An optimization model for line planning and timetabling in automated urban metro subway networks. A case study

Yolanda Hinojosa
IMUS. Universidad de Sevilla

(joint work with V. Blanco, E. Conde and J. Puerto)

SEMINARIO CONCLUSIONES PROYECTO MTM2016

FUENGIROLA, JUNE 2020

Motivation of the Project

R&D Company interested on implementing automatic subway networks in Europe.

Contract: 1853/0257 (Société Metrolab®), 2013.

Motivation of the Project

R&D Company interested on implementing automatic subway networks in Europe.

Contract: 1853/0257 (Société Metrolab®), 2013.

⇓

Omega 92 (2020) 102165

Contents lists available at ScienceDirect

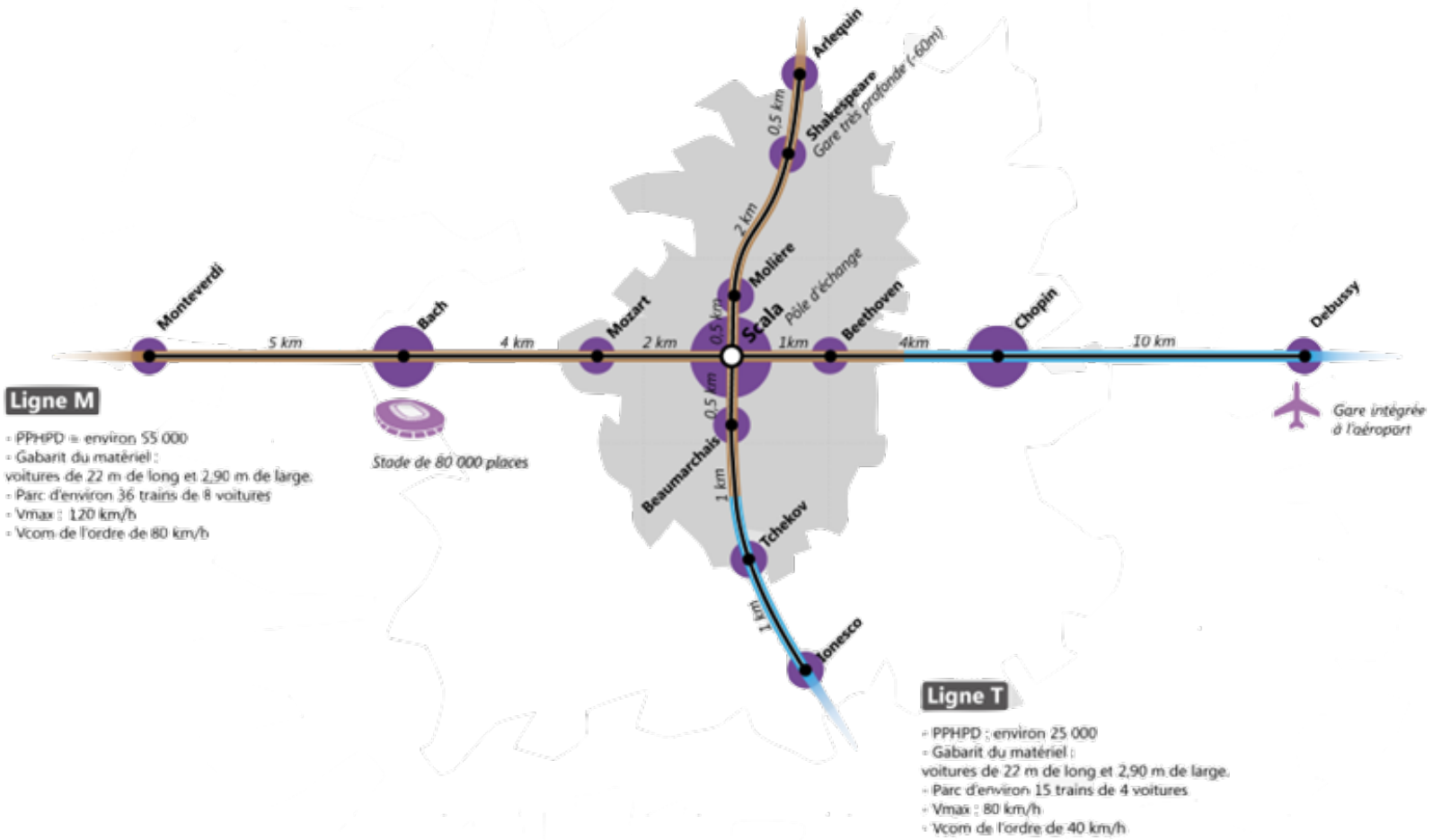
Omega

journal homepage: www.elsevier.com/locate/omega

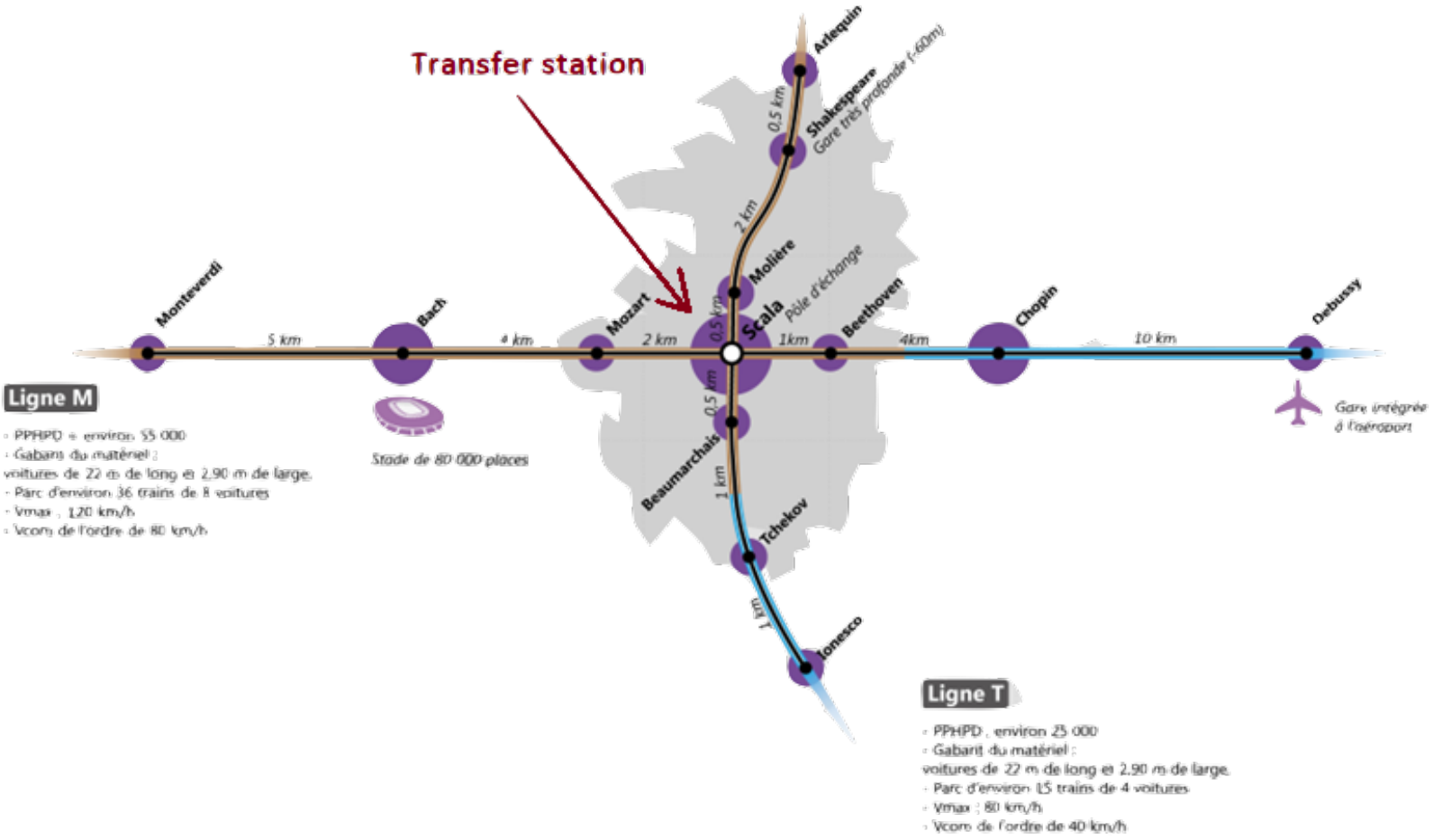
An optimization model for line planning and timetabling in automated urban metro subway networks. A case study☆,☆☆

