New advances in Covering Location and related problems

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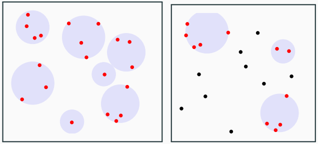
Universidad de Granada

- **Covering Location Problem:** Locate a set of facilities to give service to a finite set of users provided that the facilities are allowed to satisfy the demand of the users within certain coverage distance.
    - Full Covering Problem: all customers are served at minimum cost, or
    - Maximal Covering Problem: the (weighted) number of covered customers is maximized.



Covering Problems

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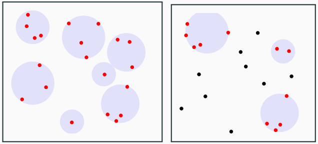
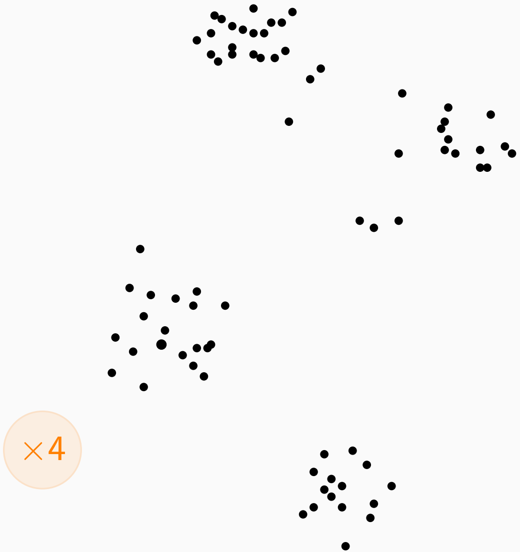


- Discrete (potential positions of the services are provided) or Continuous (the services are allowed to be located in the whole space -mostly on the plane-)
- Useful in cases in which facilities can serve only customers within a certain *coverage area*: Emergency vehicles, Wifi routers, Mobile phones antennas.

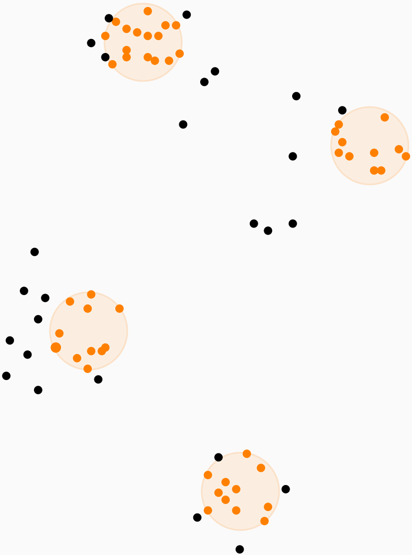
Covering Problems - Literature

- First mentions to covering problems in Berge (1957).
- First application: Police patroling in Hakimi (1965).
- First formulations:
  - Non-location context: Roth (1969).
  - Location set covering problem: Toregas et al. (1971).
  - Maximal covering problem (discrete): Church and Revelle (1974).
  - Maximal covering problem on the plane: Church (1984) (Euclidean), Younies and Wessolowsky (2004) (inclined parallelograms and block norms).

Continuous Maximal Covering Location



Continuous Maximal Covering Location



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2. Fair maximal covering location problems. In progress, joint work with Blanco.
3. Discrete-Continuous maximal covering location under uncertainty. In progress, joint work with Blanco and Saldanha-da-Gama.
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Interconnected Facilities

In many situations when locating facilities, it is desirable that they are interconnected: linked if the distance between them does not exceed a given limit.

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- Optimal design of forest fire-fighters centers that have to communicate a central server at a give radius (Demaine et al., 2009).
- Location of sensors that have to be connected to each others (Romich et al., 2015).

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Cherlesky, Landete & Laporte (2019): p-median and the p-maximal discrete covering location problems with tree-shaped interconnected facilities.

