\Rightarrow$ solve a knapsack problem with $Q^{max}, \boldsymbol{d}$

Problem Formulation - PF

Objective function

$$\min \sum_{k\in K}\sum_{(i,j)\in A^k} c_{ij}x_{ij}^k + \sum_{k\in K}\sum_{j\geq 1} rz_j^k + \sum_{j\geq 1} f_j y_j + \sum_{k\in K}\sum_{(0,i)\in A^k} v_k x_{0i}^k \qquad (1)$$

- $y_j = 1$ if station $j \in J$ is open, 0 otherwise.
- $x_{ij}^k = 1$ if arc (i,j) is traversed by vehicle $k \in K$, 0 otherwise.
- $z_j^k \geq 0$ is the amount of energy recharged at station $j \in J$ for $k \in K$.

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Lexicographic station selection

$$y_i \leq y_j, \qquad i \in J_j^A : i \neq j \qquad (2)$$

Problem Formulation - PF (cont'd.)

Sub-tour elimination - commodity flows (Yaman, 2006)

$$\sum_{(i,j)\in A^k} (l_{ij}^k - d_i x_{ij}^k) = \sum_{(j,i)\in A^k} l_{ji}^k, \qquad i \geq 1, \forall k \in K \tag{3}$$

$$\sum_{j\geq 1} l_{0j}^k = 0, \qquad \forall k \in K \tag{4}$$

$$l_{ij}^k \leq Q^k x_{ij}^k, \qquad \forall k \in K, (i,j) \in A^k \tag{5}$$

- $l_{ij}^k \geq 0$ is the cumulative load of vehicle k at node i before leaving for node j.

Routing constraints

x and **y** variables.

Battery related constraints

- $b_{ij}^k \geq 0$ is the battery level of vehicle k at node i before leaving for node j.

General algorithmic framework

Phase I

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Intermediate reduction procedure

- For each station j, calculate a lower bound Z^{LB}_{mjk} for $m \geq 2$ visits to j with vehicle k.
- If $Z^{LB}_{mjk} > Z^1$, no need for m visits to j.
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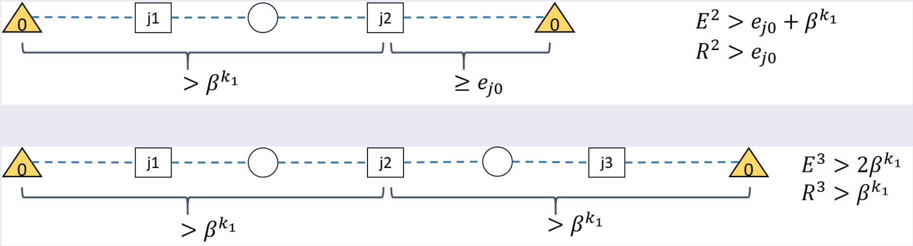
Phase II

- Solve the reduced PF to optimality.

Intermediate reduction process - a closer look

Three types of lower bounds on the total cost:

LB1: Minimum recharging need

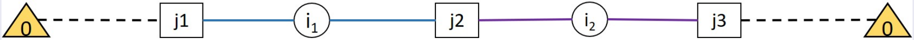


E^m: the energy consumption needed for m visits, and
R^m: the amount of recharging needed for m visits.

Intermediate reduction process - a closer look

You visit a different customer between every two visits to j.

LB2: Partial network structure



$$E^3 \geq E^C = e_{j0} + e_{ji_1} + e_{i_1 j} + e_{ji_2} + e_{i_2 j} + e_{j0} \qquad R^3 \geq E^C - \beta^{k_1}$$

E^m: the energy consumption needed for m visits, and
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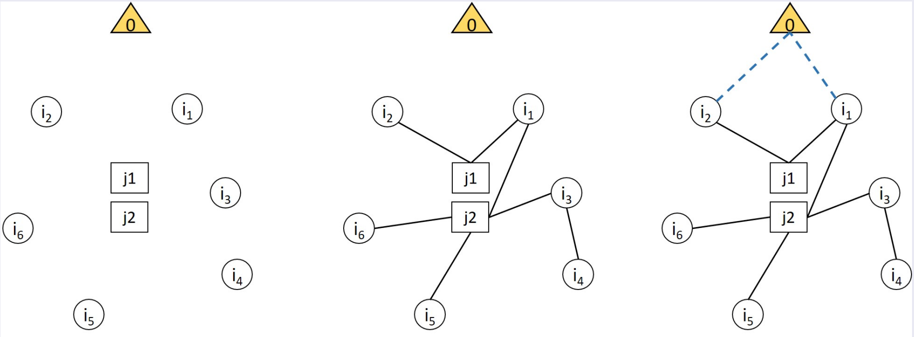
Intermediate reduction process - a closer look

Lower bounds for TSP are lower bounds for VRP as well.

