New advances in location problems and portfolio selection models

Martine Labbé[1] Lina Mallozzi[2] Justo Puerto[3]
Moisés Rodríguez-Madrena[3] Andrea Scozzari[4]

[1] Université Libre de Bruxelles, Brussels, Belgium
[2] University of Naples Federico II, Naples, Italy
[3] IMUS, Universidad de Sevilla, Spain
[4] Università degli Studi Niccolò Cusano, Roma, Italy

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0. A combinatorial optimization approach to scenario filtering in portfolio selection (Presented in the last meeting in Sevilla)

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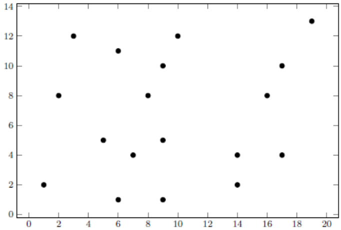
1. Continuous location among several regions with different norms
2. On Location-Allocation Problems for Dimensional Facilities
3. Clustering and portfolio selection problems: A unified framework
4. The Obstacle-Avoiding Rectilinear-Link Steiner Minimum Tree Problem

Single-facility Weber problem

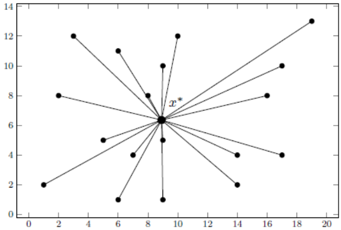
$$\min \quad \sum_{i=1}^{m} w_i \|x - a_i\|_2$$
$$\text{s.t.} \quad x \in \mathbb{R}^n$$



Single-facility Weber problem

$$\min \quad \sum_{i=1}^{m} w_i \|x - a_i\|_2$$

$$\text{s.t.} \quad x \in \mathbb{R}^n$$



Single-facility Weber problem

$$\min \quad \sum_{i=1}^{m} w_i d(x, a_i)$$
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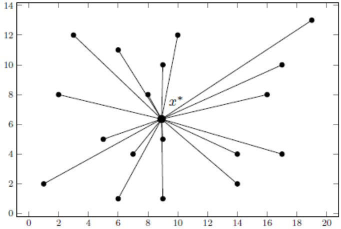

Blanco, Puerto and Ponce (2017)

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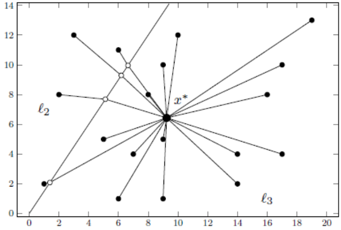
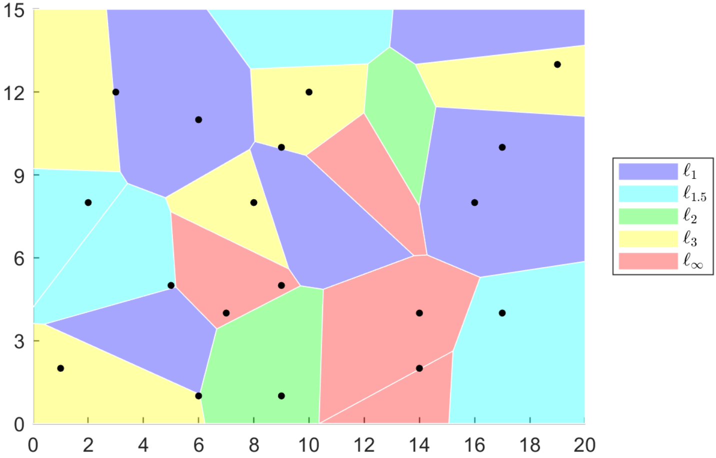


Single-facility Weber problem with different norms at different regions



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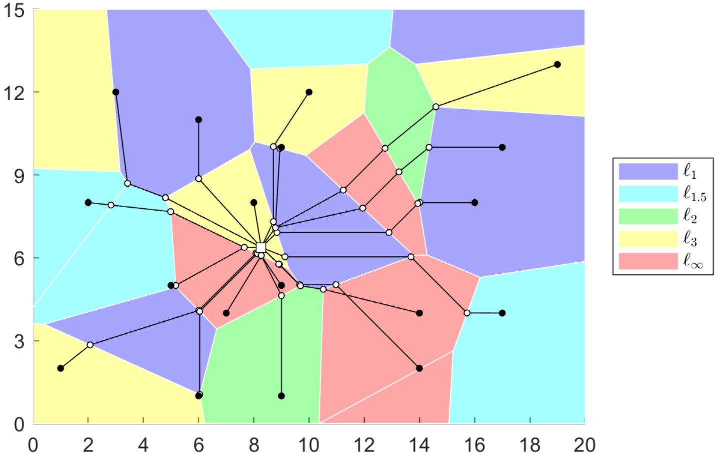