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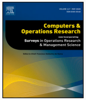

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Continuous maximal covering location problems with interconnected facilities

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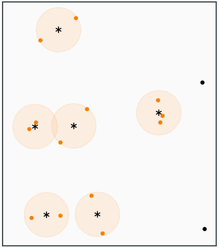
ABSTRACT

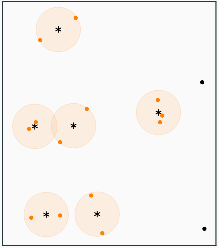
In this paper we analyze a continuous version of the maximal covering location problem, in which the facilities are required to be linked by means of a given graph structure (provided that two facilities are allowed to be linked if a given distance is not exceed). We propose a mathematical programming framework for the problem and different resolution strategies. First, we provide a Mixed Integer Non Linear Programming formulation for the problem and derive some geometrical properties that allow us to reformulate it as an equivalent pure integer linear programming problem. We propose two branch-&-cut approaches by relaxing some sets of constraints of the former formulation. We also develop a math-heuristic algorithm for the problem capable to solve instances of larger sizes. We report the results of an extensive battery of computational experiments comparing the performance of the different approaches.

MCLPIF



(a) MCLP

MCLPIF



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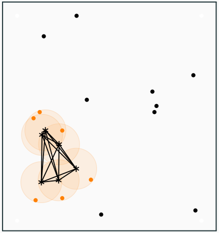
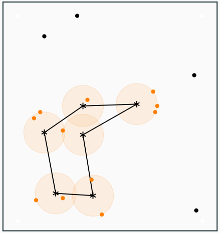
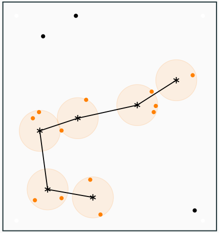


(b) Complete Graph

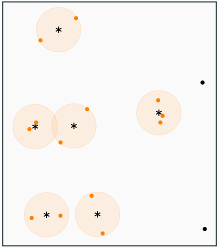


(c) Cycle Graph.



(d) Line Graph.

MCLPIF



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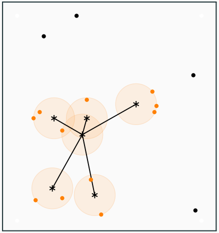
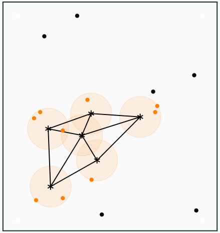
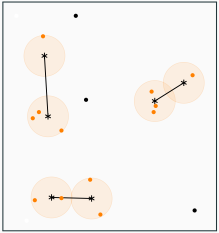


(b) Star Graph.



(c) Ring-Star Graph.



(d) Matching Graph.

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Fair MCLP

Definition.- Fairness

The quality of treating people equally or in a way that is right or reasonable (Cambridge Dictionary).

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Fairness from a facility point of view

Equitable positions for the p services to be located in the way that the difference between the facilities with maximum and minimum covering, would be reasonable.

- In some situations is preferable to slightly loss some covered demand in order to equalize the different covered demands among the open services.
- For example, in locating schools where we want to equalize attendance at each of them.
- Another example would be in the location of servers with high capacities where we want the demand to be distributed homogeneously.

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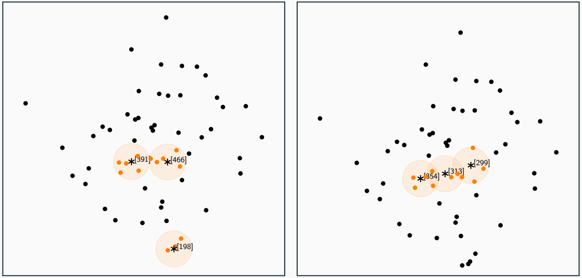

(a) `MCLP`. (b) *Fairer* `MCLP`.

- `MCLP` : [198, 391, 466] (33.46%)
- *Fairer* `MCLP`: [299, 313, 354] (30.64%)

Fair MCLP

Tools.- Ordered Weighted Averaging (OWA)

OWA operators were introduced by Yager (1988) as a powerful tool to deal with the problem of aggregating multicriteria to form an overall decision function.

- The use of OWA operators in Facility Location is not new and several authors have studied the generalization of the classical p-median, p-Weber or p-hub location problems to the so-called Ordered Median Problem (Puerto and Fernández, 1994).

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Fairness Measure.- Orness

The fairness of $\lambda = (\lambda_1, \ldots, \lambda_p)$, the associated weighting vector, can be measured by using the *orness* of the OWA operator (Ogryczak et al., 2014). This orness measure is defined as:

$$\mathrm{orness}(\lambda) = \sum_{j=1}^{p} \frac{p-i}{p-1} \lambda_i$$

Fair MCLP

Relationship of Fairness and Orness

- When $\mathrm{orness}(\lambda) \longrightarrow 1$ fairer solutions (Ogryczak et al., 2014).
- Moreover, OWA operators in which λ are monotone non increasing define an or-like OWA operator and therefore, fairer solutions (Ogryczak and Śliwiński, 2007).