Víctor Blanco[a,*], Eduardo Conde[b], Yolanda Hinojosa[c], Justo Puerto[b]

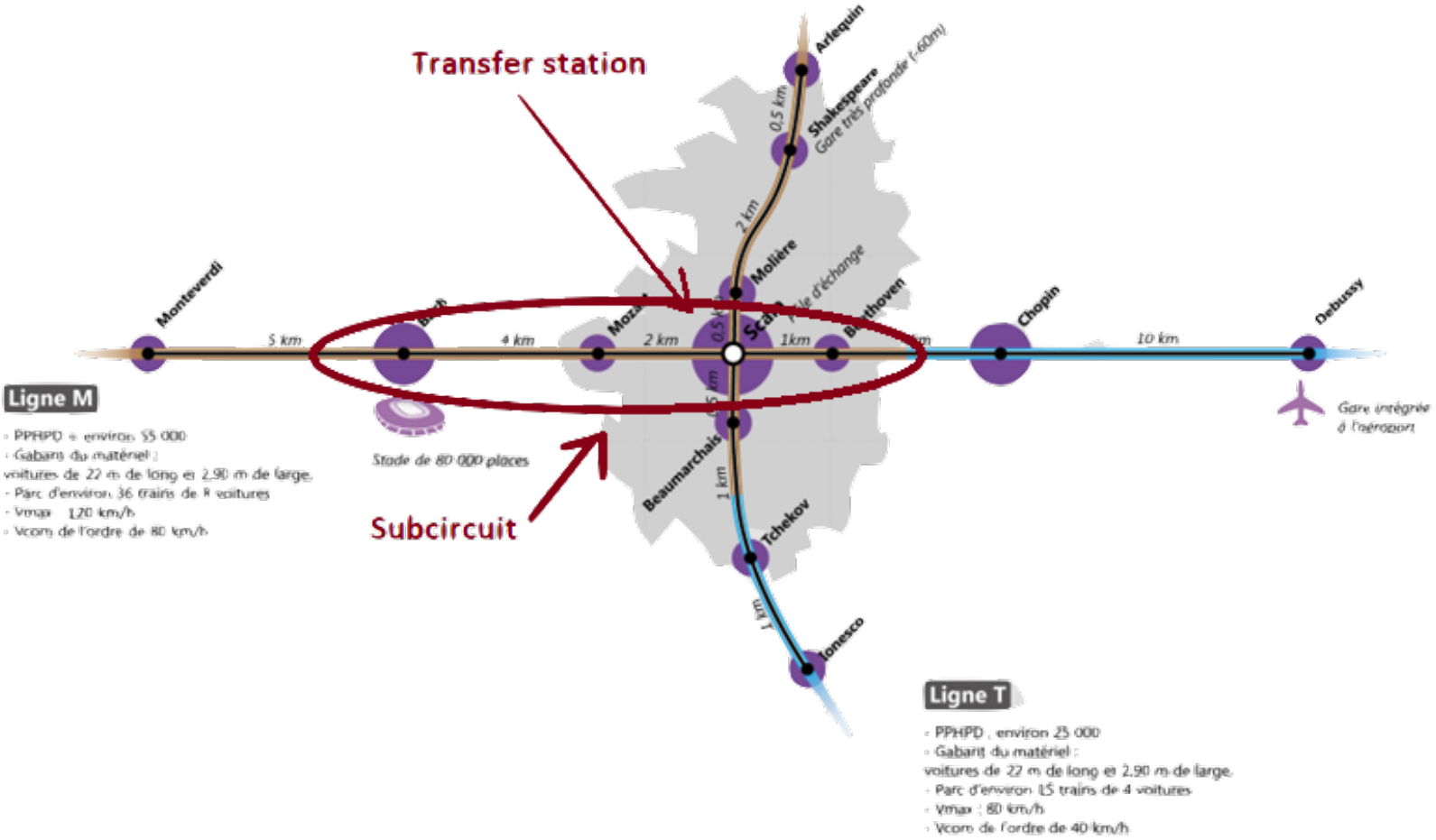
Problem Description



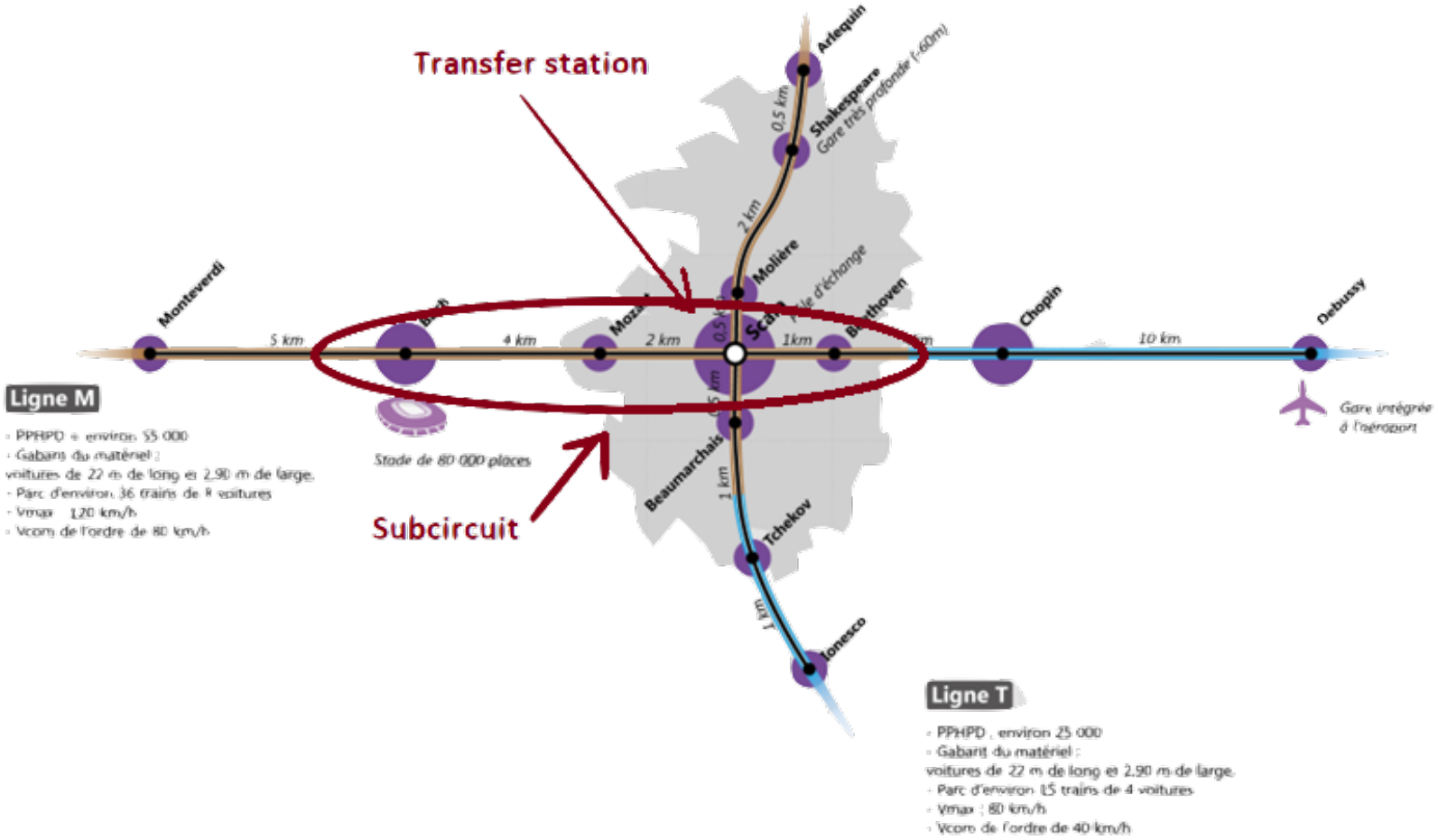
Problem Description



Problem Description



Problem Description



Goal: Optimizing the operations of the subway network.

Problem Description

Input Data

- Structure of the network (no. of lines, stations, distances, speed, stations of the short.turns,...).
- Possible Capacities for the trains (based on the carriages).
- Safe times between trains.
- Passengers flow between each O-D (can be assumed uniformly distributed in time windows of the planning horizon).