Continuous location between two regions with different norms

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- Blanco, V., Puerto, J., Ponce, D.: Continuous location under the effect of 'refraction'. Math. Program. 161(1-2), 33-72 (2017)
- Franco, L., Velasco, F., Gonzalez-Abril, L., Mesa, J. A.: Single-facility location problems in two regions with ℓ_1-and ℓ_q-norms separated by a straight line. Eur. J. Oper. Res. 269(2), 577-589 (2018)

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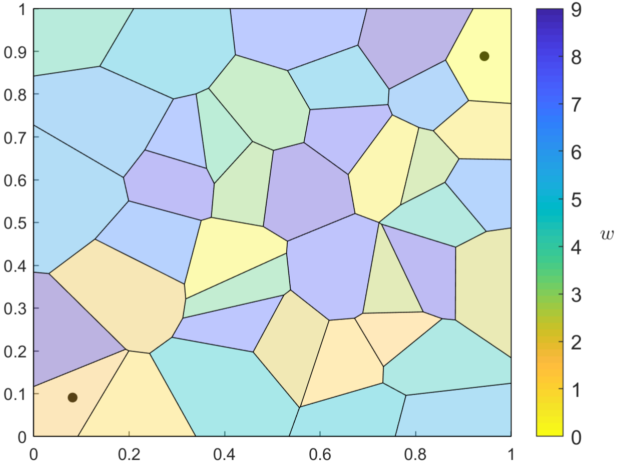
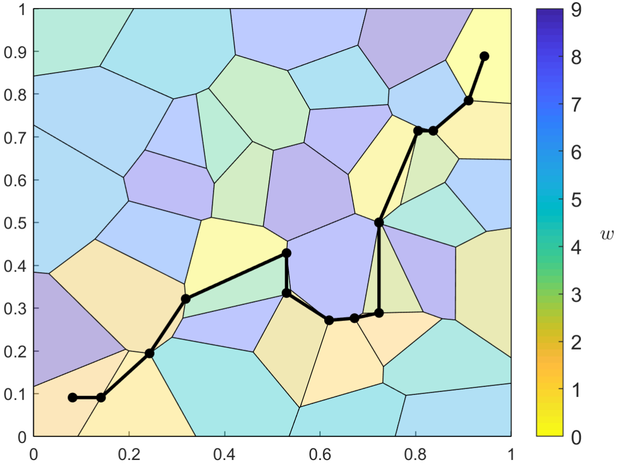
Continuous location among several regions with different norms

(joint-work with Martine Labbé and Justo Puerto)

Weighted Region Problem (WRP)



Weighted Region Problem (WRP)



Some references on the WRP and related problems

- J.S.B. Mitchell, C.H. Papadimitriou: The weighted region problem: finding shortest paths through a weighted planar subdivision. J. Assoc. Comput. Mach. 38, 18–73 (1991)
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The WRP is unsolvable in any algebraic computation model over the rational numbers

De Carufel et al. (2014)

In general, the exact solution of WRP cannot be computed in $\mathbb{Q}$ using a finite number of the operations $+$, $-$, $\times$, $\div$, $\sqrt[k]{\ }$, for any $k \geq 2$.

Our approach

$$\min \sum_{i \in V} \sum_{(h,i) \in A} \sum_{(i,j) \in A} \omega_i \|y_{ij} - y_{hi}\|_{p_i} z_{hi} z_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} z_{ij} - \sum_{(h,i) \in A} z_{hi} = b_i, \qquad \forall i \in V, \tag{2}$$

$$\sum_{(h,i) \in A} z_{hi} \leq 1, \qquad \forall i \in V, \tag{3}$$

$$\sum_{(i,j) \in A} z_{ij} \leq 1, \qquad \forall i \in V, \tag{4}$$

$$z_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A, \tag{5}$$

$$y_{ij} \in F_{ij}, \qquad \forall (i,j) \in A, \tag{6}$$

where $G = (V = \{\text{Polyhedra}\}, A = \{\text{Interfaces}\})$.

