Optimization with Quantum machines

(especially with quantum annealing)

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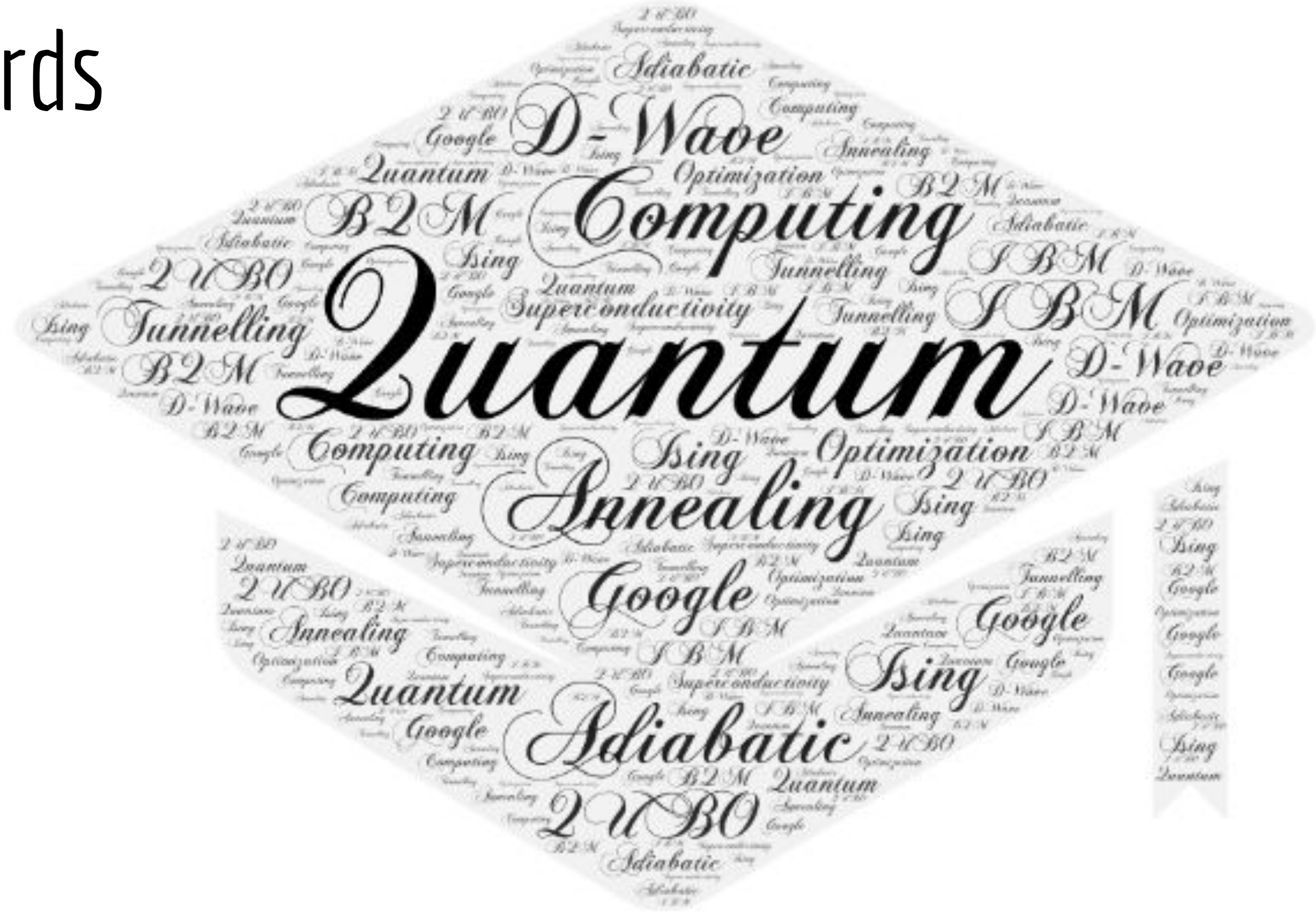
Keywords



Pic online tool: https://wordart.com/create

Outline

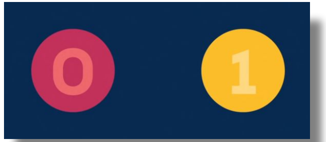
1. Quantum Bits
2. D-Wave machines overview
3. the Quantum Annealing process (overview)
4. Resolution Process
5. End-to-end example: the Max-Cut Problem
6. Hybrid Machines

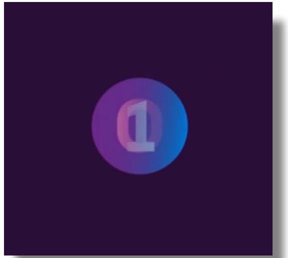
Outline

Quantum Bits

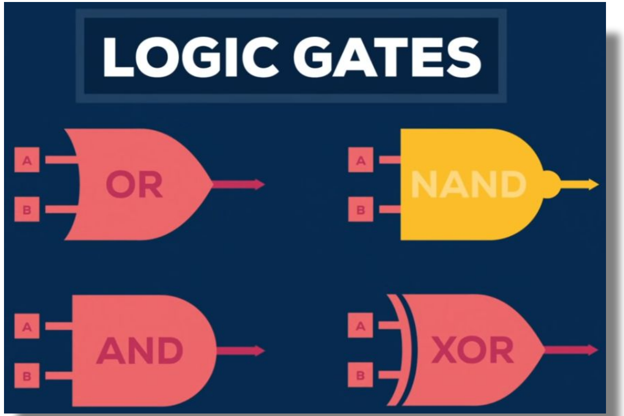
- Superposition
- Measure
- Entanglement
- Decoherence
- Overview of the 2 quantum computers paradigms

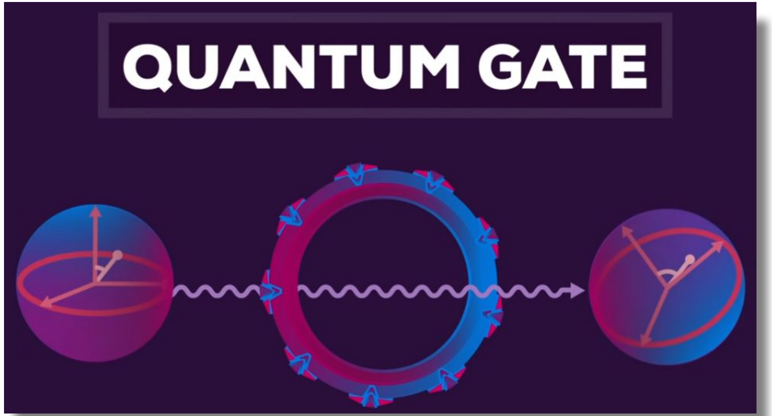
bits **Versus** qubits





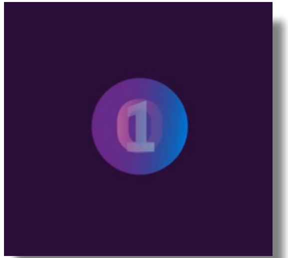
Superposition 0-1

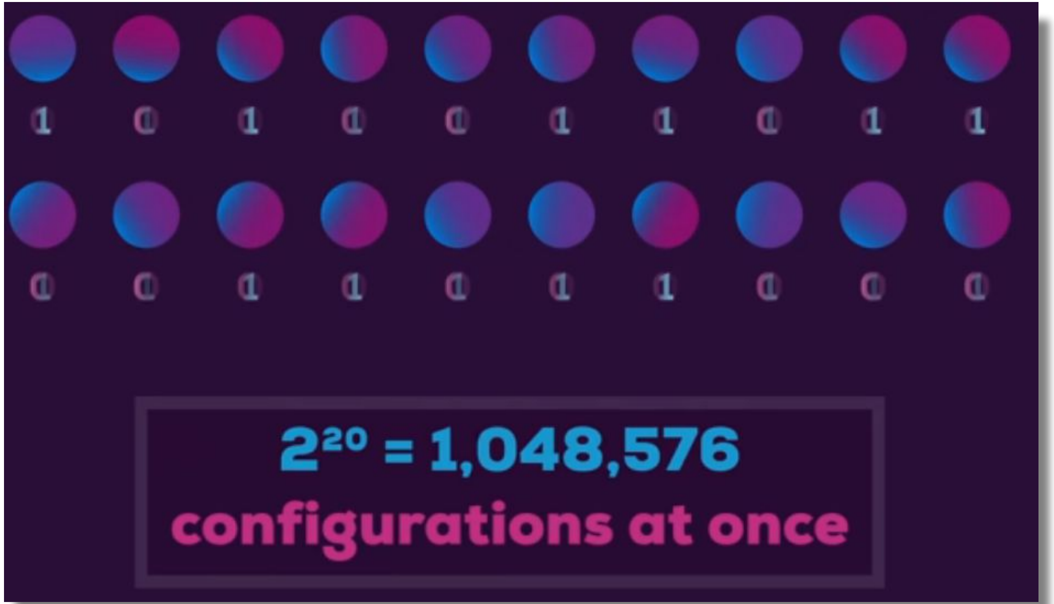




pic: https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell

Qubit Superposition





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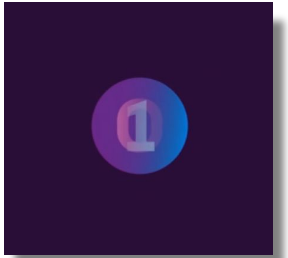
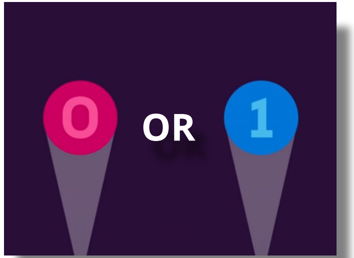
Measuring a qubit

From the superposition of states (0 **and** 1)
to the classical states (0 **or** 1)



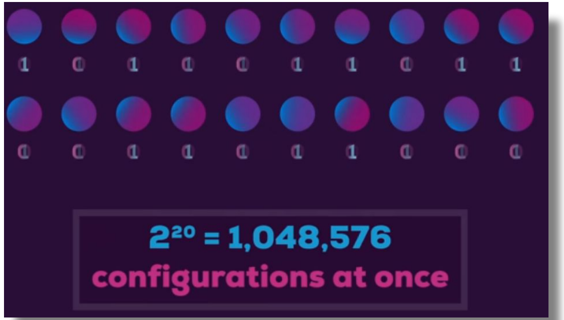
Superposition 0-1

Measure



Measure
(before the end of the coherence time)

Example:



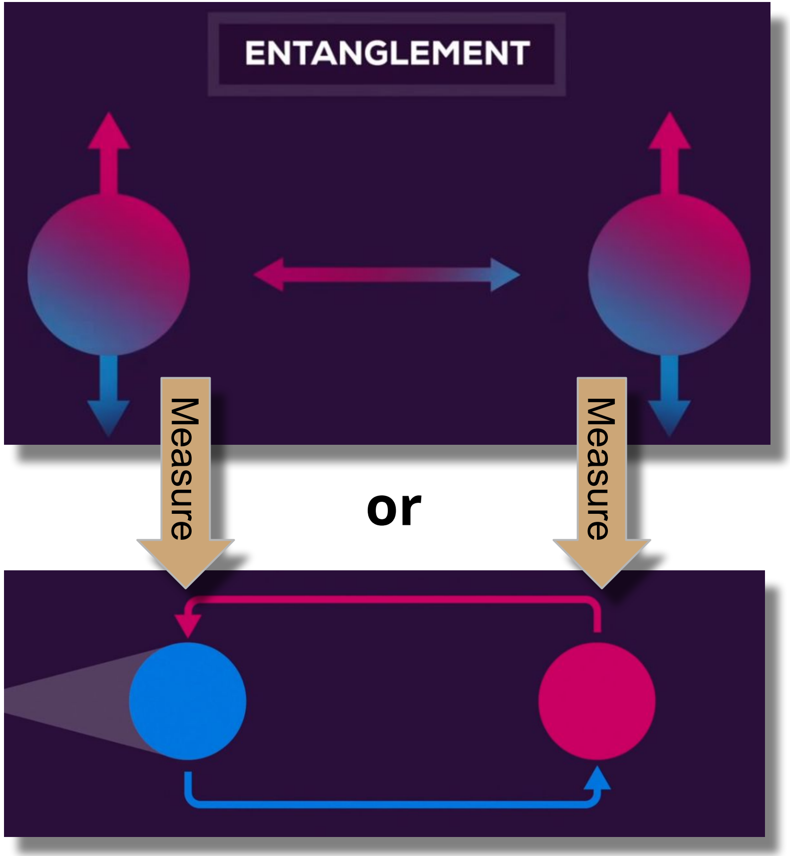
Measure

One solution
(20 qubits became 20 bits
with only one value)

pic: https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell

The entanglement

(*sp. entrelazamiento*)



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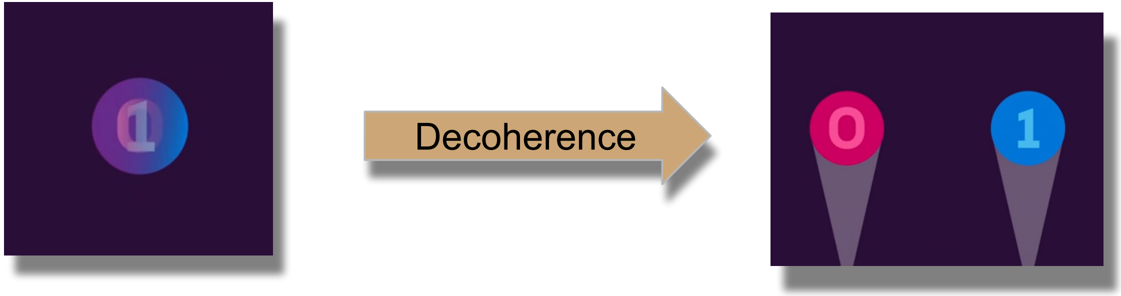
Decoherence

- Appears after a short amount of time because of:
  - radiation, light, sound, heat, magnetic fields, etc.
  - or…the measure!
- The system goes **from a quantum system to a classical one**,
- Limit the time to work on qubits.

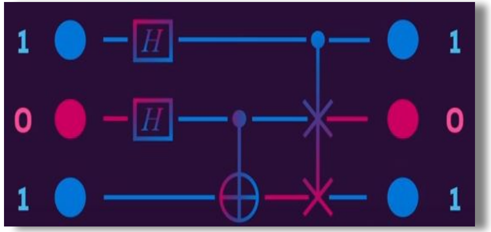
Example on a qubit: break the superposition state to ‘1 or ‘0’.



pic: https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell

The two paradigms

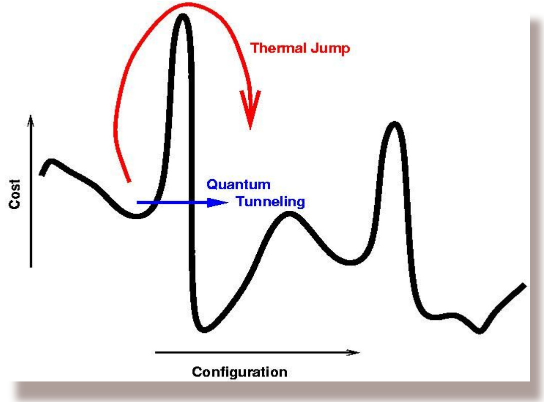
Computer based on
Quantum Logic Gates



Main actors:

- IBM
- Google

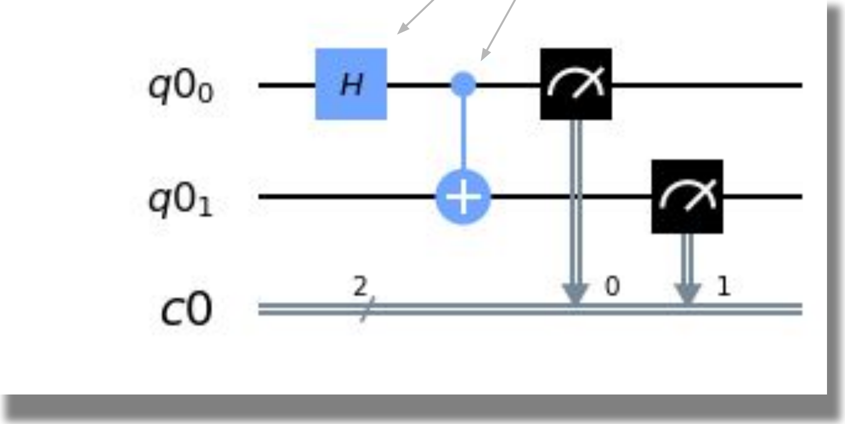
Computer based on
Quantum Annealing



Main actors:

- D-Wave
- Qilimanjaro (spain)

in short... Gates Quantum Computers



Main quantum algorithms:

- Shor (factorize an integer N in time O((log N)^3)))
- Grover (quadratic acceleration to find a value)
- QAOA (Quantum Approximate Optimization Algorithm, Nikolaj et al., 2018)

Main APIs:

- Qiskit (IBM)
- Cirq (Google)

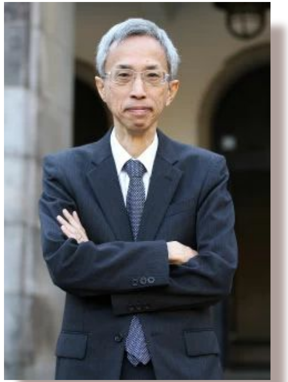
But today, even if we can express and solve a very tiny TSP with these gates (QUBO), the result is not good, there is not enough qubits and have a high error rate. **Wait and see...**

Outline

2 **D-Wave machines overview**

- Machines based on quantum annealing
- Hardware
- Software
- Some applications

D-Wave and Quantum Annealing



Hidetoshi Nishimori

- Father of the **Quantum Annealing Metaheuristic**
  - Works on Statistical physics,
  - Paper: Kadowaki, Tadashi, and Hidetoshi Nishimori. "Quantum annealing in the transverse Ising model." *Physical Review E* 58.5 published in **1998**;
- Based on classical computer Metaheuristic **Simulated Annealing**
  - Kirkpatrick, Scott, C. Daniel Gelatt, and Mario P. Vecchi. "Optimization by simulated annealing." *science* 220.4598 (1983): 671-680;
- Prove that the Quantum Annealing gives better results that the Simulated Annealing,
- A good conference from him: https://www.youtube.com/watch?v=OQ91L96YWCk.