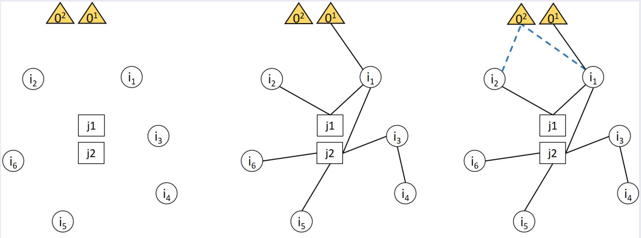
Minimum spanning tree and 1-tree relaxations.

LB3: A 1-tree lower bound (single vehicle)

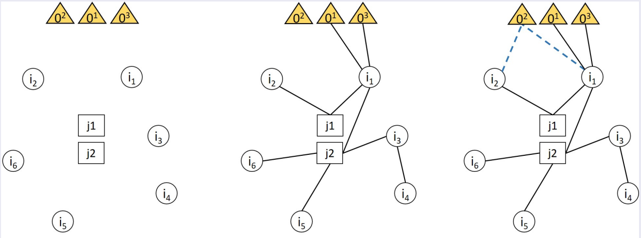


Intermediate reduction process - a closer look

1-tree lower bound for two vehicles



1-tree lower bound for three vehicles

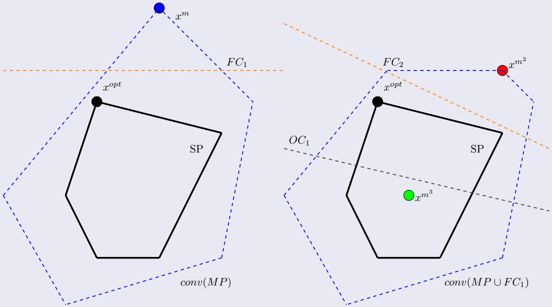


Solving the restrictive PF and reduced PF more efficiently

- Still difficult to solve (Schiffer and Walther, 2017b) ⇒ decomposition.

Benders decomposition - Benders (1962)

- First-stage variables (binary, integral) ⇒ Master problem (MP)
- Second-stage variables (continuous) ⇒ Sub-problem (SP)



- Extreme rays of dual SP (DSP) ⇒ feasibility cuts (FC)
- Extreme points of DSP ⇒ optimality cuts (OC)

Structure of our Benders implementation (BDA)

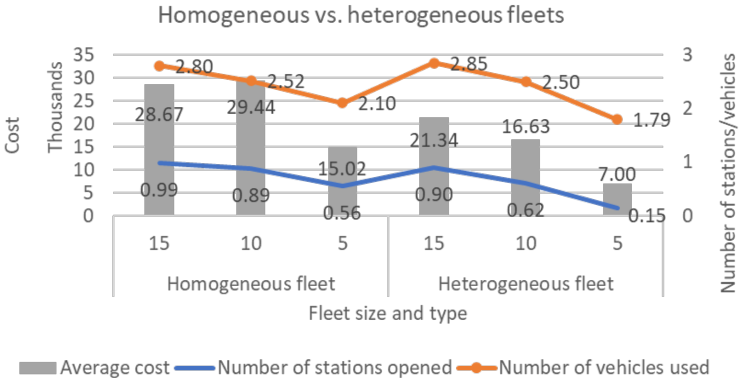
- First-stage variables $(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{l})$: binary, continuous $\Rightarrow$ Master problem (MP)
- Second-stage variables $(\boldsymbol{z}, \boldsymbol{b})$: continuous $\Rightarrow$ Sub-problem (SP)
- Feasibility cuts only
  - MP' with $w^k = \sum_{j \in J^A} z_j^k \geq 0, \forall k \in K$
  - SP decomposes into vehicle routes $\Rightarrow SP_k, \forall k \in K$
- Branch-and-cut
- Valid inequalities

Computational Study - Instances

Test instances and the environment

- Small EVRP-TW instances (Schneider et al., 2014)
- $|I|$=5, 10, 15 and $|J|$=7-22
- **Three types of vehicles** S, M, L (30000 €- 65000 €- 5 years):
- Up to 2-4 vehicles (**different fleet configurations**)
- Fast charging units (8000 €- 3 years)
- Time limit: 3600, 10800 seconds
- Memory Limit: 16 GB
- IBM ILOG CPLEX 12.8 (single thread)

Computational Study - Results



Using heterogeneous fleets

- The cost is halved for $|I| = 5, 10$.
- Fewer stations are needed.
- Fewer vehicles are needed for $|I| = 5, 10$.