Problem Description

Goal: Optimizing the operations of the subway network.

- Minimize the operative costs (no. of rounds, capacities,..).
- Maximize the profit (by passengers use).
- Minimize the no. of passengers exceeding effective capacities.

Problem Description

Decisions

- Number of trips (complete lines and short-turns) over the same line to be planned in the time horizon.
- Capacities (among the available) for each of the trains in a route.
- Timetables for each of the lines operating in the system.

Problem Description

Decisions

Planning process in public transportation

1. network design
2. **line planning**
   - Number of trips (complete lines and short-turns) over the same line to be planned in the time horizon.
   - Capacities (among the available) for each of the trains in a route.
3. **timetabling**
   - Timetables for each of the lines operating in the system.
4. scheduling

Our first Contribution

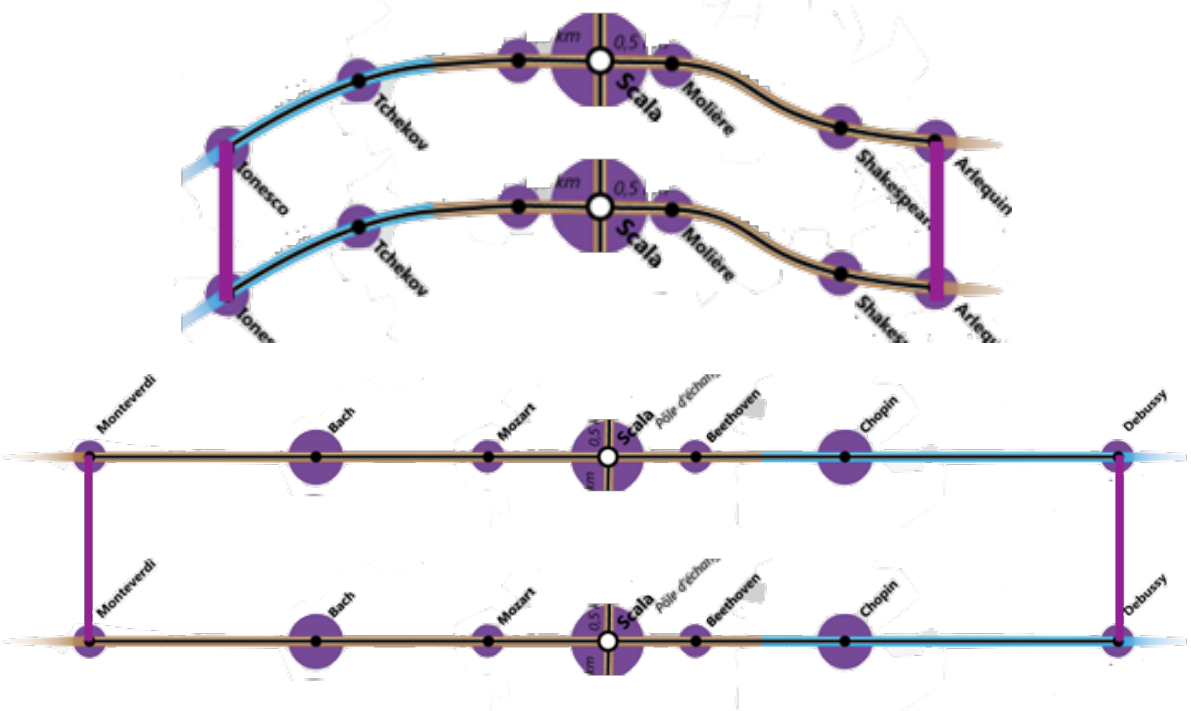
A Mixed Integer Linear Programming Model

- **line planning together with timetabling:**
  - starting times for each trip (non-periodic),
  - selection of capacities for the trips,
  - determination of the optimal number of trips,
  - activation of **short-turns**,
- **cost- and passenger-oriented objective function**,
- **interchange stations**,
- **time-dependent demands**.

A mathematical programming model

1. The whole planning is partitioned into different time windows (with homogeneous demand): peak, off-peak, etc.. hours

2. Each line is considered duplicating stations → PLATFORMS!!



Parameters : Network

- $[0, T]$: Time horizon.
- $L = LS \cup LSN$: Set of lines in the network formed by the set of lines containing short-turns and the set of lines that do not contain short-turns.
- $N_\ell = \{1, \ldots, n_\ell\}$: Stations of line $\ell \in L$.
- $S_\ell = \{1_{S_\ell}, \ldots, n_{S_\ell}\}$: Stations of short-turns $\ell \in LS$.
- d_i^ℓ: Travel distance between the stations i and $i+1$ of the line $\ell \in L$.
- e_i^ℓ: Stopping time that a train spends in the station i of the line $\ell \in L$.
- $t_{1 \mapsto 1_{S_l}}$: Time difference between the time instant in which a train departs from the first station of the subcircuit and the first station of the line.

$$t_{1 \mapsto 1_{S_l}} = \sum_{r=1}^{1_{S_l}-1} (d_r^l + e_{r+1}^\ell)$$

- $Q = \{q_1, \ldots, q_{|Q|}\}$: Possible capacities for trains operating in all the lines.
- IS^ℓ: Safety interval between consecutive rounds in line $\ell \in L$.

Parameters: Passengers Flow

- $\beta_{0_i}^{\ell}$: Passenger at the beginning of the time horizon at station i of line ℓ.
- β_i^{ℓ}: Rate of external passenger which enter to the transportation system at station i to use line ℓ.
- p_{ij}^{ℓ}: Proportion of passengers using the network starting at station i that go to the station j of line ℓ.
- $\tau_i^{\ell\ell'}$: Proportion of passengers that get off a train in a transfer-station i of the line ℓ to transfer to line ℓ'.

Parameters: Costs and profits

- b_q^ℓ: Fixed cost per complete line round of capacity $q \in Q$ on line $\ell \in L$. Largest capacities and largest lines usually involve more cost on the rounds.
- bS_q^ℓ: Fixed cost per short-turns round of capacity $q \in Q$ on line $\ell \in L$.
- γ_{ij}^ℓ: Unitary profit of transporting a passenger from the station i to the station j of the line $\ell \in L$.
- μ_1: Unitary penalty for passengers who cannot get on the first arriving train due to its limited capacity and still insist on using the system.
- μ_2: Unitary penalty for passengers who leave the system after they cannot get on the first arriving train due to its limited capacity.
- α: Proportion of passengers who decide to wait for the next train in case they cannot get on a train because of lack of capacity.

Variables

$K_\ell = \{1, \ldots, \bar{k}_\ell\}$: Trips made in the line $\ell \in L$.

(Maximum number of trips:$\bar{k}_\ell = \frac{T}{IS^\ell}$)

- t_1^{kl}: Departure time from the initial station of line $\ell \in L$ at its k-th trip.
- $y_q^{k\ell} = \begin{cases} 1 & \text{if the } k\text{-th trip of line } \ell \in L \text{ is a whole trip with capacity } q \\ 0 & \text{otherwise} \end{cases}$
- $yS_q^{k\ell} = \begin{cases} 1 & \text{if the } k\text{-th trip of line } \ell \in LS \text{ is a short-turn with capacity } q \\ 0 & \text{otherwise} \end{cases}$
- f_i^{kl}: Flow of passengers captured in the station i by the train that covers the k-th trip of the line $\ell \in L$, when k is a whole trip.
- g_i^{kl}: Flow of passengers captured in the station $i \in S_\ell \backslash \{n_\ell\}$ by the train that covers the k-th trip of the line $\ell \in LS$, when k only covers the short-turn.
- $x_i^{k\ell}$: Excess of passengers only if k is a *true* trip for station i of line $\ell \in L$.
- w^{kl}: Difference between the actual departure time from the first short-turn station of the k-th trip of line $\ell \in LS$ and the time when it should depart from this station regarding its departure time from the initial line station.

Auxiliary Variables

- $t_i^{k\ell}$: Time instant in which a train departs from station i.

$$t_i^{k\ell} = t_1^{k\ell} + \sum_{r=1}^{i-1}(d_r^\ell + e_{r+1}^\ell), \quad i > 1,\ (i,\ell) \in \overline{\mathcal{S}},\ k \in K_\ell,$$

$$t_i^{k\ell} = t_1^{k\ell} + \sum_{r=1}^{i-1}(d_r^\ell + e_{r+1}^\ell) + w^{k\ell}, \quad i > 1,\ i \in S_\ell,\ k \in K_\ell,\ \ell \in LS.$$

- $h_i^{k\ell}$: Excess of passengers at station i.