D:WAVE
The Quantum Computing Company™

- First (hardware) implementation of the Quantum Annealing,
- Starting the work in **1999**, the 1st computer based on quantum annealing released in **2007**.

D:Wave
The Quantum Computing Company™

Notable Funders

- InQTel (CIA funds),
- Lockheed Martin,
- NSA funds
  - (see Research & Technology Penetrating hard targets' Snowden File)
- and others.

≤ 2013

• (S//SI//REL TO USA, FVEY) Conduct basic research in quantum physics and architecture/engineering studies to determine if, and how, a cryptologically useful quantum computer can be built.

https://edwardsnowden.com/2014/01/04/penetrating-hard-targets-and-owning-the-net/

Notable Contracts

Volkswagen, NSA, CIA, NASA, Lockheed Martin, etc.

Scope

https://www.dwavesys.com/applications

Mainly: IA & **Optimization**.

Examples:

- Logistics,
  - trucks, boats,
  - pick-up and delivery;
- Production planning,
- Design Object,
- Chemistry simulation,
- [Geo]politics.

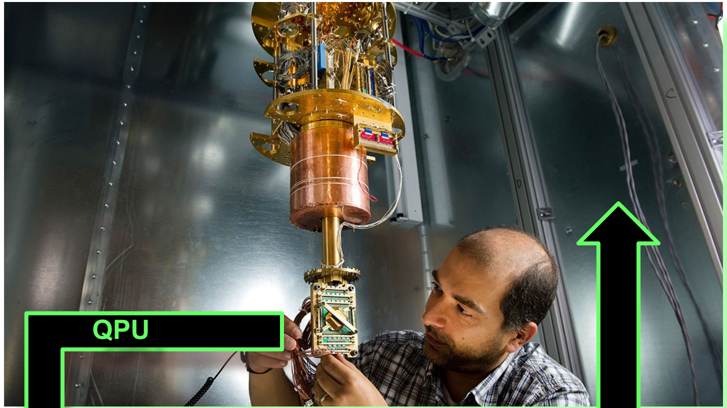
Hardware: inside a D-Wave quantum machine

Total Consumption

Example of a 1K qubits machine:

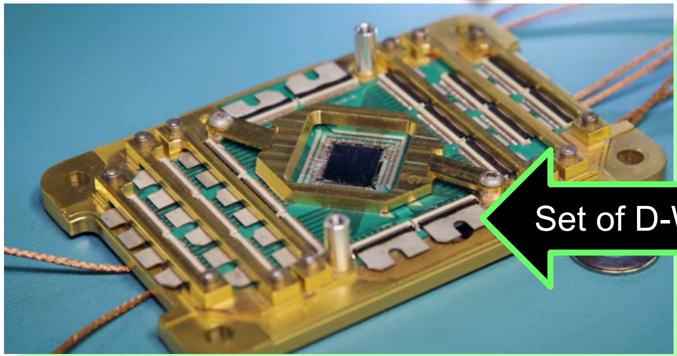
- **25kw** (16kw in cryo’).

As a comparison: **26248kW** for the Fujitsu A64FX (the most powerful in 02/2021).

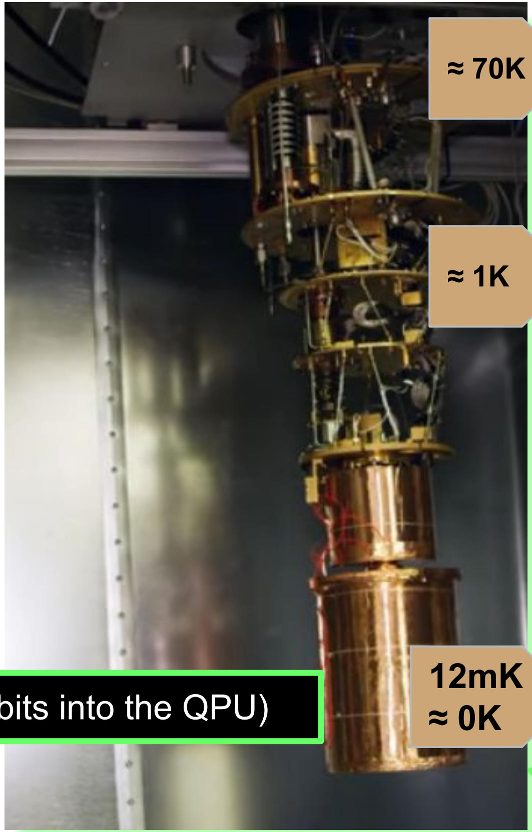


QPU

Faraday cage (for blocking block electromagnetic fields.)



Set of D-Wave Nodium superconductors (qubits into the QPU)



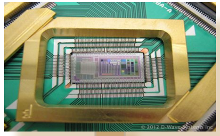
≈ 70K

≈ 1K

12mK ≈ 0K

Layers of cooling mechanisms

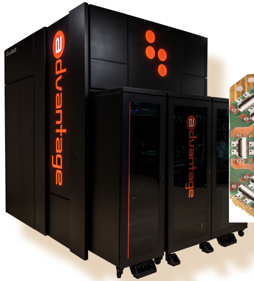
Chimera Graph
(D-Wave hardware model)

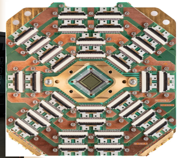


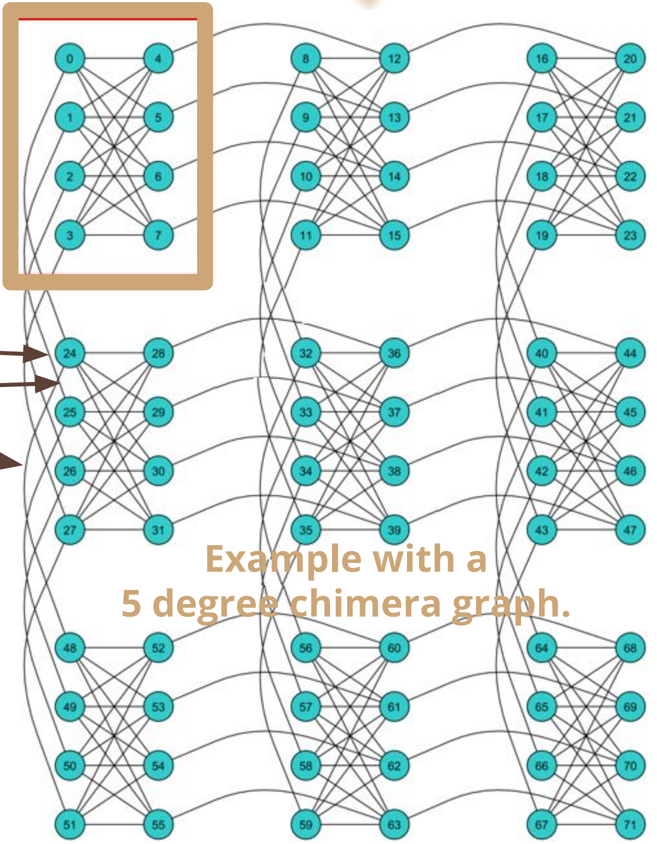
QPU

D-Wave's quantum computer hardware can be modeled as a non-oriented graph called chimera graph. This graph has:

- **qubits** as nodes,
- quantum **couplers** as edges.
- since end of 2020 (*Pegasus* topology), **the degree of the qubits is 15** (6 before).
    - > 5k qubits
    - 35000 couplers







D-Wave machines: from 2007 to 2020

Advantage QPU:
- 35,000 couplers
- 15 Connectivity

from 4 to 5640 qubits

Qubits number in D-Wave machine & Gates based Quantum Computer?

D-Wave The Quantum Computing Company™

Best:
5k qubits

- In **Quantum Annealing**, you let a sort of **natural evolution** of quantum states. **You do not have any control on this evolution**:
    - You set-up, launch, wait *ms*, and measure to obtain the solution, ***(no program!)***
    - Not able to create any possible quantum circuit.

IBM Q

Best:
127 qubits

In **Universal Gates Quantum Computers**, the aim is way more ambitious, you control the evolution of that quantum state over time:

- **More difficult to build since the quantum system is very delicate to work with**, **<<< This is why you find a big difference in the number of qubits. >>>**;

To summarize: "way more qubits, but able to solve way less types of problems".

Frameworks: Background

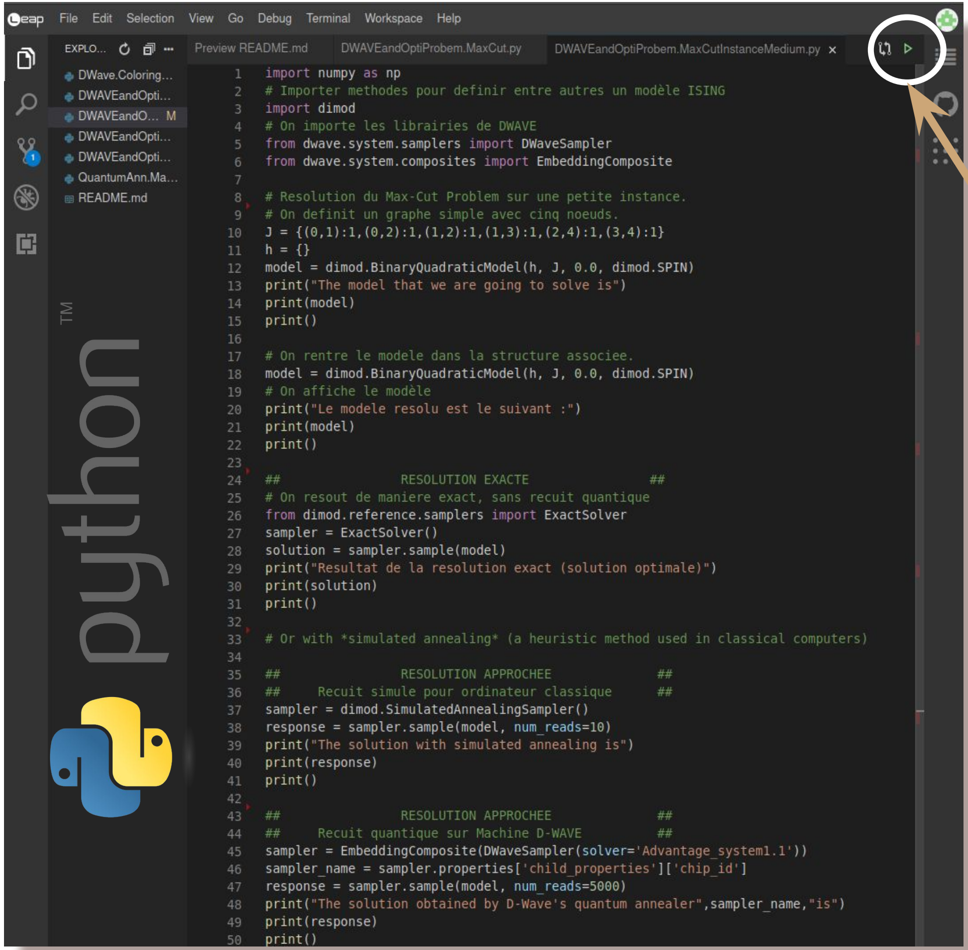
- **Cirq**
  - **Google**
  - https://cirq.readthedocs.io/en/stable/
- **Qiskit**
  - ***IBM***
  - https://quantum-computing.ibm.com/
- **Amazon Braket**
  https://aws.amazon.com/fr/braket/
- and others:
  - Rigetti (Forest/PyQuil),
  - Microsoft (AzureQuantum/LiQui/Q#),
  - From labs: PyTKET, ProjectQ, QuTip etc.)

- **D-Wave Leap**
  - **Ocean Library** (python)
  - **"High Level" programming**
  - https://www.dwavesys.com/take-leap

https://ide.dwavesys.io/#https://github.com/SaPlanq/DWAVEQuantum

Your github address

How can I solve a problem on a D-Wave machine?



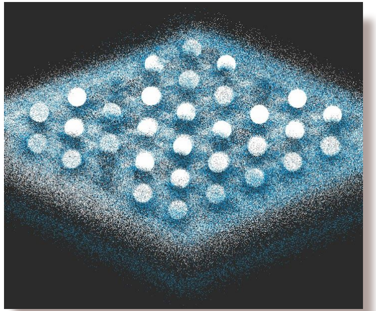
1

You launch the resolution directly here, and you will have an online terminal showing the result.