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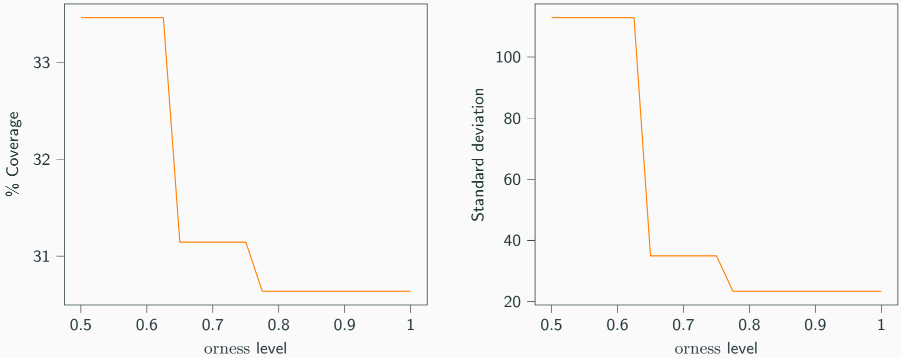
OWA	λ	orness
Average	$\lambda_j = \frac{1}{p}$	$\frac{1}{2}$
Minimum	$\lambda_1 = 1, \lambda_j = 0 \ (j \geq 2)$	1
k-Average	$\lambda_j = \frac{1}{k} \ (j \leq k),\ \lambda_j = 0 \ (j > k)$	$1 - \frac{k-1}{2(p-1)}$
α-Min-Average	$\lambda_1 = \frac{1}{1+(p-1)\alpha},\ \lambda_j = \frac{1-\alpha}{1+(p-1)\alpha} \ (j \geq 2)$	$\frac{-p\alpha+p+2\alpha}{2p\alpha-2\alpha+2}$
Gini	$\lambda_j = \frac{2(n-i)+1}{p^2}$ for all j	$\frac{4p+1}{6p}$
Harmonic	$\lambda_j = \frac{1}{p}\left(H(p) - H(j-1)\right) \ (H(k) = \sum_{\ell=1}^{k} \frac{1}{\ell})$	$\frac{3}{4}$

Table 1: Fair OWA operators.

Fair MCLP



- With λ-weights solving optimization-based methods with given $\text{orness} \in [0, 1]$ degree (Filev and Yager, 1995; Fullér and Majlender, 2001; Liu and Chen, 2004).

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Discrete-Continuous MCLP under uncertainty

- p discrete facilities *here-and-now*.
- r possible continuous facilities in the future
- Objective: Model which gives the exact position of the discrete server taking account the possible number of opened continuous facilities.

This model is useful in telecommunication networks in which p of the servers (sensors, antennas, routers, etc) must be located inside adequately prepared infrastructures (buildings, offices, air-conditioned cabins, roofs, etc) while additional r servers can be located at any place in the given space, but both with the same functions and trying to capture as much demand as possible.

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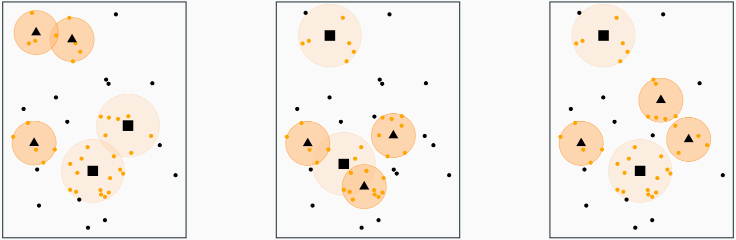
Berman and Drezner (2008): p-median problem under uncertainty consisting of locating p initial facilities plus an uncertain number of extra additional ones (q).

Discrete-Continuous MCLP under uncertainty



(a) $(D - C)$ (b) $(C - D)$ (c) Optimal

Discrete-Continuous MCLP under uncertainty

- Our scenarios are $\omega \in \Omega = \{0, 1, \ldots, r\}$ where r is the number of possible continuous facilities which can be opened in the future.
- The objective is maximize the covered demand taking account the uncertainty.
- Our objective is provide different models to deal with this particular type of uncertainty, as well as suitable mathematical programming formulations and solution methods.