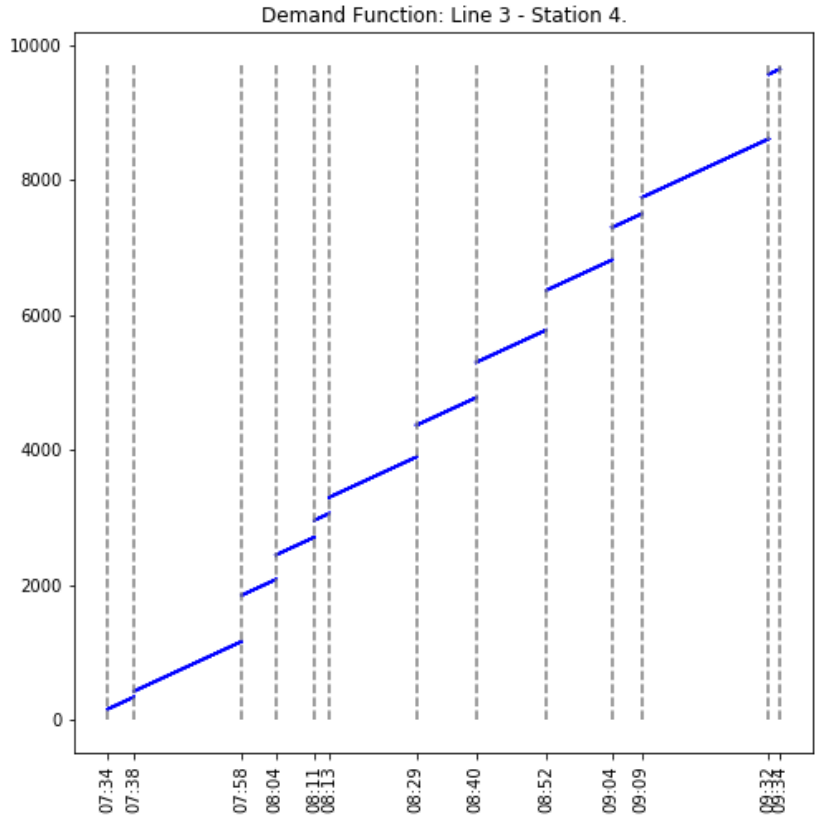
$$\begin{aligned}
& h_i^{1\ell} = D_i^\ell(t_i^{1\ell}) - f_i^{1\ell}, \quad \text{for } (i,\ell) \in \overline{\mathcal{S}}, \\
(k=1) \qquad & h_i^{1\ell} = D_i^\ell(t_i^{1\ell}) - f_i^{1\ell} - g_i^{1\ell}, \quad \text{for } i \in S_\ell \backslash \{n_{S_\ell}\},\ \ell \in LS, \\
& h_{n_{S_\ell}}^{1\ell} = D_{n_{S_\ell}}^\ell(t_{n_{S_\ell}}^{1\ell}) - f_{n_{S_\ell}}^{1\ell} + \sum_{r=1_{S_\ell}}^{n_{S_\ell}-1} \sum_{j=n_{S_\ell}+1}^{n_\ell} p_{rj} g_r^{1\ell}, \quad \text{for } \ell \in LS,
\end{aligned}$$

$$\begin{aligned}
h_i^{k\ell} &= D_i^\ell(t_i^{k\ell}) - D_i^\ell(t_i^{(k-1)\ell}) + \alpha h_i^{(k-1)\ell} - f_i^{k\ell}, \quad (i,\ell) \in \overline{\mathcal{S}} \\
h_i^{k\ell} &= D_i^\ell(t_i^{k\ell}) - D_i^\ell(t_i^{(k-1)\ell}) + \alpha h_i^{(k-1)\ell} - f_i^{k\ell} - g_i^{k\ell}, \quad i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS, \\
h_{n_{S_\ell}}^{k\ell} &= D_{n_{S_\ell}}^\ell(t_{n_{S_\ell}}^{k\ell}) - D_{n_{S_\ell}}^\ell(t_{n_{S_\ell}}^{(k-1)\ell}) + \alpha h_{n_{S_\ell}}^{(k-1)\ell} - f_{n_{S_\ell}}^{k\ell} + \sum_{r=1_{S_\ell}}^{n_{S_\ell}-1} \sum_{j=n_{S_\ell}+1}^{n_\ell} p_{rj} g_r^{k\ell},\ \ell \in LS.
\end{aligned}$$

Auxiliary Variables. The Demand function

- $D_i^\ell(t)$: Accumulated flow of passengers up to time t at station i.

$$D_i^\ell(t) = \beta_{0i}^\ell + \beta_i^\ell t + J_{i\ell}^E(t) + \sum_{\ell' \neq \ell, \ell' \ni i} J_{i\ell\ell'}^I(t), \tag{D}$$



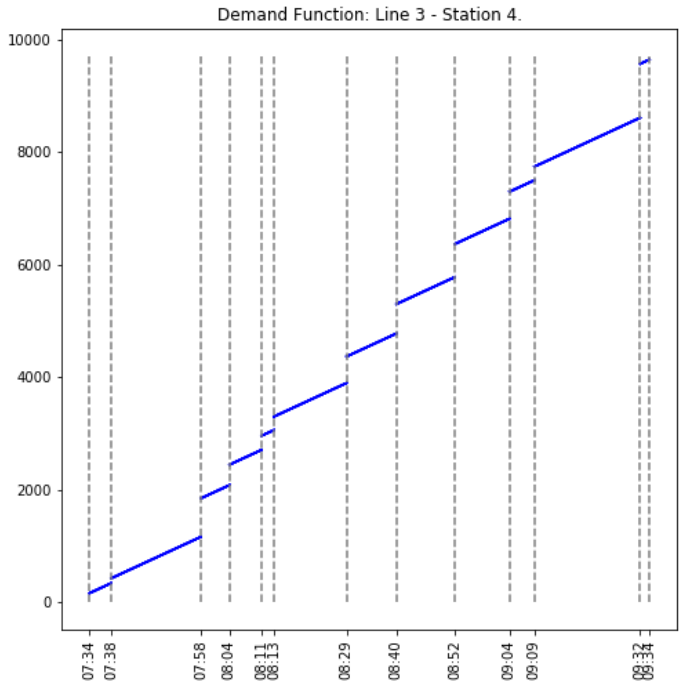
β_{0i}^ℓ: Number of passengers awaiting in the station i at the beginning of the planning horizon.

β_i^ℓ: Average rate of passengers arriving to the station i by unit of time.

$J_{i\ell}^E(t)$: Sum of the external block of arrivals of passengers up to the instant t to the station i.

$J_{i\ell\ell'}^I(t)$: Sum of the block arrivals of passengers up to the instant t to the interchange station i of line $\ell \in L$ from line $\ell' \in L$.

Auxiliary Variables. The Demand function



$se_r^{i\ell}$: Time instants when the block of arrivals occur ($r = 0, \ldots, re^{i\ell}$).

$\Psi_{ir'}^{\ell}$: Discontinuity flow jump produced at time instant $se_r^{i\ell}$.