2

```
The solution obtained by D-Wave's quantum annealer Advantage_system1.1 is
   0  1  2  3  4 energy num_oc. chain_.
0 -1 +1 +1 -1 +1   -4.0    1398     0.0
1 +1 -1 -1 +1 -1   -4.0    1029     0.0
2 -1 +1 +1 -1 -1   -4.0    1487     0.0
3 +1 -1 -1 +1 +1   -4.0    1084     0.0
4 -1 +1 +1 +1 -1   -2.0       1     0.0
5 +1 -1 -1 -1 +1   -2.0       1     0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```

Outline



3 the Quantum Annealing process (overview)

- From superposition to classical states

Quantum annealing *(fr : recuit quantique)* & Optimization Problems

"Physics can help solve [Optimization Problems] because we can frame them as energy minimization problems. "

No guaranty that quantum annealing will reach the ground state
(i.e., the global optimum of the problem / Optimal Solution minimizing the cost).
… But, it might **give a better solution than a classical heuristic.**

"The D-Wave system can be viewed as a **hardware heuristic** **that** **minimizes [..] objective functions** using a physically realized version of quantum annealing."

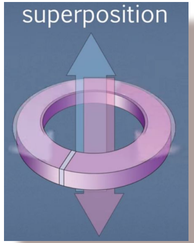
https://docs.dwavesys.com/docs/latest/c_handbook_2.html

INPUT → **Quantum Annealing** → OUTPUT

The very basic idea

First Step

The qubits start in a superposition state (**zero and one**).

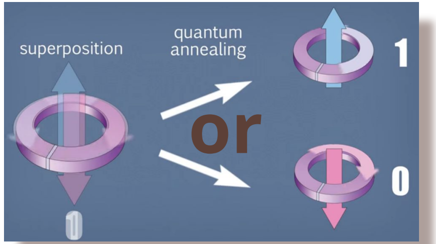


and

Work on the 0 and 1 states probabilities (amplitudes).

Last step

The measure of each qubit make them going to either the **zero state or** the **one state** (according to their probability).

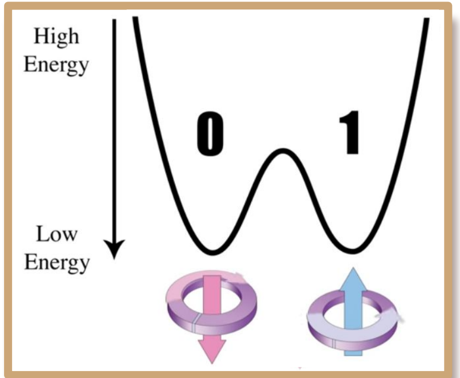


Pics: D-Wave

INPUT → **Quantum Annealing** → OUTPUT

The very basic idea

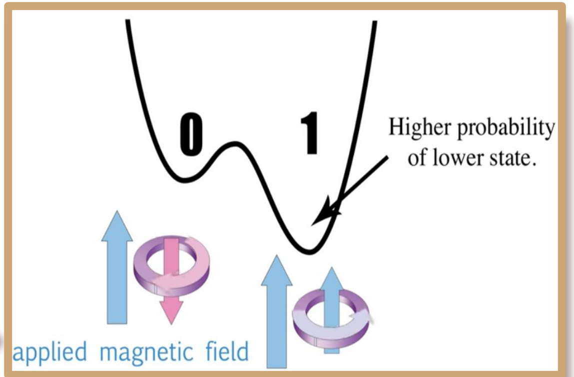
Energy Diagram (Superposition)



Starting Point. Proba = 50% for each state.

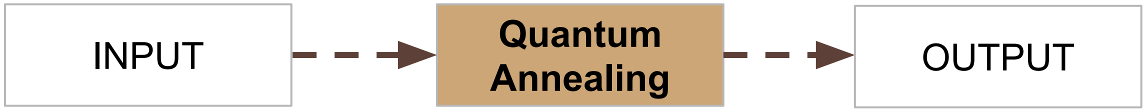
Control the probability of the state with a programmable external magnetic field (bias)

Energy Diagram (After Bias)

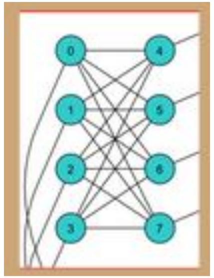


Here: higher proba to measure the state '1'.

Pics: D-Wave

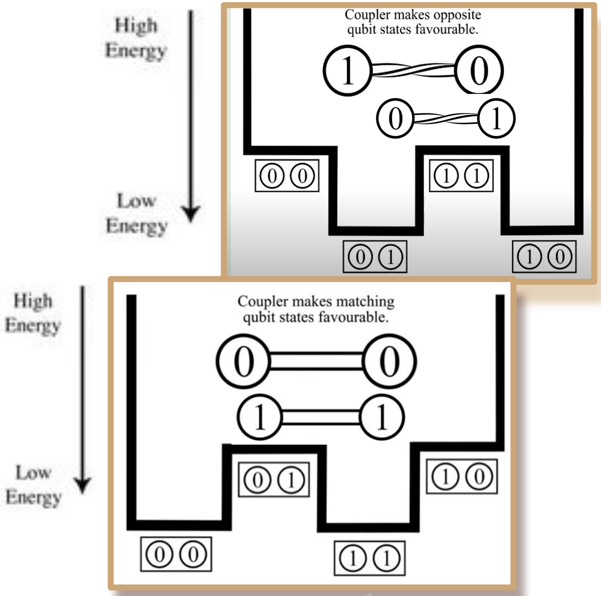


Couplers & Entanglement



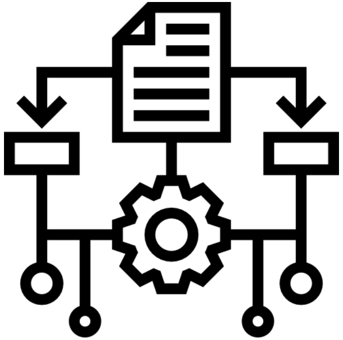
- **The "real power" of such a system appears once we start to link the qubits together**
  - by **couplers** (using **entanglement** behaviour).
    - **The couplers define how a pair of qubits ends up, in the same or in opposite state**, (0&0, 1&1 or 1&0, 0&1).

- Once 2 qubits are entangled, they can be considered as a **single object**, but with 4 states.
  - When we consider a pair of qubits to be the same (or to be opposite) **the machine will "physically" make the equality (or the inequality)** "**energetically favorable**", i.e., to "low the energy" of those states.



Pics: D-Wave

Outline

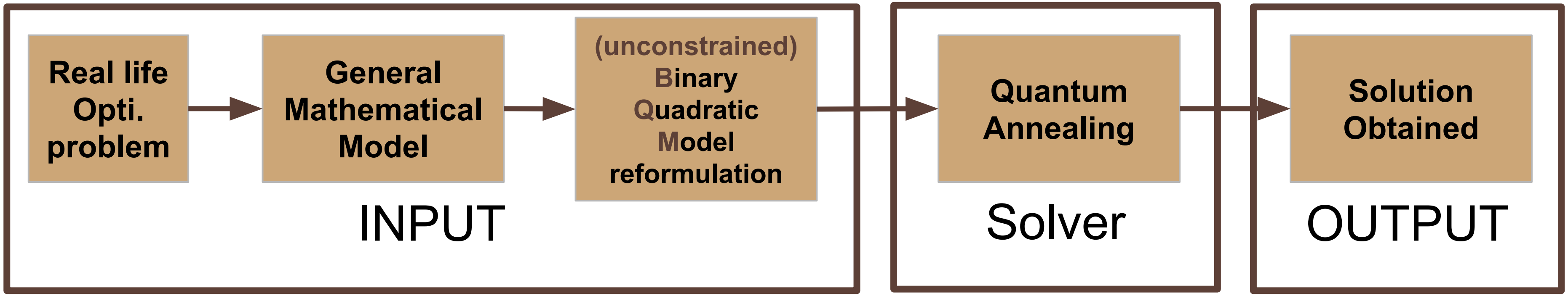


Resolution Process

- QUBO & ISING

General Process

INPUT › QUANTUM ANNEALING › OUTPUT

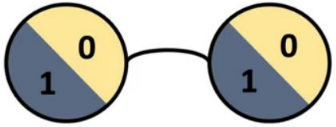


Problem formulation: 2 mathematically **equivalent** models

Binary Quadratic Model

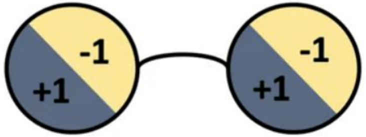
Choice n°1

Quadratic Unconstrained Binary Optimisation (QUBO)



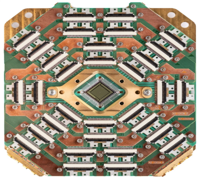
Choice n°2

Ising Problem

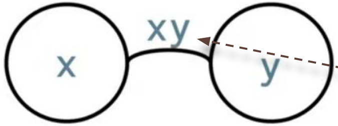


Mathematically Equivalent models

“Fit in” D-Wave QPU

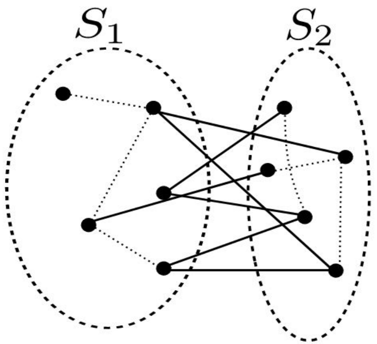


1 coupler/var prod.



2 qubits/variables

(Hardware) the **couplers** (making pairs of qubits) corresponds to (software) **quadratic expressions**.

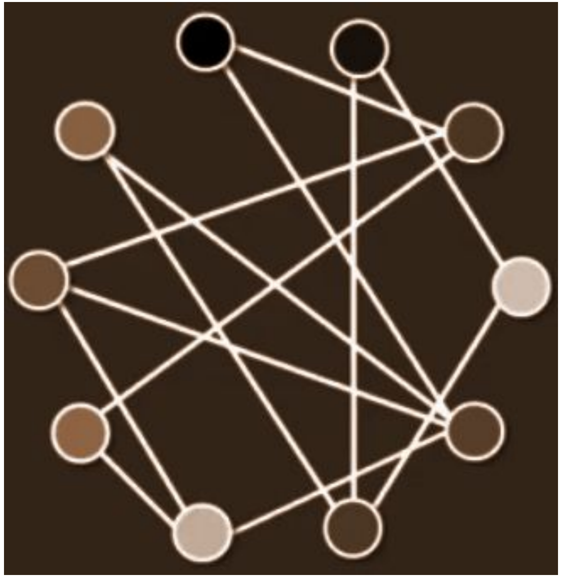


Outline

5 End-to-end example: the Max-Cut Problem

- Problem Definition
- Ising Model Formulation
- Solving one example

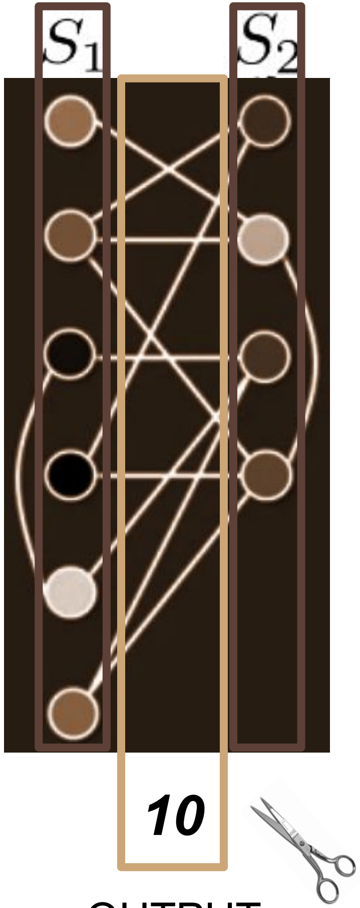
The Max-Cut Problem



INPUT

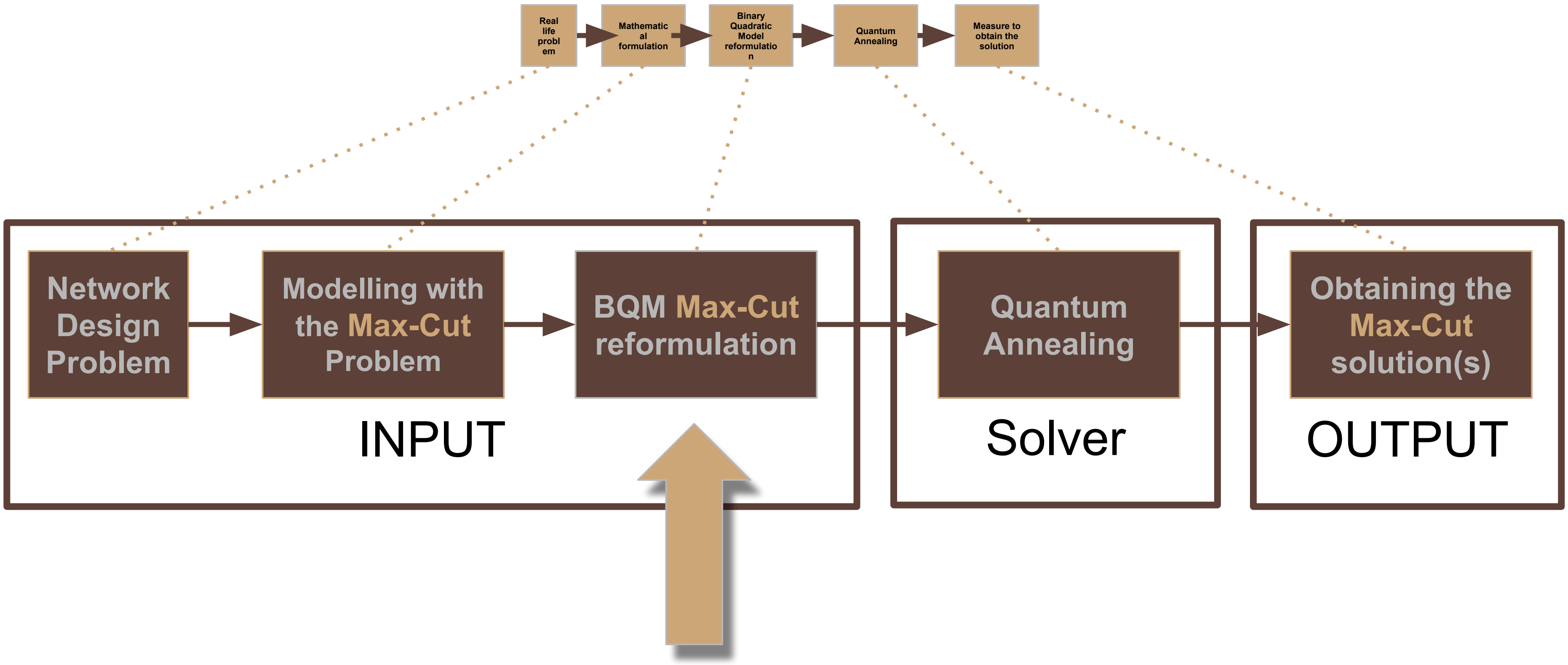
Finding 2 sets of nodes S1 and S2, such that the **number of edges** between the set S1 and the set S2 is **as large as possible**.

Complexity: NP-HARD.



OUTPUT

General Process to solve the Max-Cut Problem with a D-Wave machine

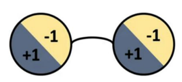


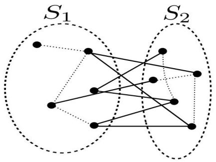
BQM Max-Cut reformulation

- Let's take a Graph **G=(X,E)**, where: **X** is the set of nodes and **E** the set of edges.
- We can identify each node **i** of the graph with a binary variable Z_i such that:

  - $Z_i = 1$
    - if **i** is in one group *S*
  - $Z_i = -1$
    - if **i** is in the other group.

Ising Model





- **The Max-Cut Problem consists in finding the vector Z of |X| binaries which MINIMIZES:**

$$\sum_{(i,j)\in E} Z_i Z_j$$

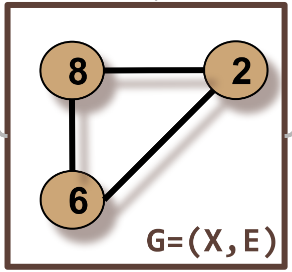
- Exemple. If two nodes (**i**,**j**) are in **different** groups:
- $Z_i Z_j = -1$,
  - $Z_i Z_j = 1$ otherwise.

Let's take a Graph G=(X,E), where **X** is the set of nodes and **E** the set of edges. We can identify each node i of the graph with a binary variable Z_i such that:

- Z_i = 1 if i is in one group,
- Z_i = -1 otherwise.

Ising Model

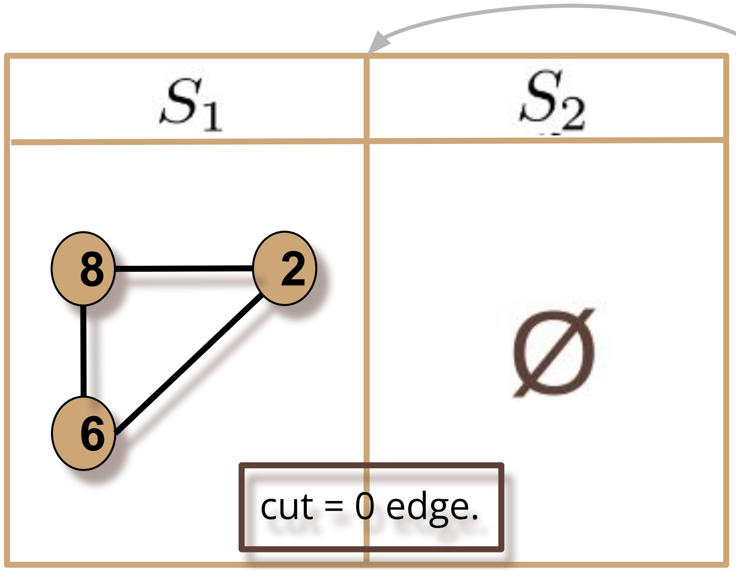
Small Example with a K3 Complete Graph



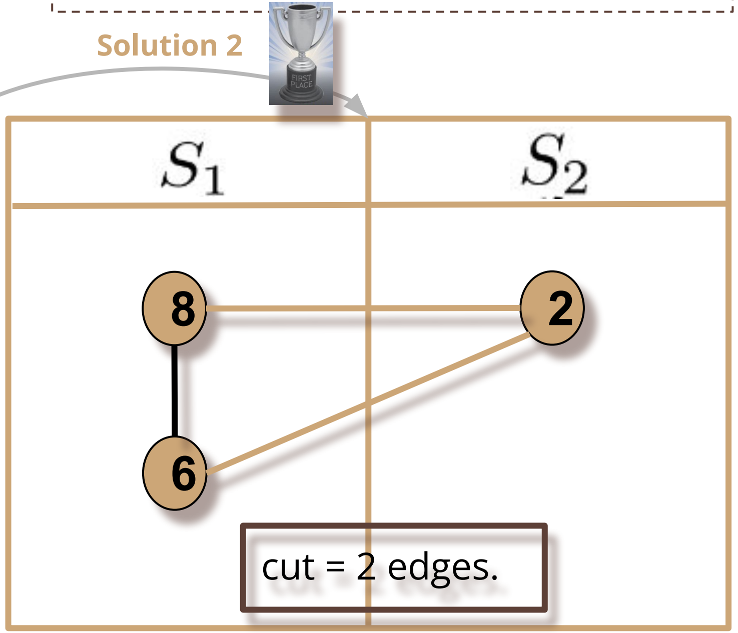
The Max-Cut Problem consists in finding the vector *z* of |X| binaries which minimizes:

$$\sum_{(i,j)\in E} Z_i Z_j$$

Solution 1



Solution 2



f(Solution 1) = 3

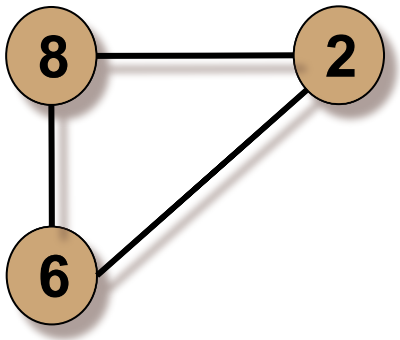
f(Solution i) = $Z_2Z_6 + Z_6Z_8 + Z_8Z_2$

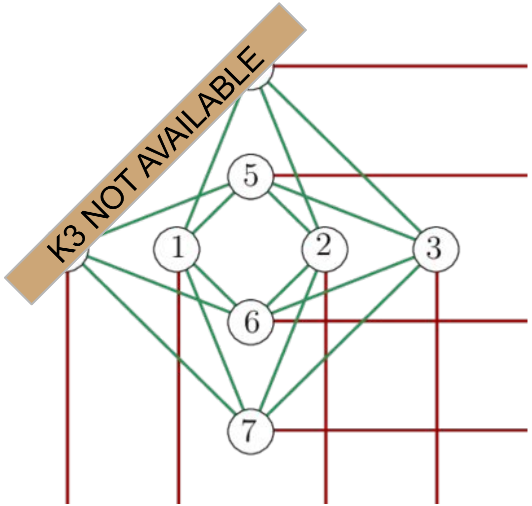
f(Solution 2) = -1

Problem of low connectivity

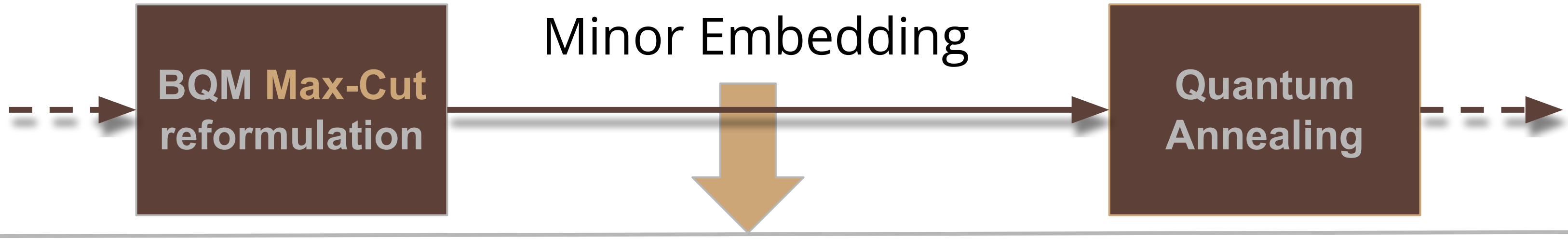
$$H = Z_2Z_6 + Z_6Z_8 + Z_8Z_2 + \text{...blablavar...}$$

That requires the following K3 complete graph in the hardware:





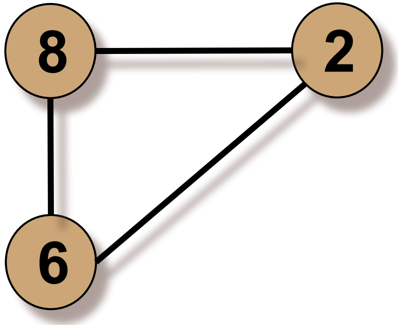
Note: From the Max-Cut Formulation as input To the (hardware) mapping into the QPU

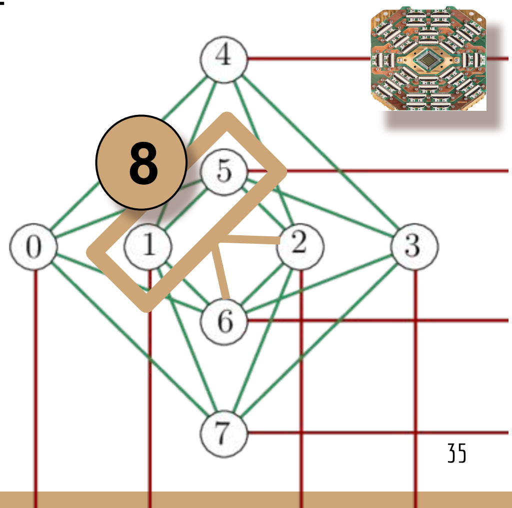


Solution: **1 logical qubit from 2 real qubits**

$H = Z_2Z_6 + Z_6Z_8 + Z_8Z_2 +$ ...blablavar...

- Generation of **logical qubits**
  - using several physical qubits to “simulate” this connectivity.





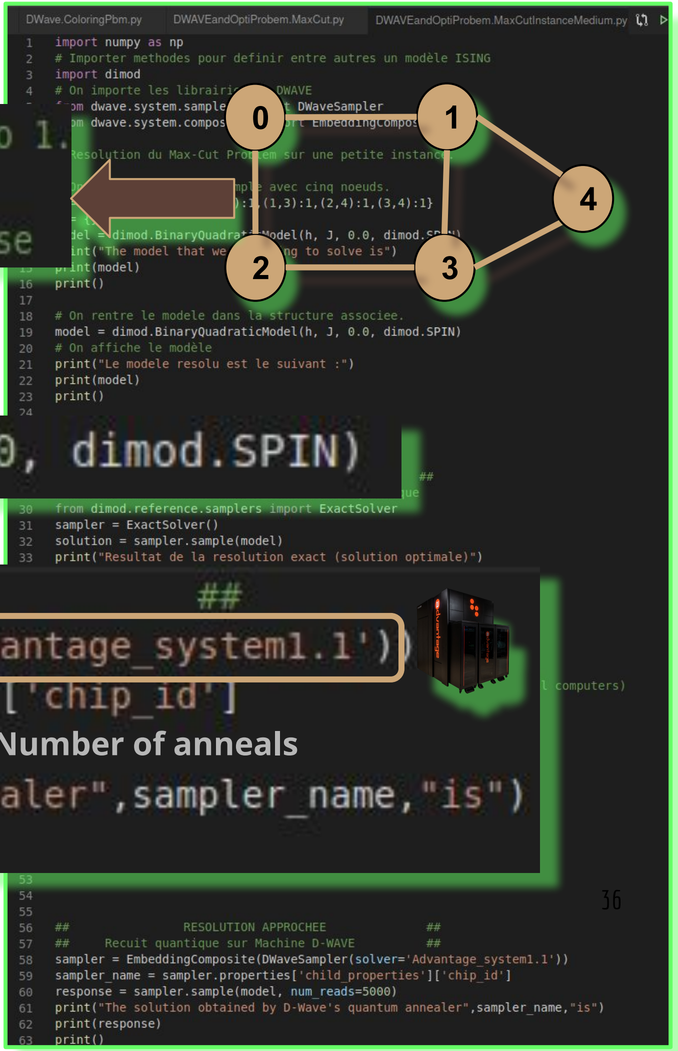
Example of **code** for the Max Cut Problem