Computational Study - Average gap and solving time

	BDA Phase I		BDA Phase II		PF	
	g	$t(s)$	g	$t(s)$	g	$t(s)$
$\|I\| = 5$	**0.00**	**1.57**	**0.00**	**1.76**	0.02	523.93
$\|I\| = 10$	**0.00**	**188.53**	**0.00**	**286.73**		
$\|I\| = 15$	0.10	4658.65	0.10	4833.38		

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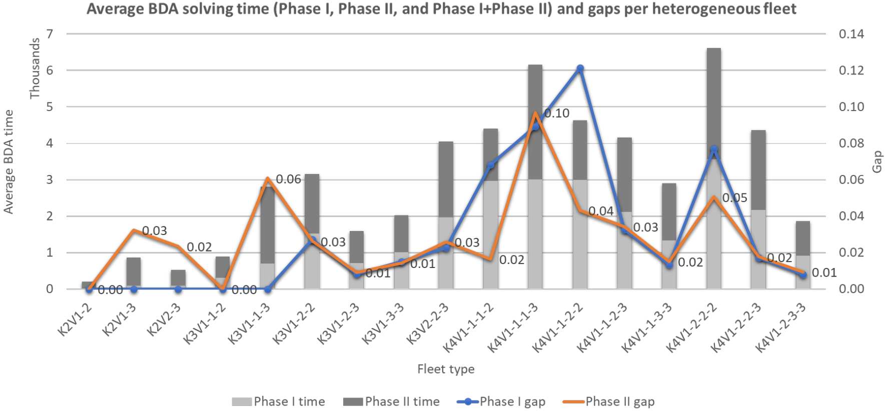

Computational Study - Average , number of stations opened and number of vehicles used

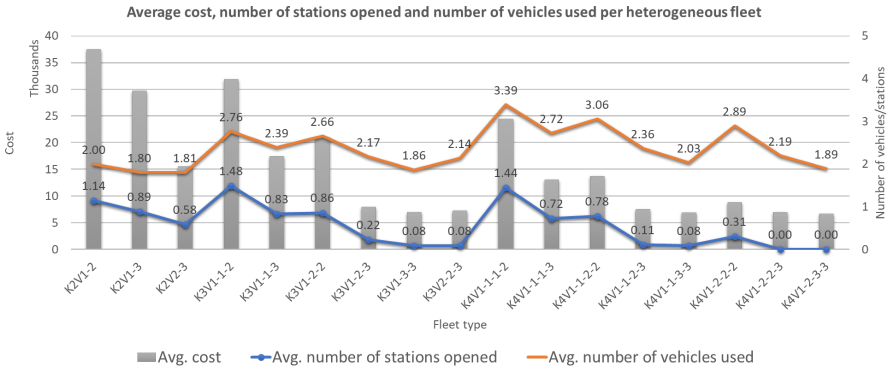

Figure: For each heterogeneous fleet type.

Fleets with smaller vehicles are usually more costly and challenging.

Computational Study - Average , number of stations opened and number of vehicles used

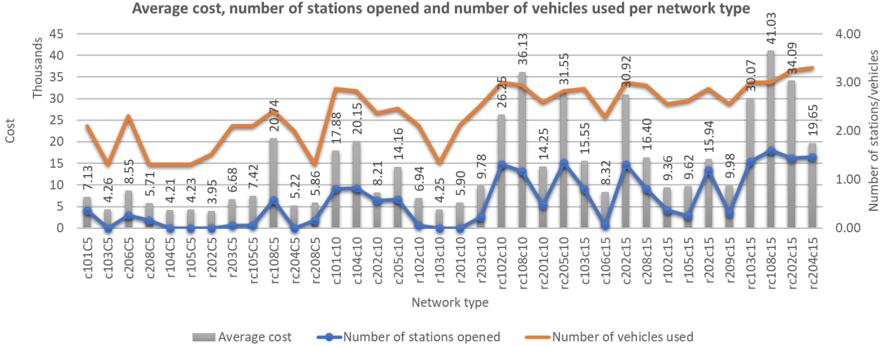

Figure: For each network type (heterogeneous fleets).

Some smaller networks are more challenging and costly as well.

Summary and Conclusions

Contribution

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- An MIP with commodity flows
- An exact decomposition framework
  Benders algorithm $\Rightarrow$ quick feasible solutions
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Conclusions

- Heterogeneous fleets are worth consideration.
- Fleet composition must be well analyzed.
- The framework with lower bounds is very successful.
  Can it be improved?

Future Research Directions

Methodological and experimental

- TSP lower bounds for LRP methods (Gandra et al., 2021)
- Matheuristic approaches
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Consideration of additional real-world components

- Periodic and/or multi-period
- Time windows
- Multi-depot with location decisions also on depots
- Stochastic or dynamic problems

Full paper and references

Full paper: https://lirias.kuleuven.be/retrieve/609757

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