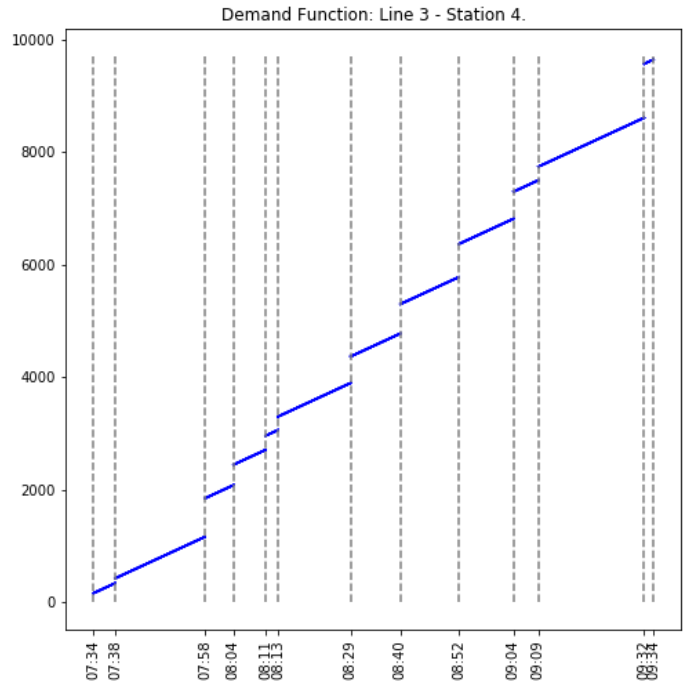
$$\delta_{ri\ell}^{E}(t) = \begin{cases} 1 & \text{if } t \in [se_r^{i\ell}, se_{r+1}^{i\ell}), \\ 0 & \text{otherwise,} \end{cases}$$

$$se_r^{i\ell}\delta_{ri\ell}^{E}(t) \leq t < se_{r+1}^{i\ell}\delta_{ri\ell}^{E}(t) + \widehat{T}_\ell(1 - \delta_{ri\ell}^{E}(t)),$$

$$\sum_{r=0}^{re^{i\ell}} \delta_{ri\ell}^{E}(t) = 1,$$

- External Arrivals: $J_{i\ell}^{E}(t) = \sum_{r=0}^{re^{i\ell}} \left(\sum_{r' \leq r} \Psi_{ir'}^{\ell} \right) \delta_{ri\ell}^{E}(t), \quad i \in N_\ell,\ \ell \in L.$

Auxiliary Variables. The Demand function



$se_r^{i\ell}$: Time instants when the block of arrivals occur ($r = 0, \ldots, re^{i\ell}$).

$\Psi_{ir'}^{\ell}$: Discontinuity flow jump produced at time instant $se_r^{i\ell}$.

$$\delta_{ri\ell}^{E}(t) = \begin{cases} 1 & \text{if } t \in [se_r^{i\ell}, se_{r+1}^{i\ell}), \\ 0 & \text{otherwise,} \end{cases}$$

$$se_r^{i\ell}\delta_{ri\ell}^{E}(t) \leq t < se_{r+1}^{i\ell}\delta_{ri\ell}^{E}(t) + \widehat{T}_\ell(1 - \delta_{ri\ell}^{E}(t)),$$

$$\sum_{r=0}^{re^{i\ell}} \delta_{ri\ell}^{E}(t) = 1,$$

- External Arrivals: $J_{i\ell}^{E}(t) = \sum_{r=0}^{re^{i\ell}} \left(\sum_{r' \leq r} \Psi_{ir'}^{\ell} \right) \delta_{ri\ell}^{E}(t), \quad i \in N_\ell,\ \ell \in L.$

- Internal Arrivals:

$$J_{i\ell\ell'}^{I}(t) = \sum_{r=0}^{\bar{k}_{\ell'}} \left(\sum_{r' \leq r} \Phi_{ir'}^{\ell\ell'} \right) \delta_{ri\ell\ell'}^{I}(t), \quad i \in N_\ell \cap N_{\ell'},\ \ell \in L, \ell' \in L.$$

Objective Function: Minimize

Capacity Costs

$$\begin{cases} \displaystyle\sum_{k\in K_\ell}\sum_{q\in Q} b_q^\ell y_q^{k\ell} & \text{if } \ell \in LNS, \\ \displaystyle\sum_{k\in K_\ell}\sum_{q\in Q} b_q^\ell y_q^{k\ell} + \sum_{k\in K_\ell}\sum_{q\in Q} b_{Sq}^\ell (y_{Sq}^{k\ell} - y_q^{k\ell}) & \text{if } \ell \in LS. \end{cases} \qquad (\mathrm{Cap}(\ell))$$

Objective Function: Minimize

–

Reward per served passenger

$$\begin{cases} \displaystyle\sum_{i \in N_\ell \setminus \{1\}} \sum_{k \in K_\ell} \sum_{r=1}^{i-1} \gamma_{ri}^{\ell} p_{ri}^{\ell} f_r^{k\ell} & \text{if } \ell \in LNS, \\ \displaystyle\sum_{k \in K_\ell} \left(\sum_{i \in N_\ell \setminus \{1\}} \sum_{r=1}^{i-1} \gamma_{ri}^{\ell} p_{ri}^{\ell} f_r^{k\ell} + \sum_{i \in S_\ell \setminus \{1_{S_\ell}\}} \sum_{r=1_{S_\ell}}^{i-1} \gamma_{ri}^{\ell} p_{ri}^{\ell} g_r^{k\ell} + \sum_{\substack{r \in S_\ell : \\ r \neq n_{S_\ell}}} \sum_{j=n_{S_\ell}+1}^{n_\ell} \gamma_{rn_{S_\ell}}^{\ell} p_{rj}^{\ell} g_r^{k\ell} \right) & \text{if } \ell \in LS. \end{cases}$$

(RewPPass(ℓ))

Objective Function: Minimize

$$+$$

Cost NonServed Passengers

$$\alpha\mu_1 \sum_{i\in N_\ell}\sum_{k\in K_\ell} x_i^{k\ell} + (1-\alpha)\mu_2 \sum_{i\in N_\ell}\sum_{k\in K_\ell} x_i^{k\ell}, \qquad \text{(NonServed}(\ell)\text{)}$$

Overall Cost:

$$(\text{Cap}(\ell)) - (\text{RewPPass}(\ell)) + (\text{NonServed}(\ell)) \qquad (\text{COST}(\ell))$$

Constraints: Capacities and true/fake trips

- For $\ell \in L$:

$$\sum_{q \in Q} y_q^{1\ell} = 1, \qquad \ell \in LNS,$$

$$\sum_{q \in Q} y_q^{k\ell} \leq 1, \qquad 1 < k < \bar{k}_\ell, \ell \in L,$$

$$\sum_{q \in Q} y_q^{\bar{k}_\ell \ell} = 1, \qquad \ell \in L,$$

- For $\ell \in LS$:

$$y_q^{k\ell} \leq y_{Sq}^{k\ell}, \qquad q \in Q, k \in K_\ell, \ell \in LS,$$

$$\sum_{q \in Q} y_q^{1\ell} + \sum_{q \in Q} y_{Sq}^{1\ell} \geq 1, \qquad \ell \in LS,$$

$$\sum_{q \in Q} y_q^{\kappa_\ell \ell} = \sum_{q \in Q} y_{Sq}^{1\ell} - \sum_{q \in Q} y_q^{1\ell}, \qquad \ell \in LS,$$

$$\sum_{q \in Q} y_{Sq}^{k\ell} \leq 1, \qquad k \in K_\ell, \ell \in LS,$$

Constraints: Time Control

- For $\ell \in L$:

$$t_1^{1\ell} = 0, \qquad t_1^{\bar{k}\ell} = T, \qquad \ell \in L,$$

$$IS\left(\sum_{q\in Q} y_q^{k\ell}\right) \leq t_i^{k\ell} - t_i^{(k-1)\ell} \leq T\left(\sum_{q\in Q} y_q^{k\ell}\right), \quad k > 1, (i,\ell) \in \overline{\mathcal{S}}$$

- For $\ell \in LS$:

$$IS\left(\sum_{q\in Q} y_{Sq}^{k\ell}\right) \leq t_i^{k\ell} - t_i^{(k-1)\ell} \leq (T + t_{1\mapsto 1_{S_l}})\left(\sum_{q\in Q} y_{Sq}^{k\ell}\right), i \in S_\ell, k > 1,$$

$$t_1^{\kappa_\ell \ell} \leq T\left(1 - \sum_{q\in Q} y_{Sq}^{1\ell} + \sum_{q\in Q} y_q^{1\ell}\right),$$

$$-t_{1\mapsto 1_{S_l}}\left(1 - \sum_{q\in Q_\ell} y_q^{k\ell}\right) \leq w^{k\ell} \leq (T + t_{1\mapsto 1_{S_l}})\left(1 - \sum_{q\in Q_\ell} y_q^{k\ell}\right), k \in K_\ell,$$

Constraints: Flow Control

- Flow determined by the capacity of the train:

$$f_i^{k\ell} + \sum_{r=1}^{i-1} f_r^{k\ell} \left(\sum_{j=i+1}^{n_\ell} p_{rj}^{\ell} \right) \leq \sum_{q \in Q} q y_q^{k\ell}, \quad k \in K_\ell, i \in N_\ell, \ell \in L,$$

$$g_i^{k\ell} + \sum_{r=1_{S_\ell}}^{i-1} g_r^{k\ell} \left(\sum_{j=i+1}^{n_{S_\ell}} p_{rj}^{\ell} \right) \leq \sum_{q \in Q} q(y_{Sq}^{k\ell} - y_q^{k\ell}), \; k \in K_\ell, i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS,$$