```
# We enter the 6 edges of our graph with a weight equals to 1.
J = {(0,1):1,(0,2):1,(1,3):1,(1,4):1,(2,3):1,(3,4):1}
h = {} # We do not have external magnetic field in this case
```

No external field (biais)

Couplers / edges no weight (':1')

```
model = dimod.BinaryQuadraticModel(h, J, 0.0, dimod.SPIN)
```




```
##     Quantum Annealing of D-WAVE Advantage Machine          ##
sampler = EmbeddingComposite(DWaveSampler(solver='Advantage_system1.1'))
sampler_name = sampler.properties['child_properties']['chip_id']
response = sampler.sample(model, num_reads=5000)
print("The solution obtained by D-Wave's quantum annealer",sampler_name,"is")
print(response)
```

Number of anneals

```
## Send problem to Sampler
sampleset = sampler.sample_qubo(Q)
```

Example for a QUBO.

D-Wave Quantum Annealing **results**

For sure we did not use all the qubits...

```
The solution obtained by D-Wave's quantum annealer Advantage_system1.1 is
   0  1  2  3  4 energy num_oc. chain_.
0 -1 +1 +1 -1 +1   -4.0    1398    0.0
1 +1 -1 -1 +1 -1   -4.0    1029    0.0
2 -1 +1 +1 -1 -1   -4.0    1487    0.0
3 +1 -1 -1 +1 +1   -4.0    1084    0.0
4 -1 +1 +1 +1 -1   -2.0       1    0.0
5 +1 -1 -1 -1 +1   -2.0       1    0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```

lower cost

Number of occurrences during the 5000 anneals

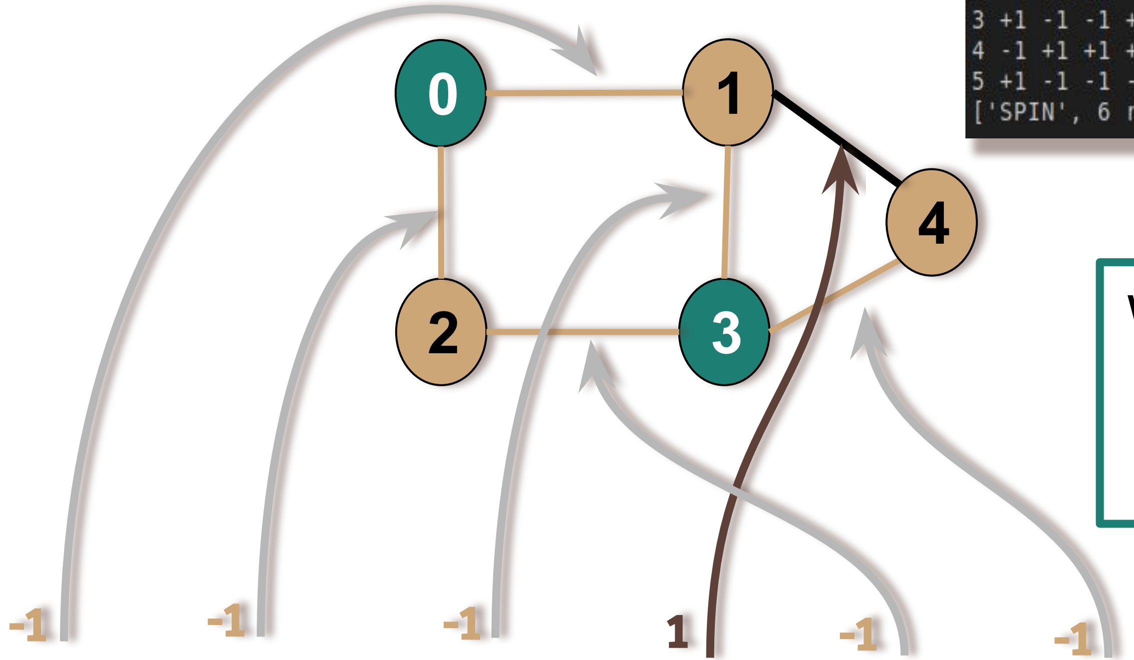
$$H = Z_0Z_1 + Z_0Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_3Z_4 = -5 + 1 = -4$$

$Z_0 = -1$ $Z_1 = 1$ $Z_2 = 1$ $Z_3 = -1$ $Z_4 = 1$

Max-cut-Problem: From the solution to the graph.

```
   0  1  2  3  4 energy num_oc. chain_.
0 -1 +1 +1 -1 +1   -4.0    1398     0.0
1 +1 -1 -1 +1 -1   -4.0    1029     0.0
2 -1 +1 +1 -1 -1   -4.0    1487     0.0
3 +1 -1 -1 +1 +1   -4.0    1084     0.0
4 -1 +1 +1 +1 -1   -2.0       1     0.0
5 +1 -1 -1 -1 +1   -2.0       1     0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```

We deduce a **5 edges** cut.

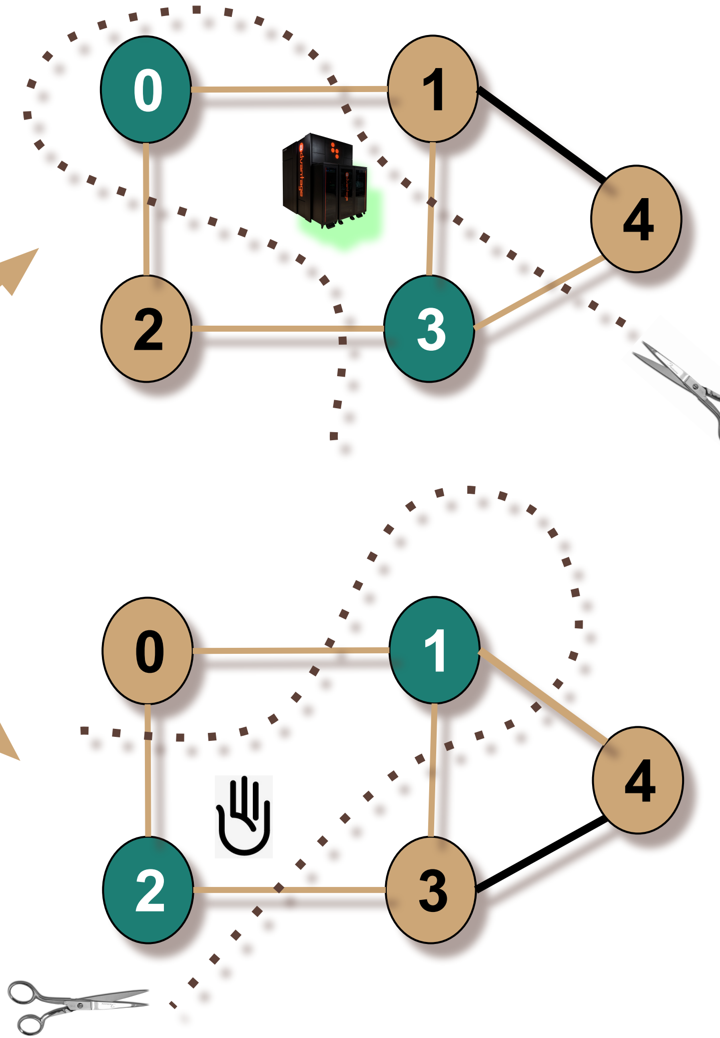


$$H = Z_0Z_1 + Z_0Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_3Z_4 = -5+1 = -4$$

$$Z_0 = \mathbf{-1} \quad Z_1 = \mathbf{1} \quad Z_2 = \mathbf{1} \quad Z_3 = \mathbf{-1} \quad Z_4 = \mathbf{1}$$

Max-cut-Problem: several optimal solutions

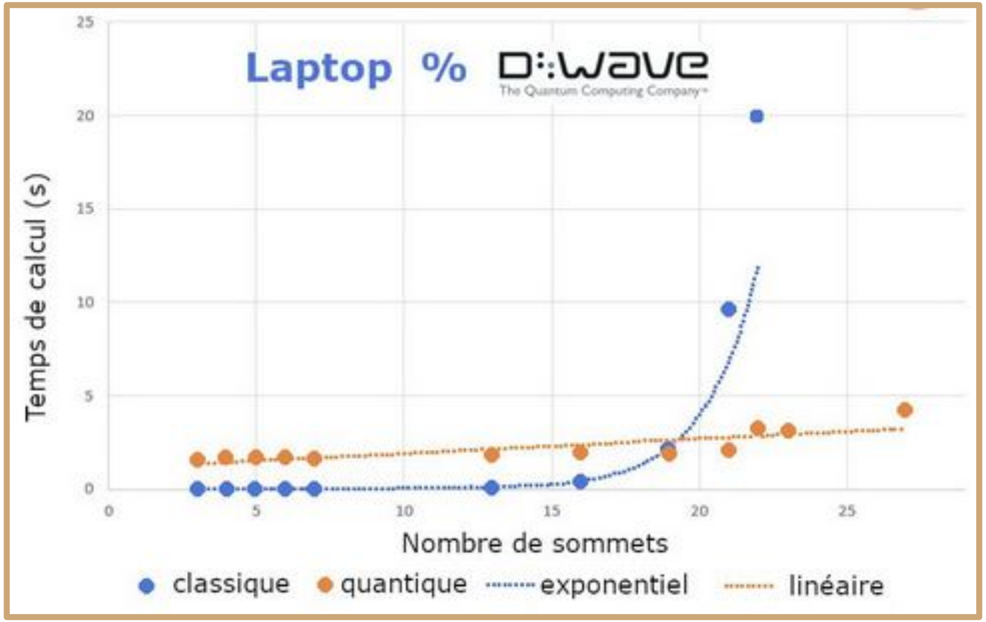
```
The solution obtained by D-Wave's quantum a
    0  1  2  3  4 energy num_oc. chain .
0 -1 +1 +1 -1 +1   -4.0    1398     0.0
1 +1 -1 -1 +1 -1   -4.0    1029     0.0
2 -1 +1 +1 -1 -1   -4.0    1487     0.0
3 +1 -1 -1 +1 +1   -4.0    1084     0.0
4 -1 +1 +1 +1 -1   -2.0       1     0.0
5 +1 -1 -1 -1 +1   -2.0       1     0.0
['SPIN', 6 rows, 5000 samples, 5 variables]
```



We obtained ALL the optimal solutions.

Some results

(Exact versus Quantum Annealing)



(next step: need to be compared with a software heuristic)

Outline

6 **Hybrid Machines**

- Focus on few projects using quantum annealing
- *Note: hybrid computers starts to be used for the gate model paradigm.*

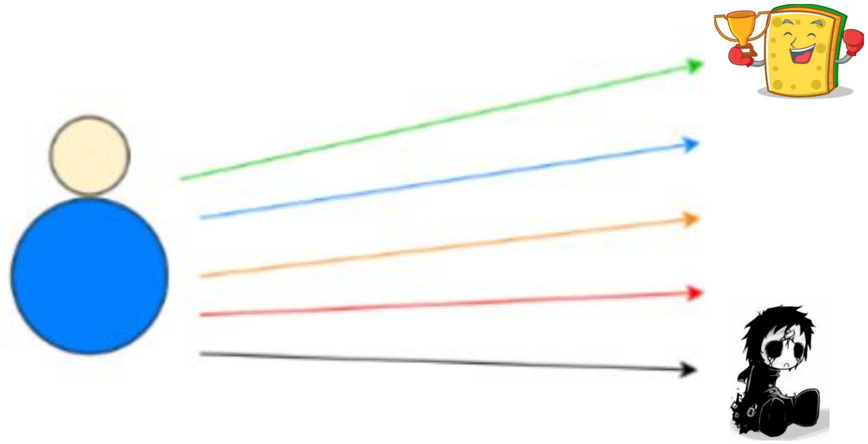
Hybrid computers

(for both paradigms)

Classical Computer

Quantum Computer

A Student Project about...Projects



- Each student select **5 projects and** sort **them from the most wanted to the 5th most wanted**.
- Each project has a window to bound the number of students.

Goal: Maximizes general contentment.

$$Obj = min \sum x_{ij} w_{ij}$$

$$\forall\ project\ j$$

$$\sum_{i}^{n\_students} x_{ij} \leq room_{max}$$

$$\forall\ student\ i$$

$$\sum_{j}^{n\_projects} x_{ij} = 1$$

$$\forall\ project\ j$$

$$\sum_{i}^{n} x_{ij} \leq M\ .\ activeProject_j$$

$$\forall\ project\ j$$

$$\sum_{i}^{n} x_{ij} \geq 3\ .\ activeProject_j$$

$$activeProject_j \in \{0,1\}\ and\ M = room_{max\ j}$$

(n <=> *n_students*)

"Classical" (generalization)

Versus

"Quantum" (example)

$$\min\left(\sum_{i}^{9}\sum_{j}^{3} x_{ij}\, w_{ij}\right) + \gamma_0\left(\sum_{i}^{9} x_{i0} - 3\right) + \gamma_1\left(\sum_{i}^{9} x_{i1} - 4\right) + \gamma_2\left(\sum_{i}^{9} x_{i2} - 2\right) +$$

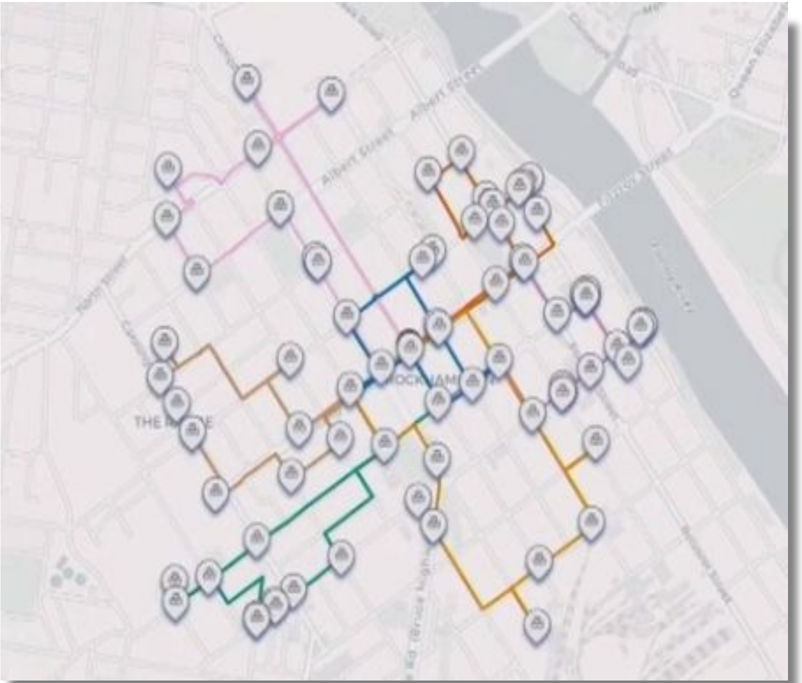
$$\gamma_3\left(\sum_{j}^{3} x_{0j} - 1\right)^2 + \gamma_4\left(\sum_{j}^{3} x_{1j} - 1\right)^2 + \gamma_5\left(\sum_{j}^{3} x_{2j} - 1\right)^2 + \ldots + \gamma_{11}\left(\sum_{j}^{3} x_{8j} - 1\right)^2$$

Relaxation of all the constraints to obtain a Quadratic Unconstrained Binary Optimization Model (QUBO).