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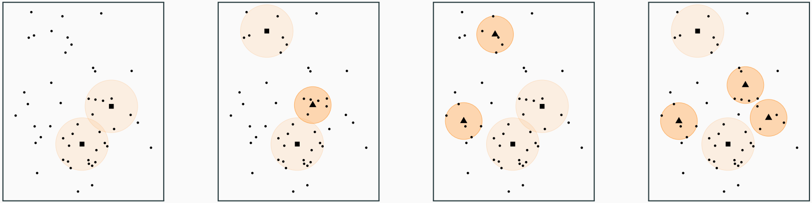

(a) Exact with $r = 0$ (b) Exact with $r = 1$ (c) Exact with $r = 2$ (d) Exact with $r = 3$

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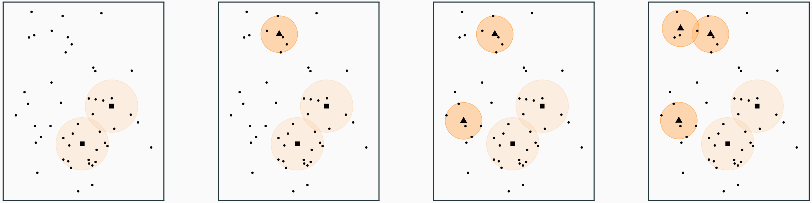

(a) Scenario 0 (b) Scenario 1 (c) Scenario 2 (d) Scenario 3

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Different models	
Robust Optimization	
Robust Worst-Case Model	Min-Max Regret
Stochastic Optimization	
Expected Coverage Model	... and with Regret Thresholds
α-Reliable Min-max Regret	... and with Regret Thresholds
α-CVaR	

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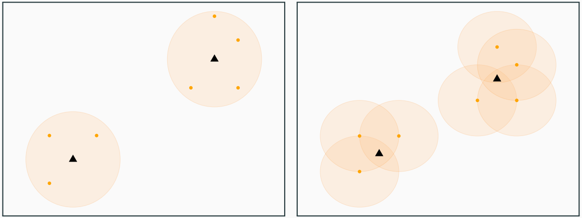

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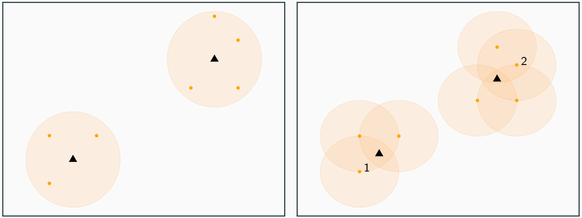
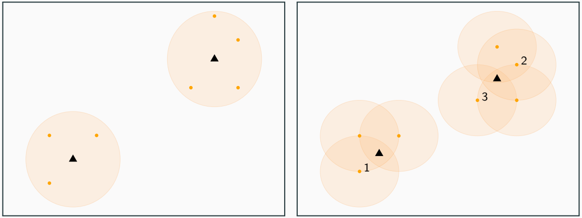


$$z_1 + z_2 \leq 1$$

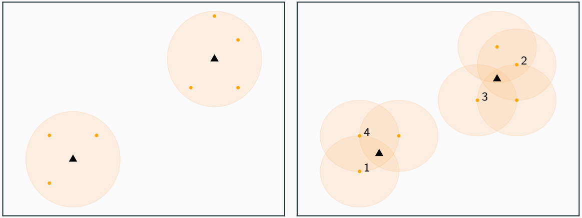
Pure IP for continuous MCLP



$$z_1 + z_2 \leq 1$$

$$z_1 + z_2 + z_3 \leq 2$$

Pure IP for continuous MCLP

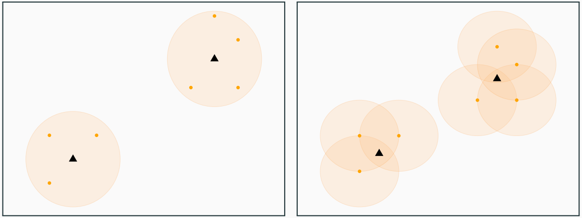


$$z_1 + z_2 \leq 1$$

$$z_1 + z_2 + z_3 \leq 2$$

$$z_1 + z_2 + z_3 + z_4 \leq 3$$

Pure IP for continuous MCLP



$$\begin{aligned} z_1 + z_2 &\leq 1 \\ z_1 + z_2 + z_3 &\leq 2 \\ z_1 + z_2 + z_3 + z_4 &\leq 3 \\ &\vdots \end{aligned} \qquad \Longrightarrow \qquad \sum_{i \in S} z_i \leq |S| - 1, \forall S \subseteq N : \bigcap_{i \in S} B_R(i) = \emptyset$$

Thank you!

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