- Flow determined by the demand function:

$$f_i^{1\ell} \leq D_i^{\ell}(t_i^{1\ell}), \quad i \in N_\ell, \ell \in L,$$

$$f_i^{kl} \leq D_i^{\ell}(t_i^{k\ell}) - D_i^{\ell}(t_i^{(k-1)\ell}) + \alpha h_i^{(k-1)\ell}, \quad k > 1 i \in N_\ell, \ell \in L,$$

$$g_i^{1\ell} \leq D_i^{\ell}(t_i^{1\ell}), \quad i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS,$$

$$g_i^{k\ell} \leq \left(D_i^{\ell}(t_i^{k\ell}) - D_i^{\ell}(t_i^{(k-1)\ell}) \right) + \alpha h_i^{(k-1)\ell}, \; k > 1, i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS.$$

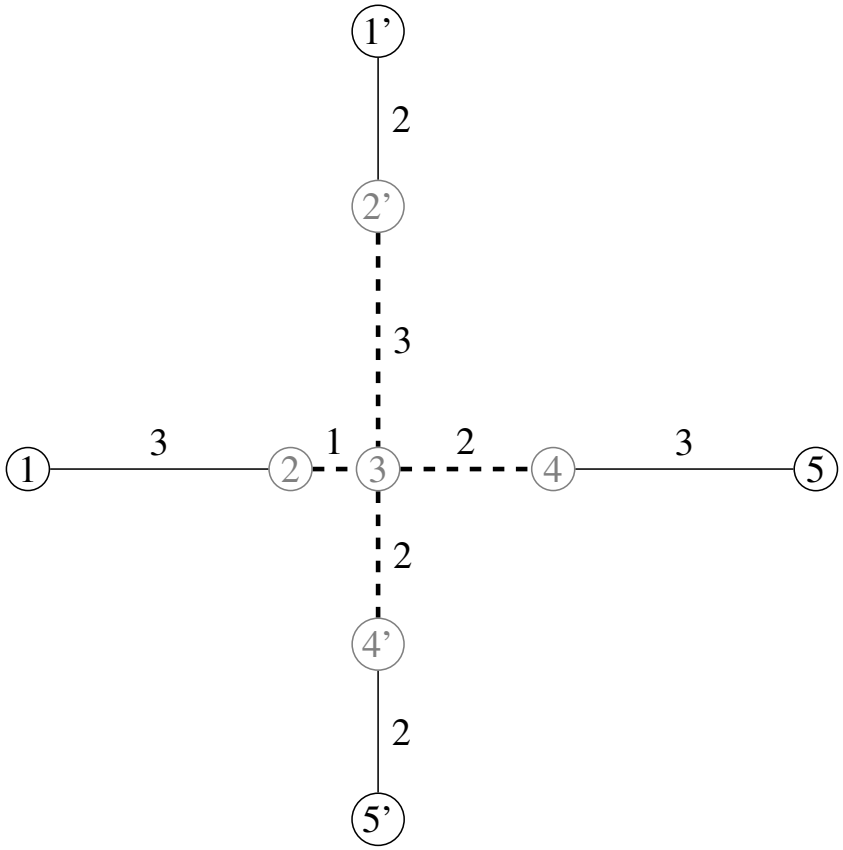
Constraints: Passenger Surplus

$$x_i^{k\ell} \geq h_i^{k\ell} - M_i^{\ell}\left(1 - \sum_{q \in Q_\ell} y_q^{k\ell}\right), (i,l) \in \overline{\mathcal{S}} \text{ or } (i = n_{S_\ell}, \ell \in LS),$$

$$x_i^{k\ell} \geq h_i^{k\ell} - M_i^{\ell}\left(1 - \sum_{q \in Q_\ell} y_{Sq}^{k\ell}\right), i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS,$$

Example (A simplified version of Metrolab network)

Dimensions: Capacities: 800 and 1600, $T = 20$ min, $\bar{k}_\ell = 7$ and 10.



Example

IS^{ℓ}: 2 minutes, $\alpha = 1$, $\mu_1 = 0.1875$, $\tau = 0.4$.

p^{ℓ}_{ij}	1	2	3	4	5
1	0	0.40	0.35	0.20	0
2	0.40	0	0.60	0.35	0
3	0.35	0.6	0	0.95	0
4	0.20	0.35	0.95	0	1
5	0.05	0.05	0.05	1	0

p^{ℓ}_{ij}	1'	2'	3	4'	5'
1'	0	0.40	0.35	0.20	0
2'	0.40	0	0.60	0.35	0
3	0.35	0.60	0	0.95	0
4'	0.20	0.35	0.95	0	1
5'	0.05	0.05	0.05	1	0

Table: O-D matrix of Example: Lines 1-2 (left) and 3-4 (right).

γ^{ℓ}_{ij}	1	2	3	4	5
1	0	0.3	0.4	0.6	1
2	0.3	0	0.1	0.3	1
3	0.5	0.2	0	0.2	1
4	0.6	0.3	0.1	0	0
5	0.9	0.6	0.4	0.3	0

γ^{ℓ}_{ij}	1'	2'	3	4'	5'
1'	0	0.2	0.5	0.7	1
2'	0.2	0	0.3	0.5	1
3	0.4	0.2	0	0.2	0
4'	0.7	0.5	0.3	0	0
5'	0.9	0.7	0.5	0.2	0

Table: Rewards of Example: Lines 1-2 (left) and 3-4 (right).

Example

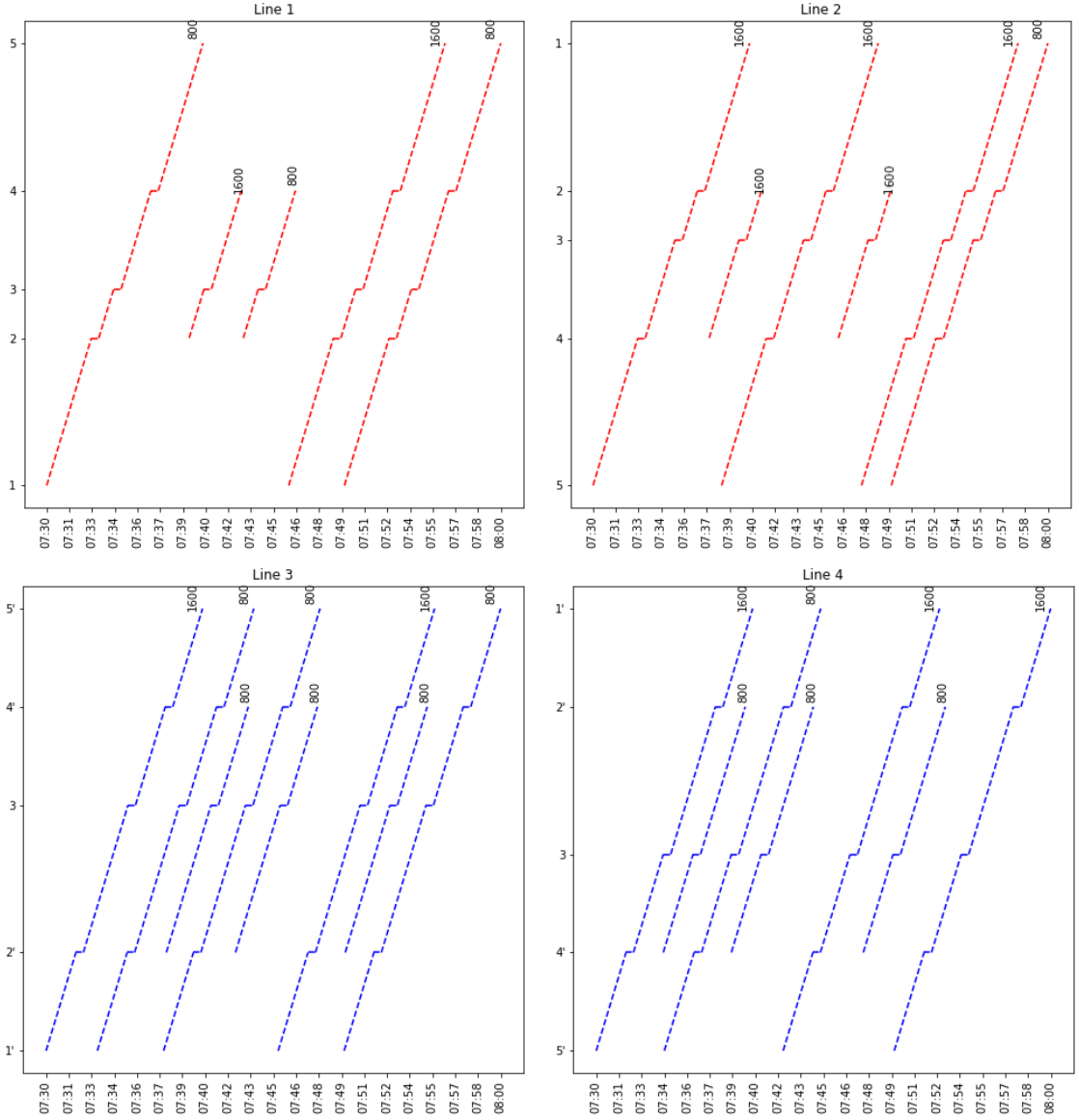
	Lines									
	$\ell = 1$					$\ell = 2$				
Stations (i)	1	2	3	4	5	5	4	3	2	1
β_{0i}^{ℓ}	50	50	50	50	0	50	50	50	50	0
β_{i}^{ℓ}	10	100	120	90	0	10	160	180	150	0
	$\ell = 3$					$\ell = 4$				
Stations (i)	1'	2'	3	4'	5'	5'	4'	3	2'	1'
β_{0i}^{ℓ}	50	50	50	50	50	50	50	50	50	50
β_{i}^{ℓ}	10	150	170	160	0	10	100	180	150	0