```
Pour le projet 0 (3/4)
   étudiant : 92 => anthony.monvoisin@isen.yncrea.fr
   étudiant : 93 => martin.valet@isen.yncrea.fr
   étudiant : 98 => victorien.roussel@isen.yncrea.fr
Pour le projet 1 (5/5)
   étudiant : 19 => thibaut.blasselle@isen.yncrea.fr
   étudiant : 52 => joshua.ofori@student.yncrea.fr
   étudiant : 120 => theo.ruchot@isen.yncrea.fr
   étudiant : 125 => nicolas.defoort@isen.yncrea.fr
   étudiant : 153 => lilian.houplin@isen.yncrea.fr
```

NEC experiment: CVRP

(David Garvin, Nec Australia at Qubits conf. 2021-10)



Constraints: from one depot node, supply of the other nodes with 1 of the **K vehicles** with a capacity,
Objective: Min total distance.

NEC experiment: CVRP

(David Garvin, Nec Australia at Qubits conf. 2021-10)

- Resolution Scheme:
  - 1. **Clustering** (for each vehicle)
    - quadratic formulation
    - quantum computer
  - 2. **Routing** (for each vehicle)
    - basic heuristic solving the TSP
    - classical computer

min problem

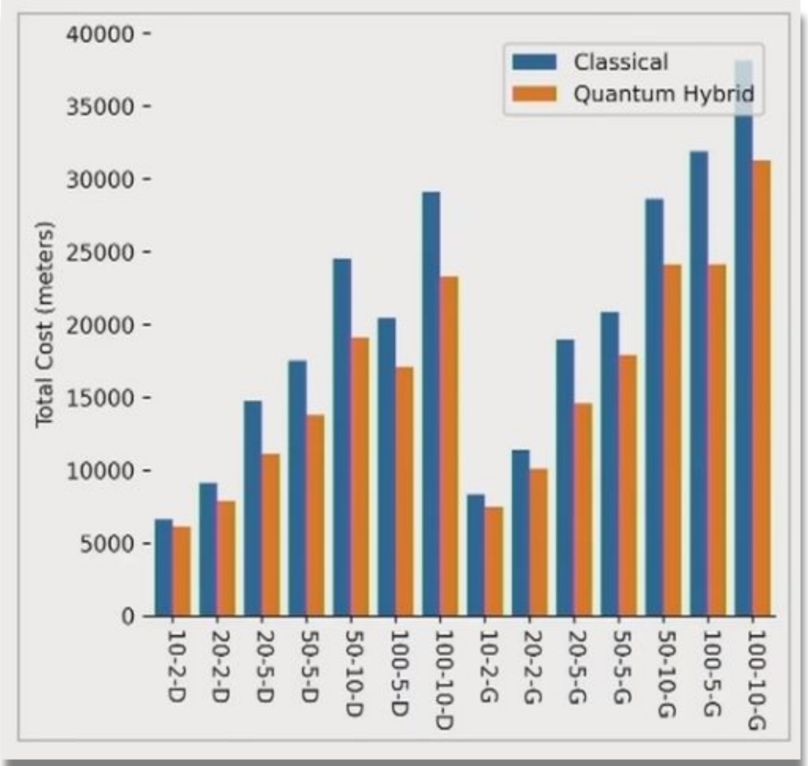
$$f_{clustering} = \sum_{ij} \sum_{k} c_{ij} x_{ik} x_{jk}$$

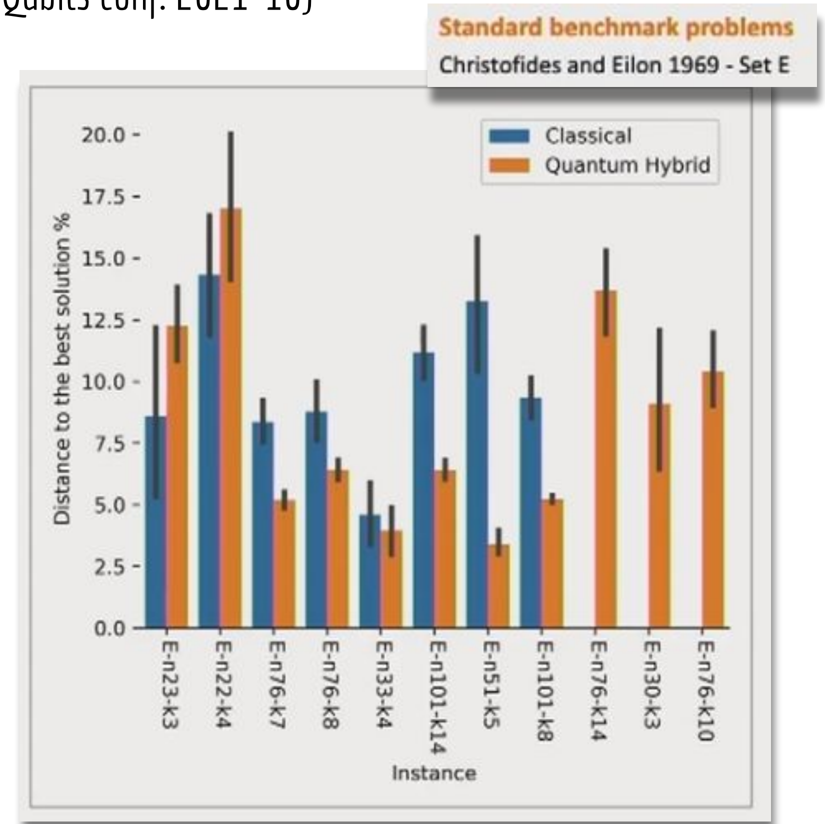
$$\sum_{i} x_{ik} u_i \leq C_k$$

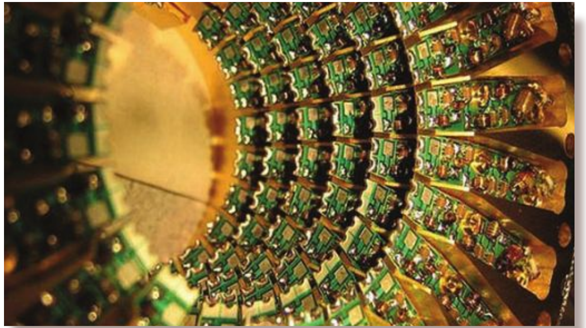
$$\sum_{k} x_{ik} = 1$$

NEC experiment: CVRP

(David Garvin, Nec Australia at Qubits conf. 2021-10)







Outline

Summary, General Results, and Conclusions

Summary

- **Another paradigm**: Method NOT based on universal gates models,
    - But work on smaller number of problems as ones in machine learning and optimization fields;
- **Huge number of qubits, but not controllable as with the universal quantum computers**,
    - Not programmable, no universal gates to apply during the process,
- The Quantum Annealing could be called a "**Hardware Heuristic**" letting the physics solve the problem.
- Last Evolution: MILP API (but how do they map' ?)

Literature: Some Results on a 1K and 2K qubits Machine

- **Vert, D., Sirdey, R., & Louise, S. (2021). Benchmarking Quantum Annealing Against "Hard" Instances of the Bipartite Matching Problem. SN Computer Science, 2(2), 1-12.**
  - "we argue that unless much denser qubits interconnects are developed, it will be difficult for the approach to compete with classical algorithms on real-world problems".
  - (But using a 6-connectivity graph with 2K qubits).
- (other document -french thesis)
  - Étude des performances des machines à recuit quantique pour la résolution de problèmes combinatoires. 2021, thèse de Daniel Vert.

Literature: Some Results on a 1K and 2K qubits Machine

- D-Wave 2X again (1K qubits) runs up to **100M times faste**r than an optimized implementation of the quantum Monte Carlo algorithm **on a single core**.
  - Denchev, V.S., Boixo, S., Isakov, S.V., Ding, N., Babbush, R., Smelyanskiy, V., Martinis, J. and Neven, H., 2016. What is the computational value of finite-range tunneling?. Physical Review X, 6(3), p.031015.
- D-Wave newsletter of the **18/02/2021** about results on 2K machine:
  - "Today D-Wave marks a major milestone on the journey to quantum advantage in a new peer-reviewed paper published in *Nature Communications*. The new research uses a D-Wave lower noise system to demonstrate **3 million times speed-up over classical alternatives in a real-world problem**. [...]"
  - The Paper: King, A.D., Raymond, J., Lanting, T. et al. Scaling advantage over path-integral Monte Carlo in quantum **simulation of geometrically frustrated magnets**. Nature Communication 12, 1113 (2021).
  - **They compared the QA against the best known classical simulation algorithm for this problem (PIMC).**
  - To know which CPU is used in comparison (also the number of threads used), we need to go in the supplementary materials (they use a light **i7-8650U on one core**! To compare, if it is still possible, and if the algo is fully parallelizable, the Fujitsu A64FX will win since it has more than 7 millions of (ARM) cores)!
- Be careful with all the announcements in the media...and also from the companies!
- But still… waiting for results on **Advantage**(5k qubits and 15 connectivity)

Maybe not for all optimization problems…

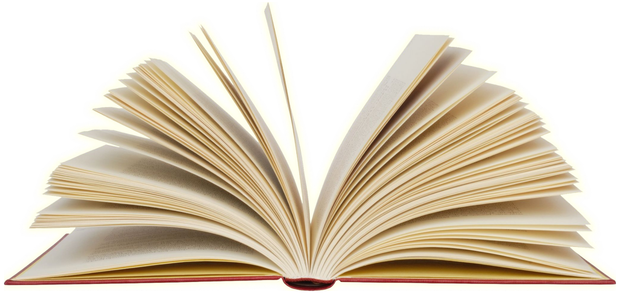
$$
\begin{aligned}
\min \quad & \sum_{r\in R} z_{n_r} && (1)\\
\text{s.t.} \quad & && \\
& x_e^{end} = x_{e+1}^{begin}, \quad e \in RE_r, r \in R : e = 1..n_r - 1, && (2)\\
& x_e^{end} \geq x_e^{begin} + d_e, \quad e \in E, && (3)\\
& x_e^{begin} \geq \tau_e^{begin}, \quad e \in E : i_e = 1, && (4)\\
& x_e^{end} - \tau_e^{end} \leq z_e, \quad e \in E, && (5)\\
& \sum_{t\in T_s} q_{et} = 1, \quad e \in SE_s, s \in S, && (6)\\
& q_{et} + q_{\hat{e}t} - 1 \leq \lambda_{e\hat{e}} + \gamma_{e\hat{e}}, \quad e, \hat{e} \in SE_s, t \in T_s, s \in S : e < \hat{e}, && (7)\\
& x_{\hat{e}}^{begin} - x_e^{end} \geq \Delta_s^M \gamma_{e\hat{e}} - M(1 - \gamma_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s \in S, o_e \neq o_{\hat{e}}, && (8)\\
& x_{\hat{e}}^{begin} - x_e^{end} \geq \Delta_s^F \gamma_{e\hat{e}} - M(1 - \gamma_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s \in S, o_e = o_{\hat{e}}, && (9)\\
& x_e^{begin} - x_{\hat{e}}^{end} \geq \Delta_s^M \lambda_{e\hat{e}} - M(1 - \lambda_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s \in S, o_e \neq o_{\hat{e}}, && (10)\\
& x_e^{begin} - x_{\hat{e}}^{end} \geq \Delta_s^F \lambda_{e\hat{e}} - M(1 - \lambda_{e\hat{e}}), e < \hat{e} \text{ in } SE_s, s \in S, o_e = o_{\hat{e}}, && (11)\\
& \lambda_{e\hat{e}} + \gamma_{e\hat{e}} \leq 1, \quad e, \hat{e} \in SE_s, s \in S : e < \hat{e}, && (12)\\
& x_e^{begin} \geq w_t^{end} q_{et} - M\alpha_e^{w_t}, \quad e \in SE_s, t \in T_s, s \in S, && (13)\\
& x_e^{end} \leq w_t^{begin} q_{et} + M(1 - \alpha_e^{w_t}), \quad e \in SE_s, t \in T_s, s \in S, && (14)\\
& x_e^{begin}, x_e^{end}, z_e \geq 0, \quad e \in E, && (15)\\
& \gamma_{e\hat{e}}, \lambda_{e\hat{e}}, \in \{0,1\}, \quad e, \hat{e} \in SE_s, s \in S : e < \hat{e}, && (16)\\
& q_{et}, \alpha_e^{w_t} \in \{0,1\}, \quad e \in SE_s, t \in T_s, s \in S, && (17)
\end{aligned}
$$

Not a relaxing relaxation…

Future of the quantum annealing tech

- Several Projects in Europe:
  - **AVaQus**
    - the European project developing the first superconducting quantum annealer
    - https://www.quantaneo.com/AVaQus-the-European-project-to-develop-the-first-superconducting-coherent-quantum-annealer_a479.html
  - **ATOS**
    - Quantum Annealing **Simulator**
    - Scope: Machine Learning
    - https://atos.net/en/2020/press-release_2020_07_07/atos-opens-up-a-new-path-to-quantum-annealing-simulation
  - **Qilimanjaro**

Outline



References & Co

- Some Help to install D-Wave Ocean API
- Several types of references:
  - Books, Vidéos, Conf', Papers, Courses,
  - From vulgarisation to theoretical.

Some help to install the D-Wave Ocean API on Debian based machine

- Linux commands :
  - sudo apt-get install python<version>
  - sudo pip install virtualenv
  - virtualenv ocean
  - pip install dwave-ocean-sdk
  - git clone https://github.com/dwavesystems/dwave-ocean-sdk.git
  - cd dwave-ocean-sdk ou cd ocean
  - python setup.py install ou ./python ./easy_install install
  - Add to your cod the token available in your dashboard

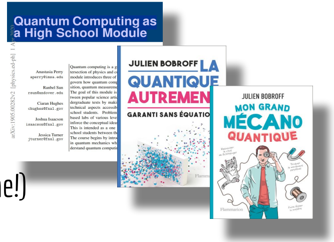
- More detail: https://docs.ocean.dwavesys.com/en/stable/overview/install.html