Table: Coefficients of the Demand functions of Example.

Model Coded in `Python 3.6` + `Gurobi 8.0` in a Mac OSX with an Intel Core i7 processor at 3300 MHz and 16GB of RAM.

CPU: 12 hours MIP GAP: 1.51%.

Example

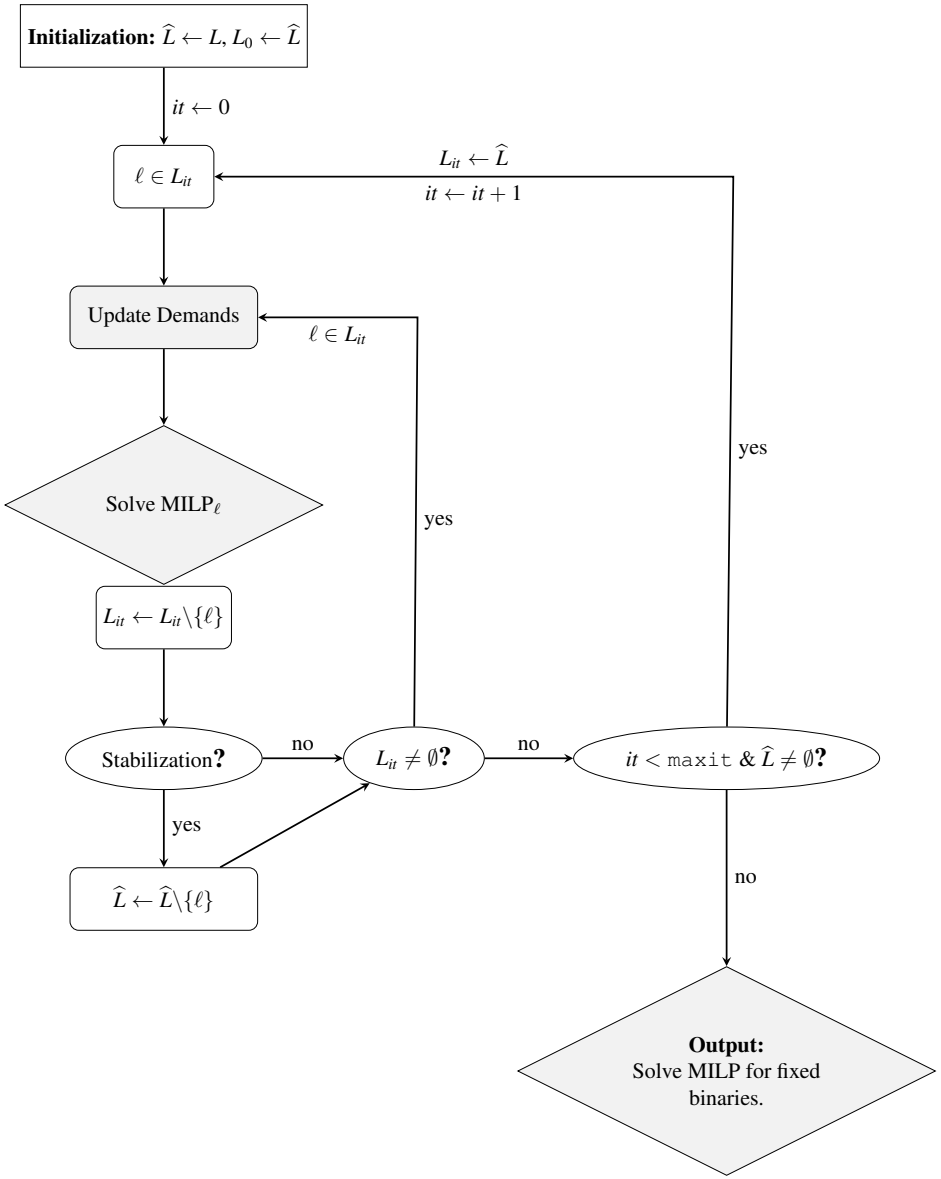

Line 1
Line 2
Line 3
Line 4
800
1600
07:30
07:31
07:33
07:34
07:36
07:37
07:39
07:40
07:42
07:43
07:45
07:46
07:48
07:49
07:51
07:52
07:54
07:55
07:57
07:58
08:00

Timetable (Line 1)

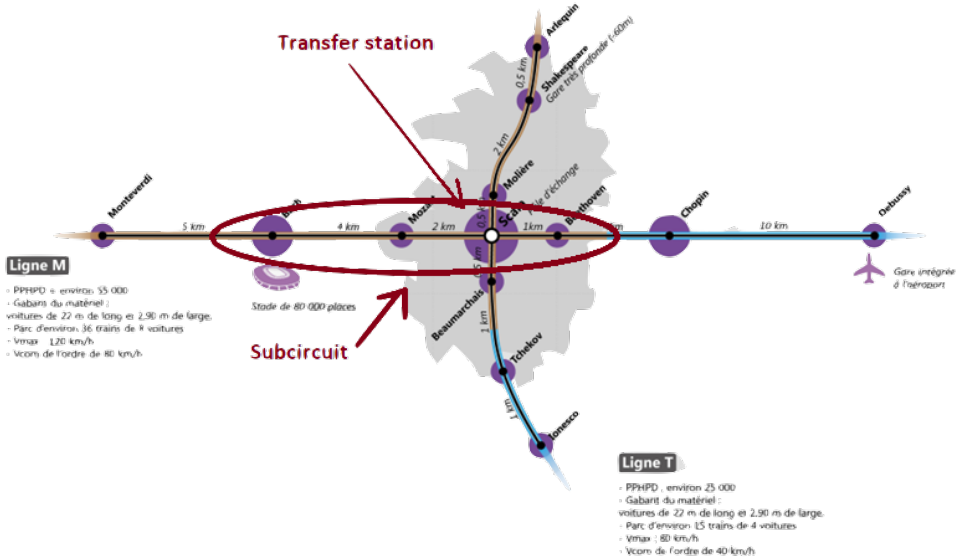
k: Capacity	i	DepTime	Get-Off	$f_i^{k\ell}$ ($g_i^{k\ell}$)	$h_i^{k\ell}$	$x_i^{k\ell}$	Load
1: 800	1	07:30:00	0.00	50.00	0.00	0.00	50.00
	2	07:33:30	20.00	400.00	0.00	0.00	430.00
	3	07:35:00	257.50	627.50	101.50	101.50	800.00
	4	07:37:30	746.13	725.00	0.00	0.00	778.88
	5	07:40:30	778.88	0.00	0.00	0.00	0.00
2S: 1600	2	07:39:34	0.00	606.94	0.00	0.00	606.94
	3	07:41:04	364.17	1231.59	0.00	0.00	1474.36
	4	07:43:34	1474.36	0.00	638.18	0.00	0.00
3: **0**	1	07:30:00	0.00	0.00	0.00	0.00	0.00
	2	07:39:34	0.00	0.00	0.00	0.00	0.00
	3	07:41:04	0.00	0.00	0.00	0.00	0.00
	4	07:43:34	0.00	0.00	638.18	0.00	0.00
	5	07:40:30	0.00	0.00	0.00	0.00	0.00
4S: 800	2	07:43:12	0.00	364.02	0.00	0.00	364.02
	3	07:44:42	218.41	603.05	0.00	0.00	748.66
	4	07:47:12	748.66	0.00	1014.15	0.00	0.00
5: **0**	1	07:30:00	0.00	0.00	0.00	0.00	0.00
	2	07:43:12	0.00	0.00	0.00	0.00	0.00
	3	07:44:42	0.00	0.00	0.00	0.00	0.00
	4	07:47:12	0.00	0.00	1014.15	0.00	0.00
	5	07:40:30	0.00	0.00	0.00	0.00	0.00
6: 1600	1	07:46:15	0.00	162.60	0.00	0.00	162.60
	2	07:49:45	65.04	655.05	0.00	0.00	752.61
	3	07:51:15	449.94	1297.33	0.00	0.00	1600.00
	4	07:53:45	1494.25	1494.25	109.44	109.44	1600.00
	5	07:56:45	1600.00	0.00	0.00	0.00	0.00
7: 800	1	07:50:00	0.00	37.40	0.00	0.00	37.40
	2	07:53:30	14.96	373.99	0.00	0.00	396.43
	3	07:55:00	237.48	641.05	0.00	0.00	800.00
	4	07:57:30	747.38	446.03	0.00	0.00	498.65
	5	08:00:30	498.65	0.00	0.00	0.00	0.00