```
(ocean) spydel@spydel-NUC10i5FNH:~/Documents/vracCode/ocean/bin$ ls
activate       activate.ps1      easy_install      easy_install3.8  pip-3.8  python3    wheel3
activate.csh   activate_this.py  easy_install3     pip              pip3.8   python3.8  wheel-3.8
activate.fish  activate.xsh      easy_install-3.8  pip3             python   wheel      wheel3.8
(ocean) spydel@spydel-NUC10i5FNH:~/Documents/vracCode/ocean/bin$ dwave config create
Configuration file not found; the default location is: /home/spydel/.config/dwave/dwave.conf
Configuration file path [/home/spydel/.config/dwave/dwave.conf]:
Configuration file path does not exist. Create it? [y/N]: y
Profile (create new) [prod]: spydel
API endpoint URL [skip]:
Authentication token [skip]:
Default client class [skip]:
Default solver [skip]:
Configuration saved.
(ocean) spydel@spydel-NUC10i5FNH:~/Documents/vracCode/ocean/bin$
```

General popularization works

(from the easiest to the hardest)

- Article (100p): Quantum Computing as a High School Module https://arxiv.org/abs/1905.00282
- Book (fr): La quantique autrement. Julien Bobroff.
- Book (fr): Mon grand mécano quantique. Julien Bobroff.



- Video: "Les Ordinateurs Quantiques Expliqués - Limites de la technologie humaine" (so well done!) https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell
- Website article: "You don't need to be a mathematician to master quantum computing" https://towardsdatascience.com/you-dont-need-to-be-a-mathematician-to-master-quantum-computing-161026af8878
- Slides: US Department of energy: How about quantum computing. Bert de Jong https://cs.lbl.gov/assets/CSSSP-Slides/20190624-deJong.pdf

- Videos: Understanding Quantum Mechanics. Sabine Hossenfelder. https://www.youtube.com/watch?v=XJSfgE9LUJw&list=PLwgQsqtH9H5djIfFhXE6We207beTgUnyL
- Book: Quantum computation and quantum information. Nielsen & Chuang

Research Papers

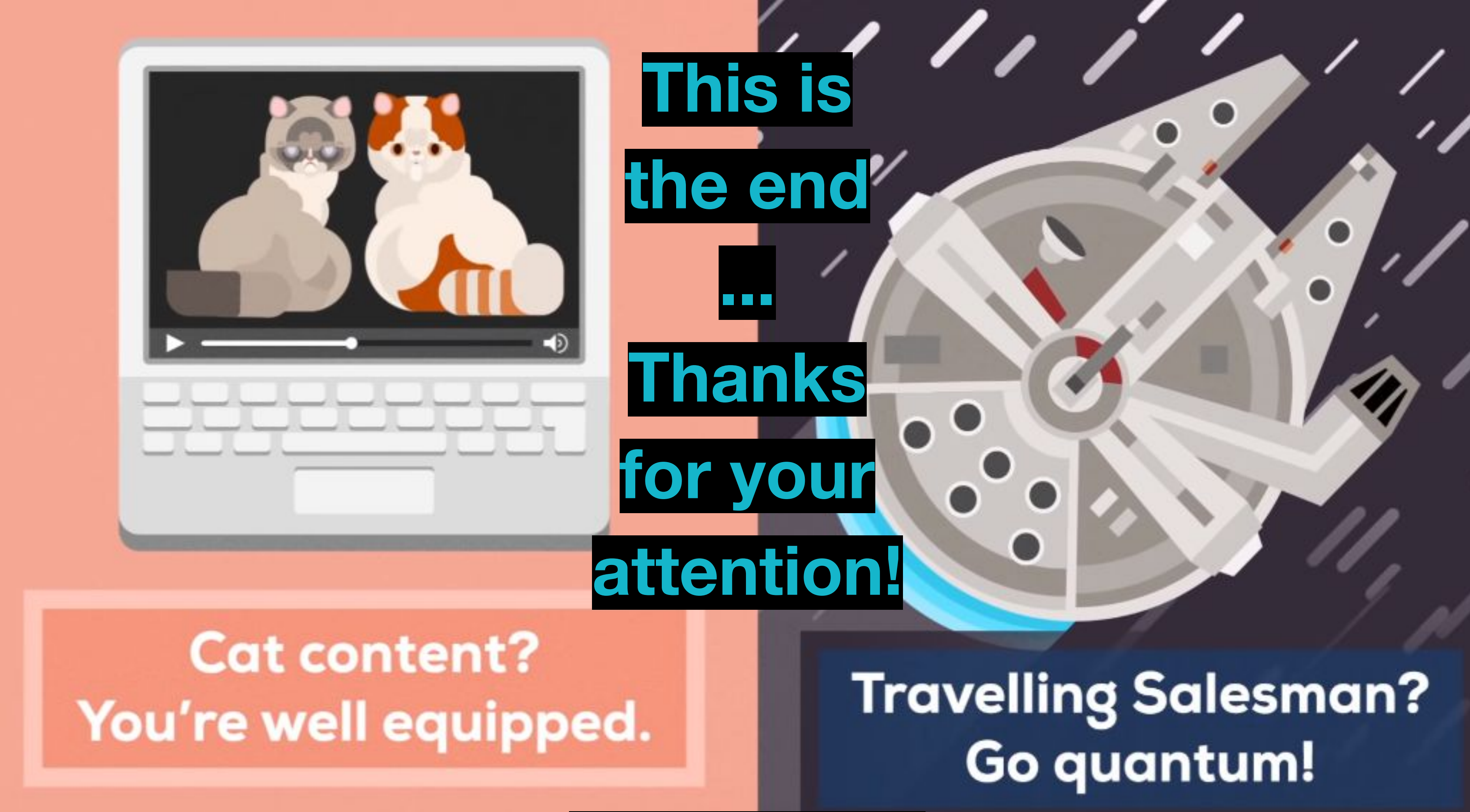
- Simulated Annealing: Kirkpatrick, Scott, C. Daniel Gelatt, and Mario P. Vecchi. "Optimization by simulated annealing." science 220.4598 (1983): 671-680.
- **Vert, D., Sirdey, R., & Louise, S. (2021). Benchmarking Quantum Annealing Against "Hard" Instances of the Bipartite Matching Problem. SN Computer Science, 2(2), 1-12.**
- Smelyanskiy, Vadim N., et al. "A near-term quantum computing approach for hard computational problems in space exploration." arXiv preprint arXiv:1204.2821 (2012).
- Hamerly, R., Inagaki, T., McMahon, P.L., Venturelli, D., Marandi, A., Onodera, T., Ng, E., Langrock, C., Inaba, K., Honjo, T. and Enbutsu, K., 2019. Experimental investigation of performance differences between coherent Ising machines and a quantum annealer. *Science advances*, *5*(5), p.eaau0823.
    - Here the MaxCut problem is solved on DW2Q has 2048 qubits (lets try with a 5K!)
- Barahona, Francisco; Grötschel, Martin; Jünger, Michael; Reinelt, Gerhard (1988). "An Application of Combinatorial Optimization to Statistical Physics and Circuit Layout Design". *Operations Research*. **36** (3): 493–513.
- Barahona. "On the Computational Complexity of Ising Spin Glass Models." J. Phys. A 15 (1982), pp. 3241–3253.
    - About the NP-Hard complexity of Ising and QUBO Problems
- Mengoni, Riccardo, Daniele Ottaviani, and Paolino Iorio. "Breaking RSA Security With A Low Noise D-Wave 2000Q Quantum Annealer: Computational Times, Limitations And Prospects." *arXiv preprint arXiv:2005.02268* (2020).
- King, James, et al. "Benchmarking a quantum annealing processor with the time-to-target metric." arXiv preprint arXiv:1508.05087 (2015).
- Analytical and numerical evidence suggests that quantum annealing outperforms simulated annealing under certain conditions. Heim, B., Rønnow, T. F., Isakov, S. V., & Troyer, M. (2015). Quantum versus classical annealing of Ising spin glasses. Science, 348(6231), 215-217.
- Albash, Tameem, Victor Martin-Mayor, and Itay Hen. "Temperature scaling law for quantum annealing optimizers." Physical review letters 119.11 (2017): 110502.

Courses

- "A practical introduction to quantum computing: from qubits to quantum machine learning and beyond (5/7)"
  - 7 courses from the CERN Quantum Technology Initiative.
  - (5/7): "Quantum algorithms for combinatorial optimization. Quantum adiabatic computing and quantum annealing. Introduction to D-Wave Leap. Quantum Approximate Optimization Algorithm."
  - https://cds.cern.ch/record/2746545
- John Preskill lectures:
  - http://theory.caltech.edu/~preskill/ph229/
- Serge Haroche (collège de France lectures)
  - https://www.college-de-france.fr/site/serge-haroche/_course.htm

D-Wave ressource

- Leap:
  - https://cloud.dwavesys.com/leap/
- Qubo & Ising Models
  - https://docs.dwavesys.com/docs/latest/c_gs_3.html
- D-Wave examples on Github
  - https://github.com/dwave-examples
- Tutos (Videos and articles) for developers
  - https://www.dwavesys.com/practical-quantum-computing-developers
- Tuto for installing the Ocean API:
  - https://docs.ocean.dwavesys.com/en/stable/overview/install.html
- A tons of videos from D-Wave:
  - https://www.youtube.com/user/dwavesystems/playlists



This is the end … Thanks for your attention!

pic: https://www.youtube.com/watch?v=JhHMJCUmq28&ab_channel=Kurzgesagt%E2%80%93InaNutshell

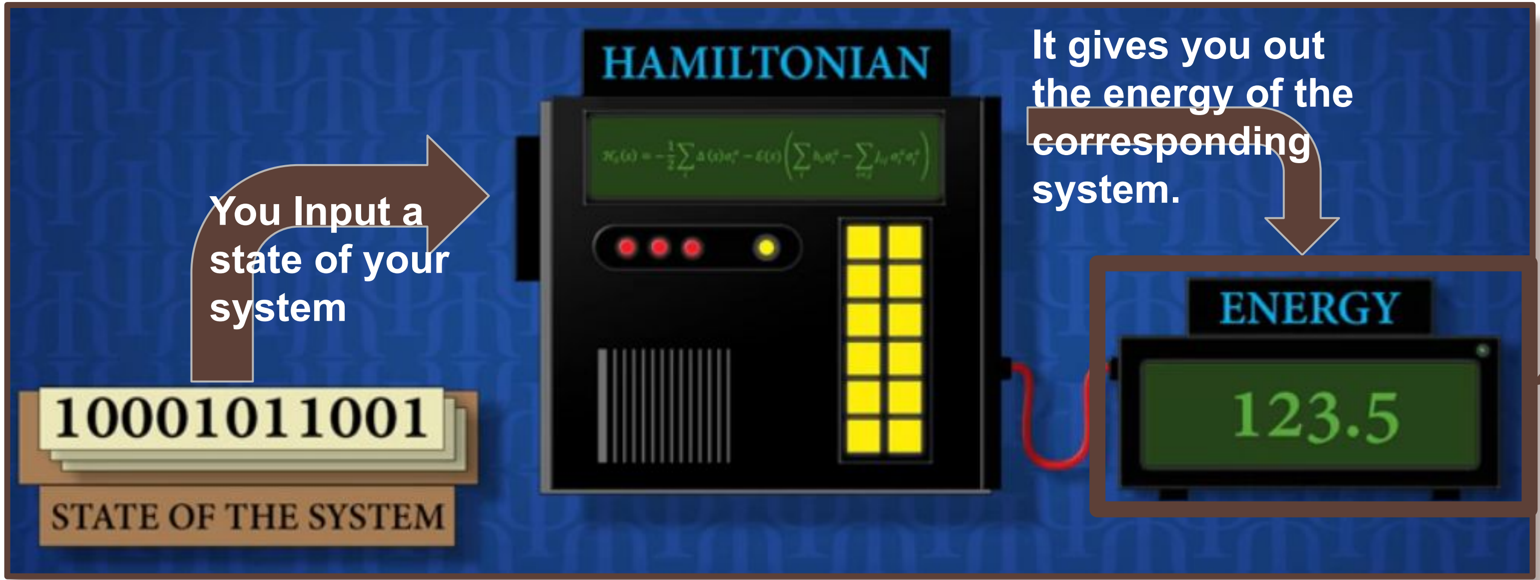
(samuel.deleplanque@junia.com)

And now...
The rejected slides!

The Hamiltonian during the quantum annealing....

Quantum Annealing

- The **Hamiltonian ($\mathcal{H}$)** is (here) a mathematical <u>description of the energy of a physical system</u>. (here we focus on the energy inside the QPU).



Goal? To Find the **minimum** energy state

pics: https://www.youtube.com/watch?v=tnikftltqE0&t=125s

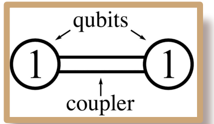
The Hamiltonian during the quantum annealing....

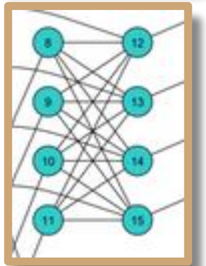
Quantum Annealing

- The **Hamiltonian (H)** includes 2 types of interaction:
  - ***h_i*** : the "programmable" external magnetic field called a **biais**,
  - ***J_{ij}*** : the interaction between each neighboring qubits ***i*** and ***j*** **(coupling)**.

$$\mathcal{H}_S(s) = -\frac{1}{2}\sum_i \Delta(s)\sigma_i^x + \mathcal{E}(s)\left(-\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z\right)$$

Initial Hamiltonian | **Final Hamiltonian**





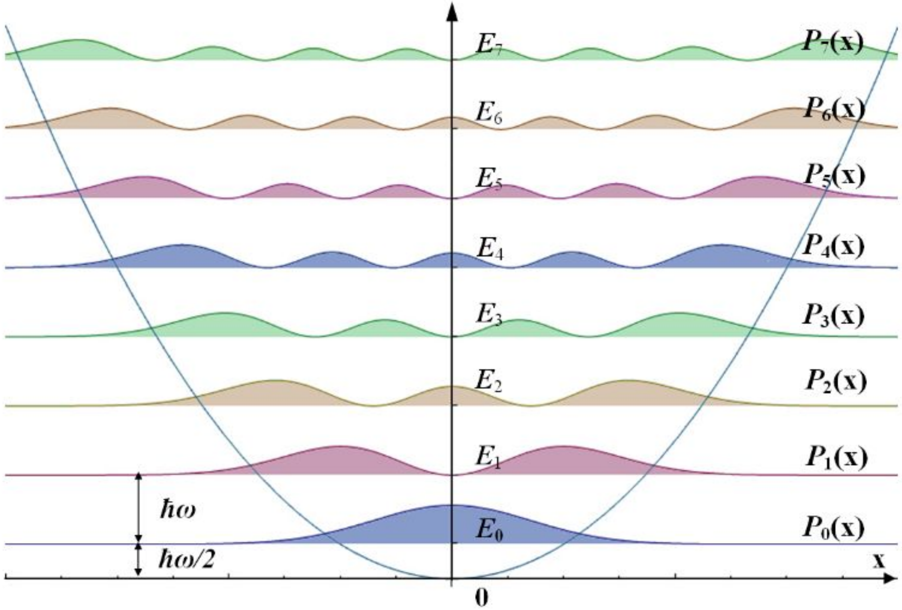
pics: https://www.youtube.com/watch?v=tnikftltqE0&t=125s

Slow Final Hamiltonian Integration

(Adiabatic Theorem)

Principle: we need to integrate slowly the final Hamiltonian step by step to stay in the ground state (P0(x) in the figure) and to avoid the excited states ((Pi>0(x) in the figure)

In some way: we map slowly the qubits and couplers from our model.



(very) Few words on Discrete Optimization problems

Goal of an optimization problem

Finding a solution that minimizes the cost of the problem

- an **exact method** is trying to find the best solution(s) with **the lowest cost**.

If it is not possible:

- an **approximate method**, also named **heuristic**, will try to find a good solution with a **low cost.** A heuristic can find the best solution but it is not guaranteed and it will not be proven.

Discrete optimization problems involve discrete variables (**integers and/or binaries**).

Discrete Optimization problems (MILP): Example 1

Traveling Salesman Problem Example

Given a list of cities and the distances between each pair of cities, <u>what is the shortest possible route that visits each city exactly once and returns to the origin city</u>?

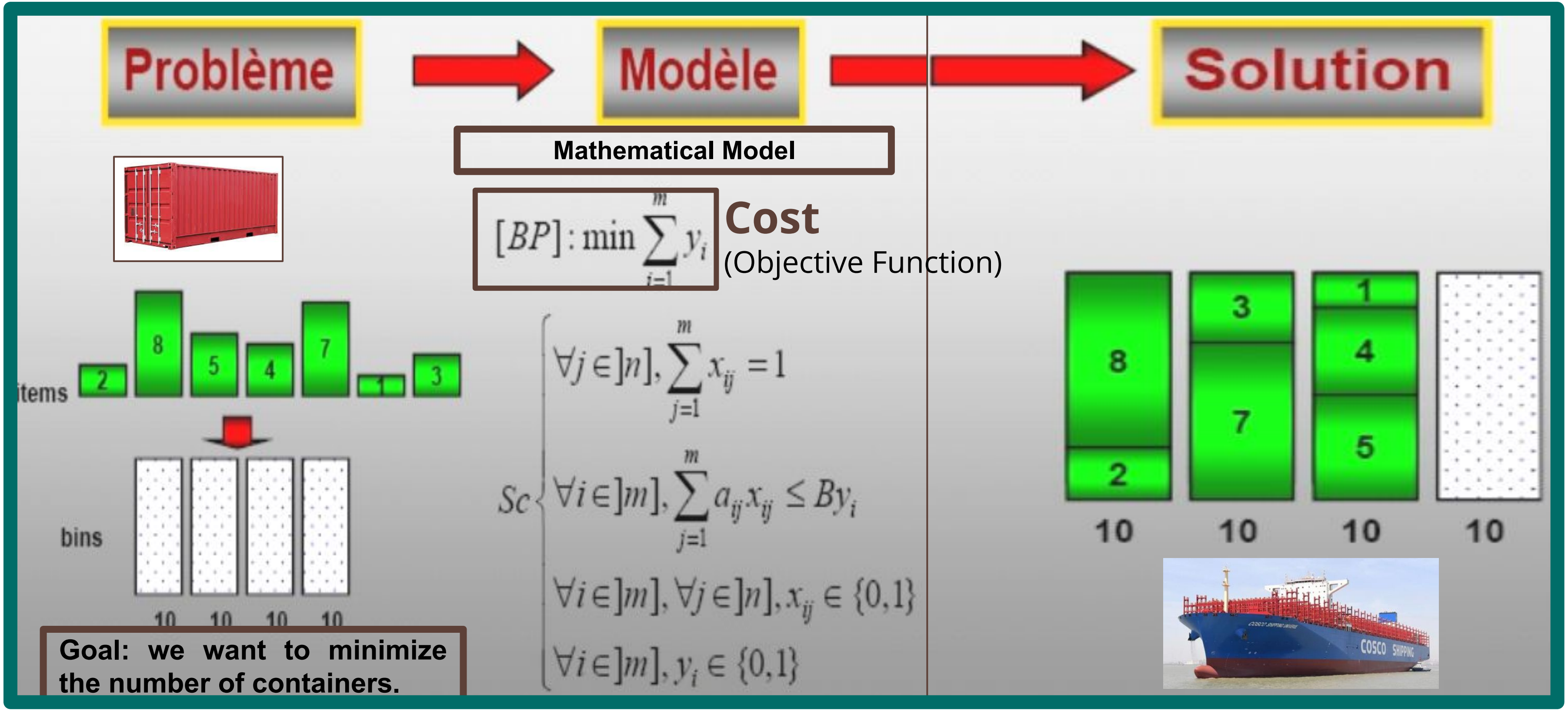
$$\min \sum_{i=1}^{n} \sum_{j\neq i, j=1}^{n} c_{ij} x_{ij}:$$

Cost
(Objective Function)

Mathematical Model

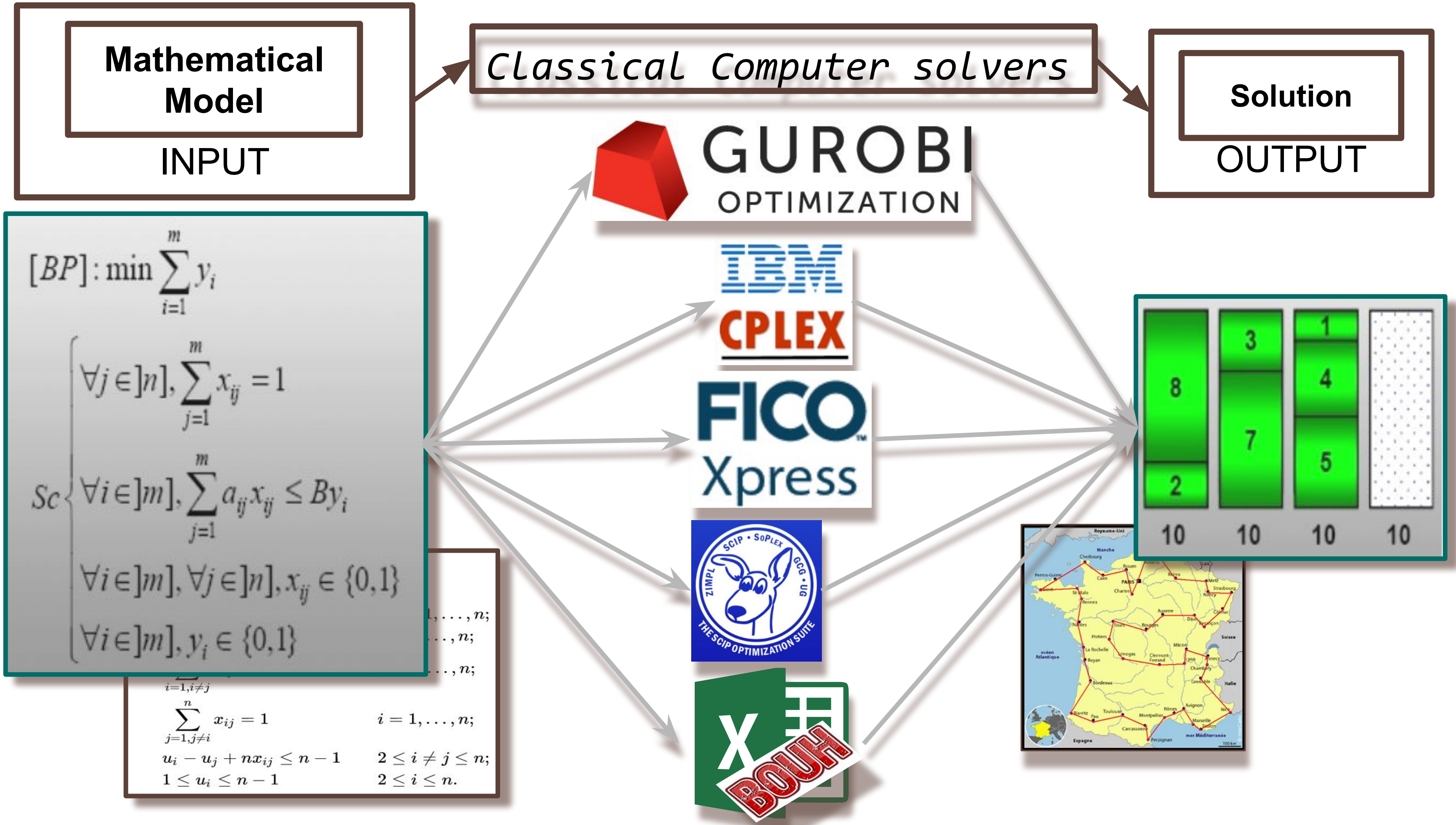
$$
\begin{aligned}
& x_{ij} \in \{0,1\} && i,j = 1,\ldots,n; \\
& u_i \in \mathbf{Z} && i = 2,\ldots,n; \\
& \sum_{i=1, i\neq j}^{n} x_{ij} = 1 && j = 1,\ldots,n; \\
& \sum_{j=1, j\neq i}^{n} x_{ij} = 1 && i = 1,\ldots,n; \\
& u_i - u_j + n x_{ij} \leq n-1 && 2 \leq i \neq j \leq n; \\
& 1 \leq u_i \leq n-1 && 2 \leq i \leq n.
\end{aligned}
$$

Discrete Optimization problems (MILP): Example 2

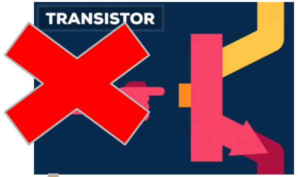


$$[BP] : \min \sum_{j=1}^{m} y_i$$

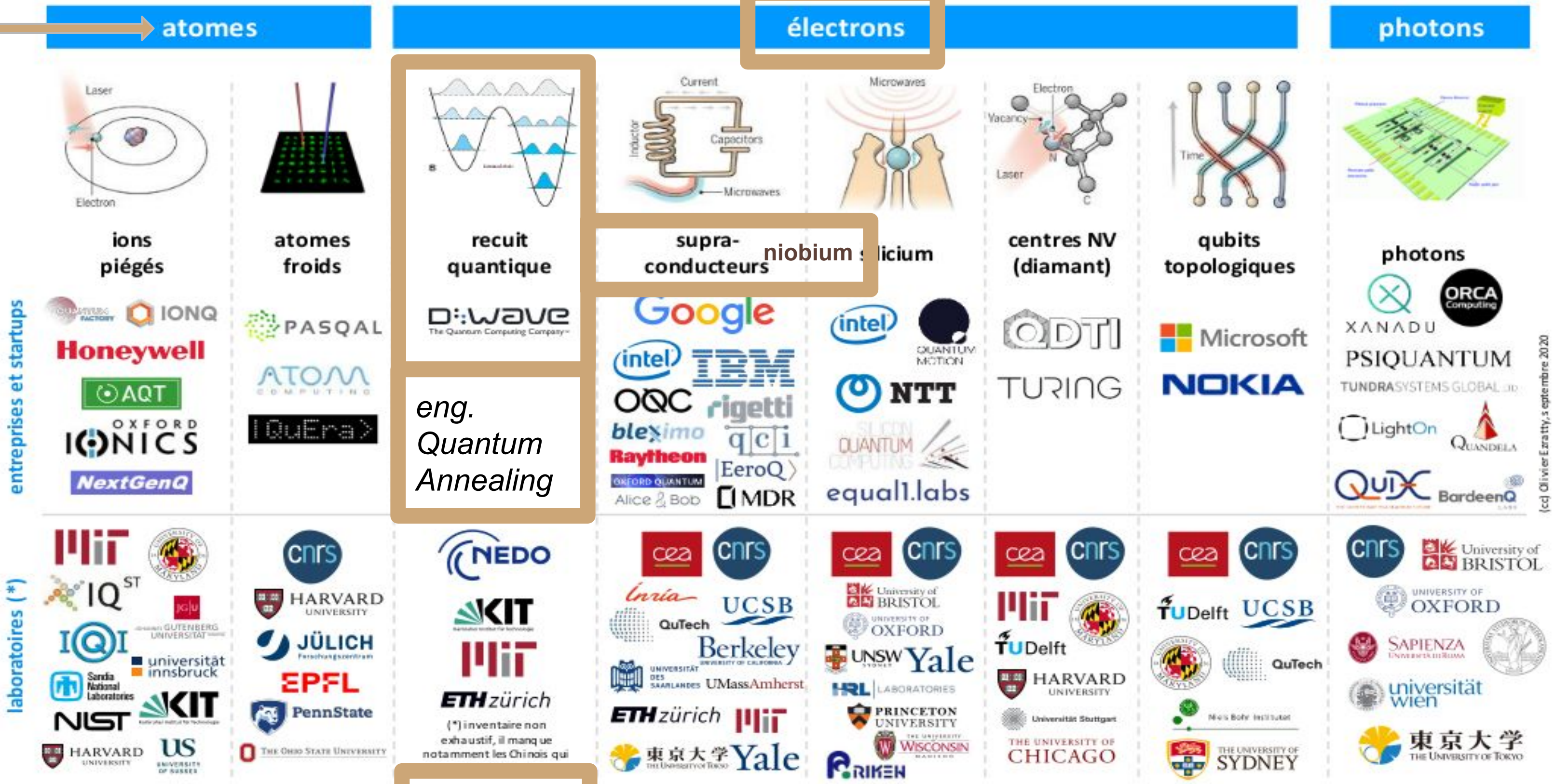
$$Sc \begin{cases} \forall j \in]n], \sum_{j=1}^{m} x_{ij} = 1 \\ \forall i \in]m], \sum_{j=1}^{m} a_{ij} x_{ij} \leq B y_i \\ \forall i \in]m], \forall j \in]n], x_{ij} \in \{0,1\} \\ \forall i \in]m], y_i \in \{0,1\} \end{cases}$$



Mathematical Model
INPUT
Classical Computer solvers
Solution
OUTPUT
GUROBI
OPTIMIZATION
IBM
CPLEX
FICO
Xpress
SCIP • SoPLEX
GCG • UG
ZIMPL
THE SCIP OPTIMIZATION SUITE
BOUH
$[BP]: \min \sum_{i=1}^{m} y_i$
$\forall j \in]n], \sum_{j=1}^{m} x_{ij} = 1$
$Sc \quad \forall i \in]m], \sum_{j=1}^{m} a_{ij} x_{ij} \le B y_i$
$\forall i \in]m], \forall j \in]n], x_{ij} \in \{0,1\}$
$\forall i \in]m], y_i \in \{0,1\}$
$\sum_{j=1, j \neq i}^{n} x_{ij} = 1 \qquad i = 1, \dots, n;$
$u_i - u_j + n x_{ij} \le n - 1 \qquad 2 \le i \ne j \le n;$
$1 \le u_i \le n - 1 \qquad 2 \le i \le n.$
8
2
3
7
1
4
5
10
10
10
10



D-Wave qubits among the quantum computing world

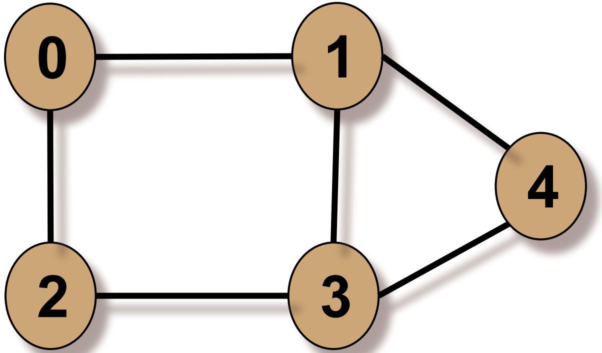


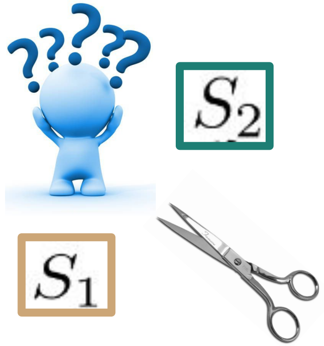
→ https://atos.net/wp-content/uploads/2020/07/PR-Atos-opens-up-a-new-path-to-quantum-annealing-simulation.pdf
& https://atos.net/fr/2020/communiques-de-presse_2020_07_07/atos-ouvre-la-voie-a-la-simulation-du-recuit-quantique

original pic: https://www.oezratty.net/wordpress/2020/comprendre-informatique-quantique-edition-2020/

Max-cut-Problem

Let’s solve another example by hand





Try to find “by hand” the maximal max cut of this simple graph.

Let's take a Graph `G=(X,E)`, where **X** is the set of nodes and **E** the set of edges. We can identify each node `i` of the graph with a binary variable Z_i such that:

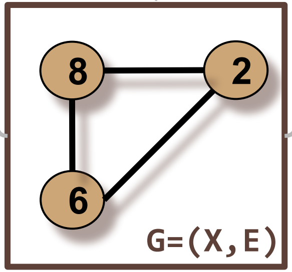
- Z_i = 1 if i is in one group,
- Z_i = -1 otherwise.

Ising Model

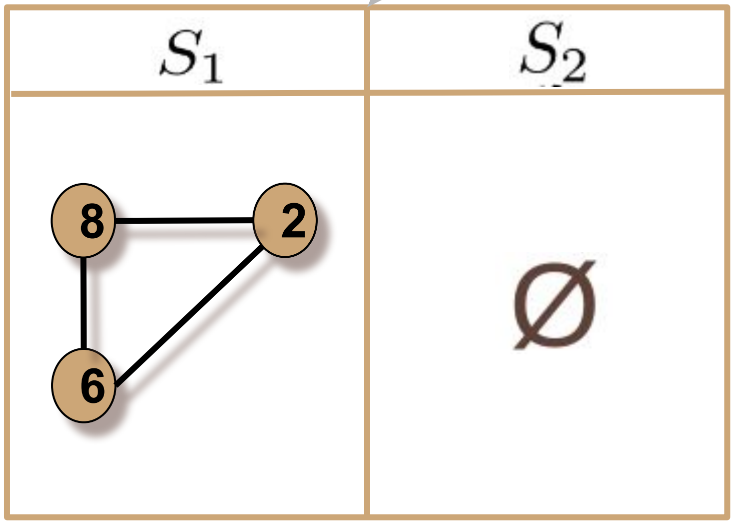
Small Example with a K3 Complete Graph

The Max-Cut Problem consists in finding the vector Z of $|X|$ binaries which minimizes:

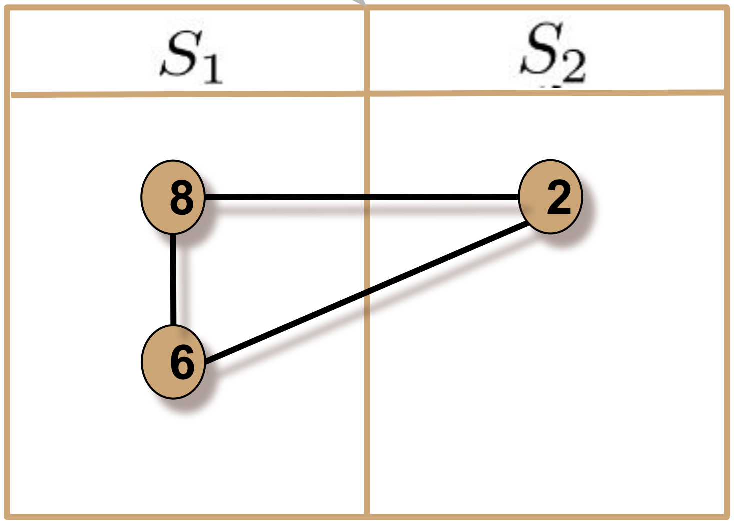
$$\sum_{(i,j)\in E} Z_i Z_j$$



Solution 1

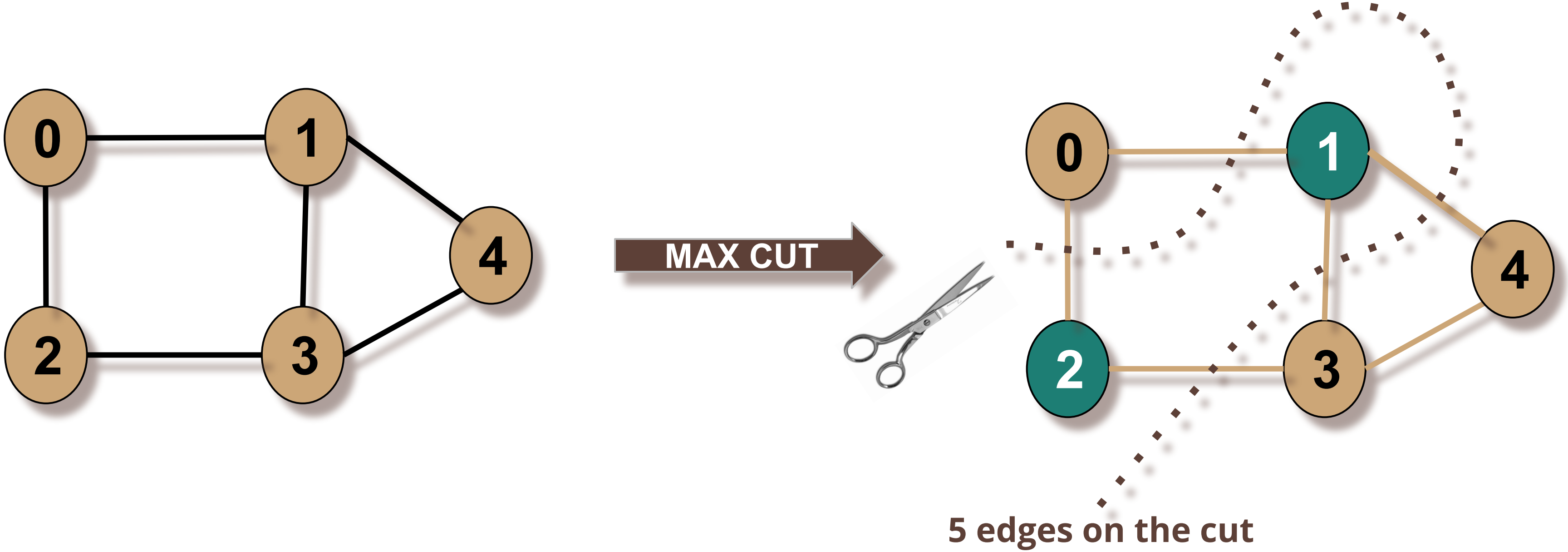


Solution 2



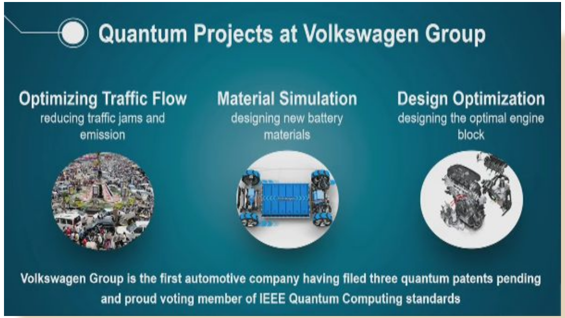
Here you have 2 solutions...You might know which one is the best...
But **try to be sure by calculating the (cost) Objective function f() of each.**

Max-cut-Problem: a first solution

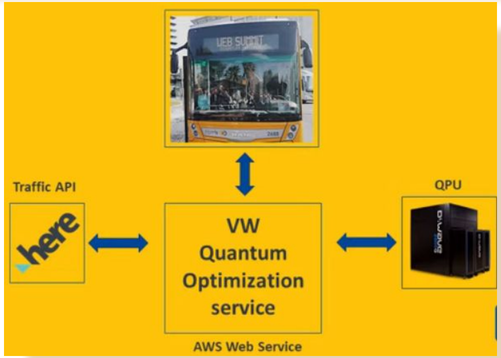


An optimal solution of the Max Cut problem is: {**1**;**2**} and {0;3;4} giving **5 edges.**
The solution can be written as $x = (x_0\ \mathbf{x_1}\ \mathbf{x_2}\ x_3\ x_4) = (-1\ \mathbf{1}\ \mathbf{1}\ -1\ -1)$.

D-Wave & "Volkswagen"



Mobility might be one of the field the most impacted by Quantum Computing in the coming years (with D-Wave machines).



Example (volkswagen/D-wave partnership, 2019):

- **First quantum traffic management system** is Lisbon. 9 buses in Lisbon with a management based on D-Wave quantum computer results (finding almost in real time the fastest route for each of the buses).

D-Wave quantum computers: early applications

Materials Properties

- Atomic Magnetometer,
- Solid state materials simulation,
- Quantum chemistry computation;

Machine Learning

- Image recognition,
- Individual cancer drugs,
- Finding Higgs Boson;

Cyber Security & Fault Detection

- Formation of Terrorist Networks,
- Facial recognition,
- Fault Detection in circuits;

Optimization

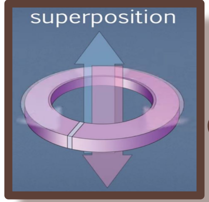
- Location problems
  - Satellites, Network devices, stations;
- Multi-period portfolios (finance),
- Traffic Flow,

and so many others!

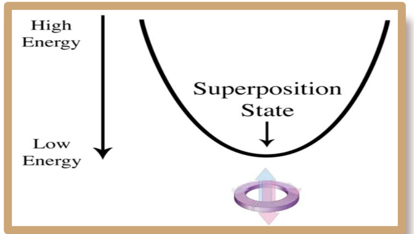
https://www.dwavesys.com/applications

First Step

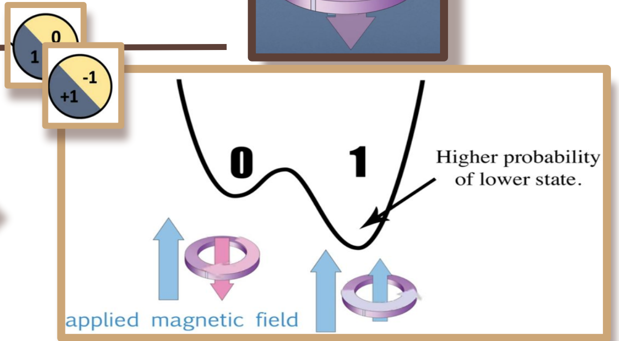
The Qubits start in a superposition state (zero and one).

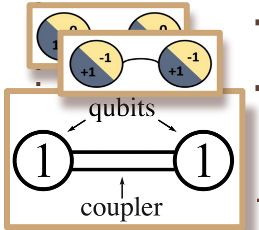


and

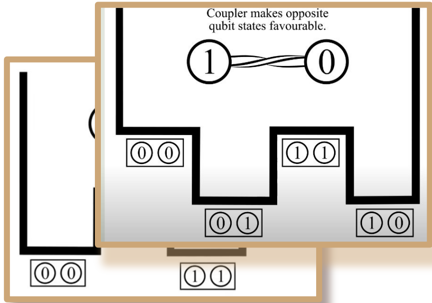


Control the probability of the state with a programmable external magnetic field



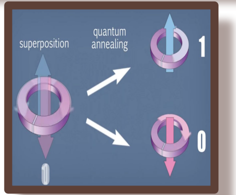


- **link the qubits together** by **couplers** (entanglement)
- The couplers define how a pair of qubits **ends up**, in the same or in opposite state, resp.: 0 & 0 ; 1 & 1 or, 1 & 0 ; 0 & 1.
- **the machine will "physically" make the equality (or the inequality)** "**energetically favorable**", i.e., to "low the energy" of those states.

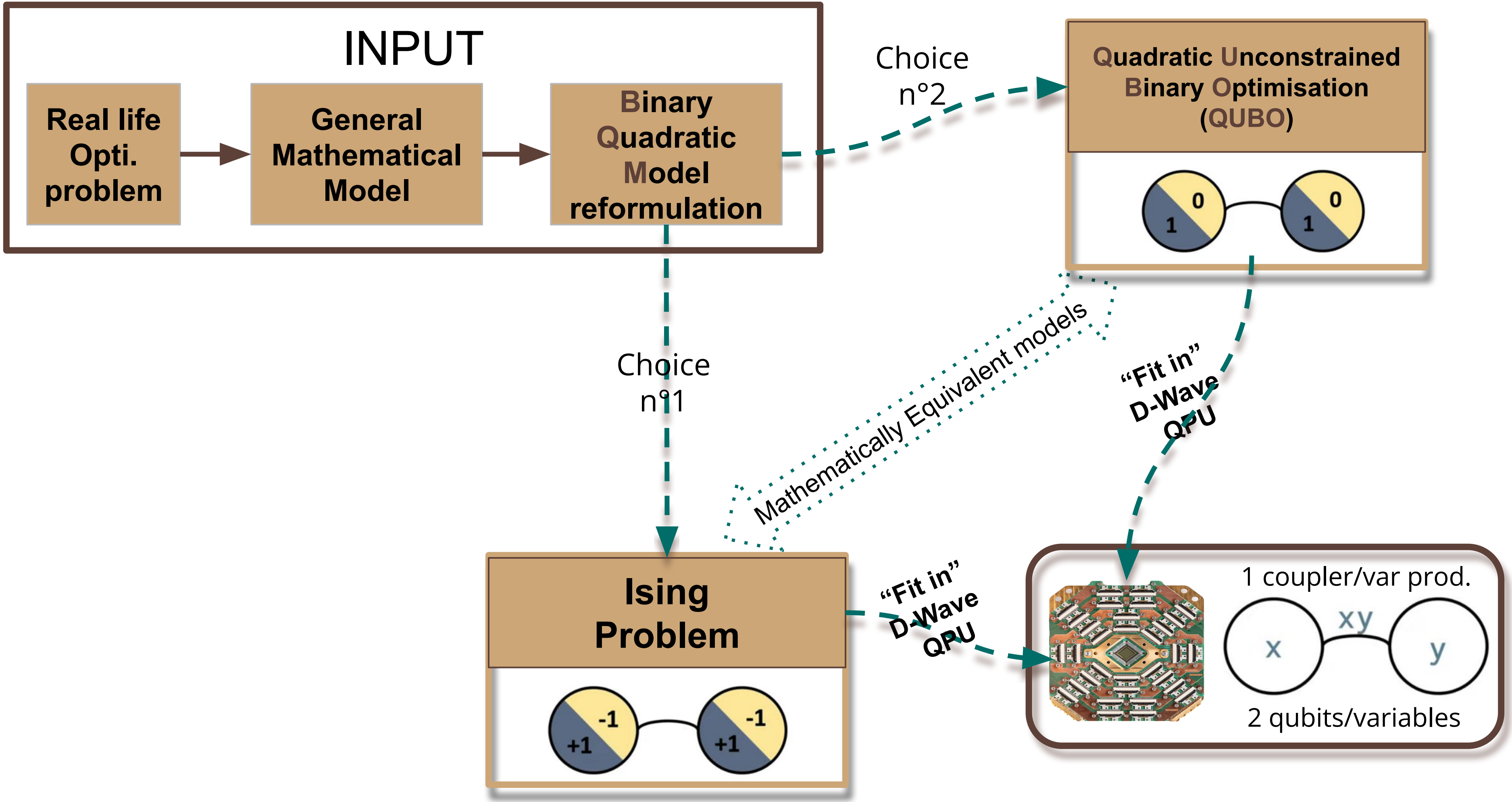


Last step

At the end, the measure of each qubit make them going to (with a high probability) either the **zero state** or the **one state**.

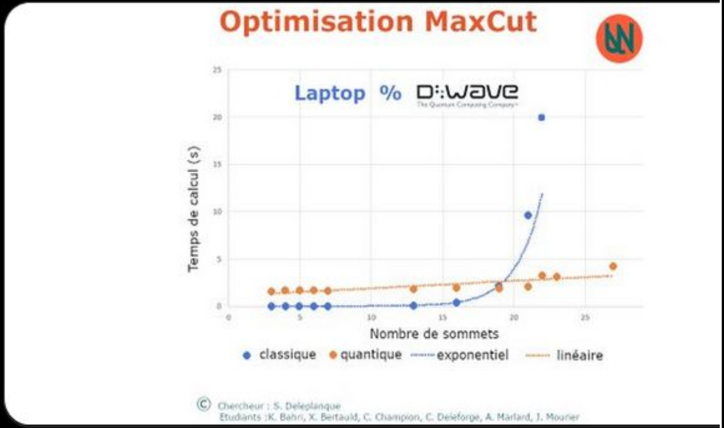


or


INPUT
Real life Opti. problem
General Mathematical Model
Binary Quadratic Model reformulation
Choice n°2
Quadratic Unconstrained Binary Optimisation (QUBO)
0
1
0
1
Choice n°1
Mathematically Equivalent models
"Fit in" D-Wave QPU
Ising Problem
-1
+1
-1
+1
"Fit in" D-Wave QPU
1 coupler/var prod.
x
xy
y
2 qubits/variables

Isabelle LEFEBVRE @ilefebvre_iemn · Mar 24

Projet M1 à @isenlille en informatique quantique: optimisation avec @dwavesys. 1er résultat des étudiants sur la comparaison de temps de calcul. Ca marche!
@DeleplanqueSam2
#QuantumComputing #EnseignementSupérieur #quantique

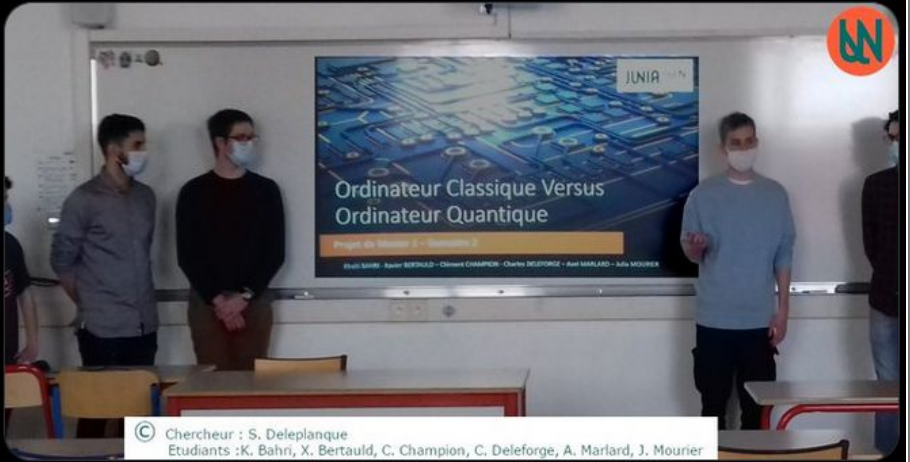


1 6

Isabelle LEFEBVRE @ilefebvre_iemn · Apr 26

Excellente soutenance de projet M1 à @isenlille: optimisation quantique de la répartition des projets selon les préférences des étudiants.
@dwavesys
@DeleplanqueSam2

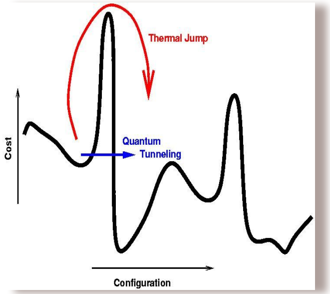
#QuantumComputing #EnseignementSupérieur #quantique

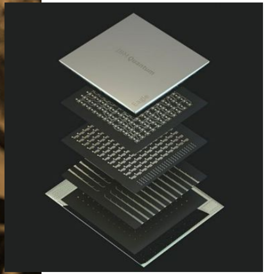


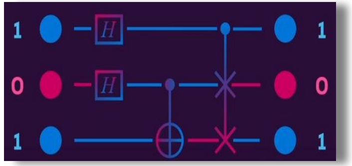
1 5

5k qubits… versus 127 ???

Differences between a D-Wave machine and a gates quantum computer?



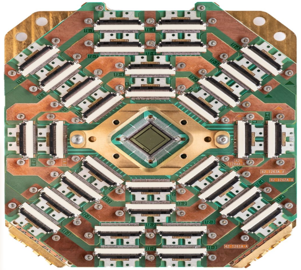




There is no match…they are not in the same competition

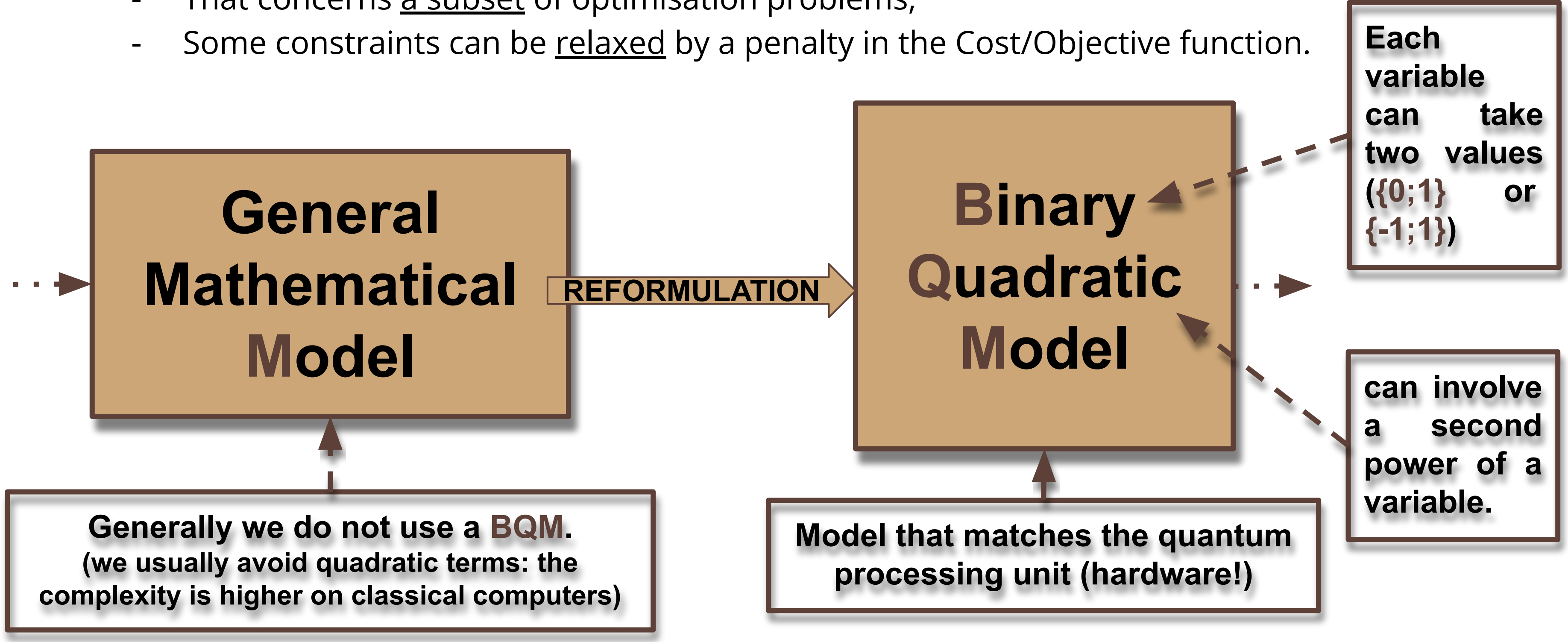
End of 2020, the Advantage D-Wave machine is released...

- A new **Q**PU with:
  - more than **5,000 qubits**
  - **35,000 couplers**
    - To link pair of qubits
- qubits technology:
  - Superconducting qubits (*SQUID*)
  - Metal used: **niobium**
    - We obtain quantum mechanical effects when the **niobium** becomes a superconductor (once it is cooled down to ≈0K).

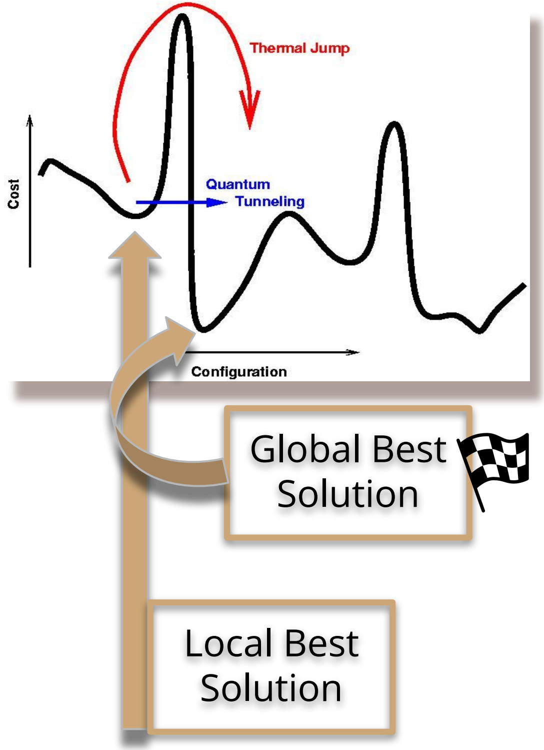


Preparing the input...

- To model an optimisation problem to a Binary Quadratic Model
  - That concerns a subset of optimisation problems,
  - Some constraints can be relaxed by a penalty in the Cost/Objective function.



Simulated Annealing & Quantum annealing: 2 Metaheuristics



Analogy

While the temperature is directly related to the probability of moving to a new solution in simulated annealing, in quantum annealing, the strength of transverse field determines the quantum-mechanical probability to **change the amplitudes of all states in parallel**.

The advantage of QA compared to SA is due to the quantum mechanics allowing for **an additional escape route from local minima**. While SA must climb over energy barriers to escape traps, **QA can penetrate these barriers without any increase in energy**. This effect is known as **quantum tunneling**.