Our second contribution: Math-Heuristic Algorithm

Divide and conquer



Case Study



q	b_q^ℓ & bS_q^ℓ
400	48.8× length(ℓ)
800	97.4× length(ℓ)
1600	194.8× length(ℓ)

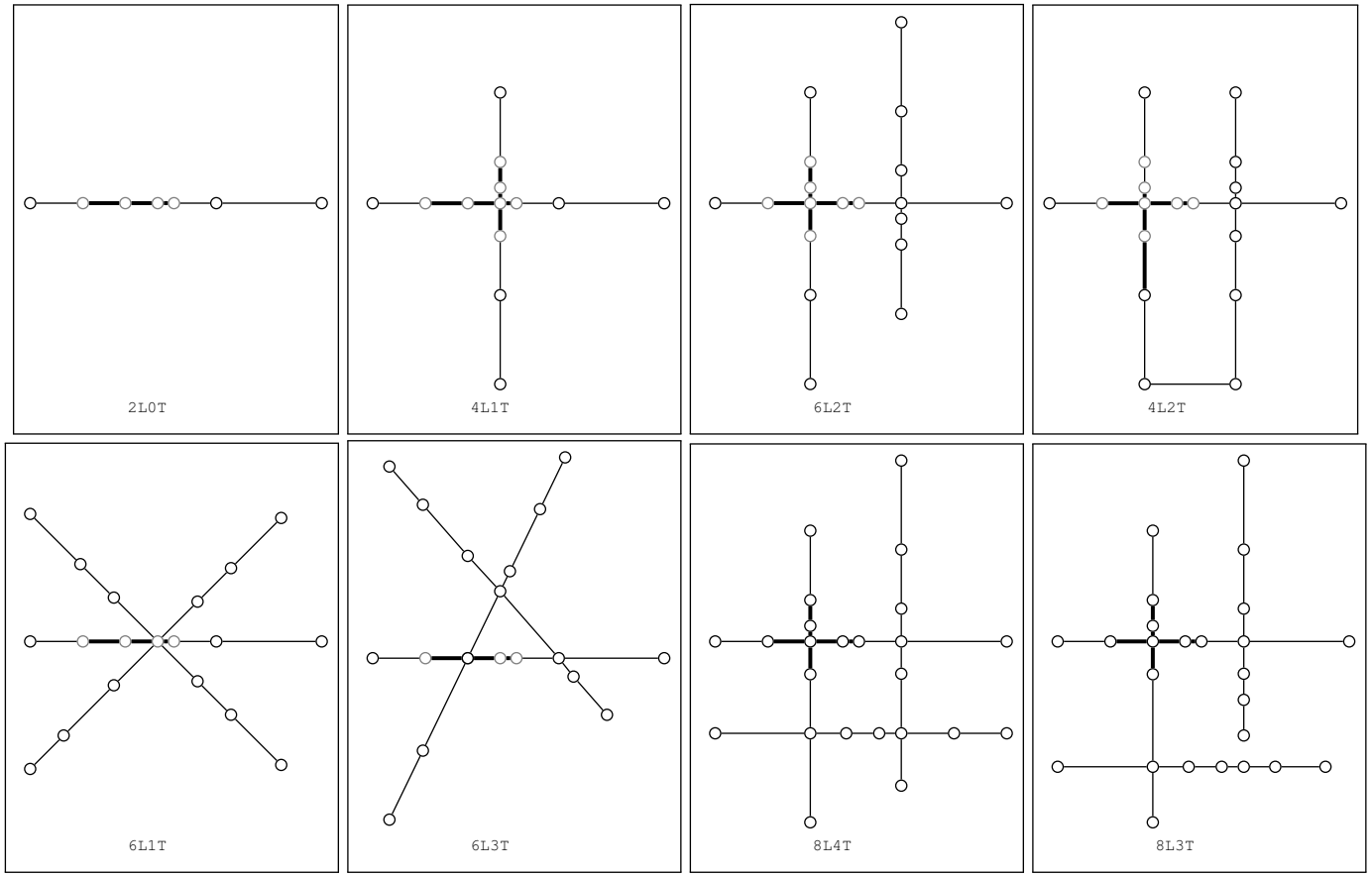
Line	μ	IS^ℓ
M	0.1875	1.66
T	0.1875	1.25

$$\star\ \gamma_{ij}^\ell = 0.075 \times d_{ij}^\ell + 0.075$$

Time window: 7:30 - 9:30 a.m.

Line M					Line T				
	d_i^ℓ	e_i^ℓ	$\overleftarrow{\beta_i^\ell}$	$\overrightarrow{\beta_i^\ell}$		d_i^ℓ	e_i^ℓ	$\overleftarrow{\beta_i^\ell}$	$\overrightarrow{\beta_i^\ell}$
Debussy - Chopin	6.32	0	3.3	138.7	Arlequin-Shak.	0.88	0	3.3	100
Chopin-Beeth.	2.53	0.5	36.7	130	Shak.-Molière	2.22	0.3	5	93.3
Beeth.-Scala	0.97	0.3	108.3	21.7	Molière-Scala	0.88	0.3	38.3	68.3
Scala-Mozart	1.94	0.6	125	99.6	Scala-Beaum.	0.88	0.5	25	140
Mozart Bach	2.53	0.3	120	13	Beaum.-Tchekov	1.11	0.3	50	41.6
Bach-Mont.	3.16	0.5	40	8.7	Tchekov-Ionescu	1.11	0.3	53.3	3.33
Max Rounds			40	30	Max Rounds			20	40

More Experiments



More Experiments

		Matheuristic		MILP		
Network	**|LS|**	**BestObj**	**CPU**(sec.)	**BestObj**	**CPU** (sec.)	**GAP** (%)
2L0T	0	145702	< 0.1	145573	7	0.09
	2	114145	11	112916	TL	1.08
4L1T	0	206729	132	206242	TL	0.24
	4	152890	631	152016	TL	0.57
4L2T	0	348267	3522	347224	TL	0.30
	4	333102	4892	332665	TL	0.13
6L1T	0	276961	1130	276545	TL	0.15
	2	235488	1521	234589	TL	0.38
6L2T	0	249038	606	248854	TL	0.07
	4	203080	2882	203080	TL	0.00
6L3T	0	248988	520	248979	TL	< 0.01
	2	217004	2688	216909	TL	0.04
8L3T	0	404627	432	404147	TL	0.12
	4	362916	1467	362908	TL	< 0.01
8L4T	0	404469	1191	404211	TL	0.06
	4	374067	1144	374064	TL	< 0.01

TL= 10 hours.

Extensions

1. Introduce the average speed between consecutive stations and the stopping time that a train spends in a station as variables.
2. Flexibilize the use of short-turns.
3. Integration of the scheduling phase.
4. Consider stochastic demands.
5. ...

Muchas gracias.