MISOCP Formulation 1

$$\min \sum_{i \in V} \omega_i d_i \tag{7a}$$

$$\text{s.t.} \quad \sum_{(s,j)\in A} z_{sj} - \sum_{(h,s)\in A} z_{hs} = 1, \tag{7b}$$

$$\sum_{(i,j)\in A} z_{ij} - \sum_{(h,i)\in A} z_{hi} = 0, \qquad \forall i \in V \setminus \{s,t\}, \tag{7c}$$

$$\sum_{(t,j)\in A} z_{tj} - \sum_{(h,t)\in A} z_{ht} = -1, \tag{7d}$$

$$\sum_{(h,i)\in A} z_{hi} \le 1, \qquad \forall i \in V, \tag{7e}$$

$$\sum_{(i,j)\in A} z_{ij} \le 1, \qquad \forall i \in V, \tag{7f}$$

$$d_s \ge \| \sum_{(s,j)\in A} \sum_{e \in \textbf{Ext}(F_{sj})} \lambda_{sje} e - x_s \|_{p_s}, \tag{7g}$$

$$d_i \ge \| \sum_{(i,j)\in A} \sum_{e \in \textbf{Ext}(F_{ij})} \lambda_{ije} e - \sum_{(h,i)\in A} \sum_{e \in \textbf{Ext}(F_{hi})} \lambda_{hie} e \|_{p_i}, \qquad \forall i \in V \setminus \{s,t\}, \tag{7h}$$

$$d_t \ge \| x_t - \sum_{(h,t)\in A} \sum_{e \in \textbf{Ext}(F_{ht})} \lambda_{hte} e \|_{p_t}, \tag{7i}$$

$$\sum_{e \in \textbf{Ext}(F_{ij})} \lambda_{ije} = z_{ij}, \qquad \forall (i,j) \in A, \tag{7j}$$

$$d_i \ge 0, \qquad \forall i \in V, \qquad z_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A, \tag{7k}$$

$$\lambda_{ije} \ge 0, \qquad \forall (i,j) \in A, e \in \textbf{Ext}(F_{ij}). \tag{7l}$$

MISOCP Formulation 2

$$\min \sum_{i\in V} \omega_i d_i \tag{8a}$$

$$\text{s.t.} \quad \sum_{(s,j)\in A} z_{sj} - \sum_{(h,s)\in A} z_{hs} = 1, \tag{8b}$$

$$\sum_{(i,j)\in A} z_{ij} - \sum_{(h,i)\in A} z_{hi} = 0, \qquad \forall i \in V \setminus \{s,t\}, \tag{8c}$$

$$\sum_{(t,j)\in A} z_{tj} - \sum_{(h,t)\in A} z_{ht} = -1, \tag{8d}$$

$$\sum_{(h,i)\in A} z_{hi} \le 1, \qquad \forall i \in V, \tag{8e}$$

$$\sum_{(i,j)\in A} z_{ij} \le 1, \qquad \forall i \in V, \tag{8f}$$

$$d_s \ge \sum_{(s,j)\in A} \Big\| \sum_{e\in \textbf{Ext}(F_{sj})} \lambda_{sje} e - x_s z_{sj} \Big\|_{p_s}, \tag{8g}$$

$$d_i \ge \sum_{(h,i)\in A} \sum_{(i,j)\in A} \Big\| \sum_{e\in \textbf{Ext}(F_{ij})} \Psi_{hije} e - \sum_{e\in \textbf{Ext}(F_{hi})} \Phi_{hije} e \Big\|_{p_i}, \qquad \forall i \in V \setminus \{s,t\}, \tag{8h}$$

$$d_t \ge \sum_{(h,t)\in A} \Big\| x_t z_{ht} - \sum_{e\in \textbf{Ext}(F_{ht})} \lambda_{hte} e \Big\|_{p_t}, \tag{8i}$$

$$\sum_{e\in \textbf{Ext}(F_{ij})} \lambda_{ije} = z_{ij}, \qquad \forall (i,j) \in A, \tag{8j}$$

MISOCP Formulation 2

$$\sum_{(i,j)\in A} \rho_{hij} = z_{hi}, \qquad \forall i \in V \setminus \{s,t\}, (h,i) \in A, \tag{9a}$$

$$\sum_{(h,i)\in A} \rho_{hij} = z_{ij}, \qquad \forall i \in V \setminus \{s,t\}, (i,j) \in A, \tag{9b}$$

$$\sum_{e\in \textsf{Ext}(F_{hi})} \Phi_{hije} = \rho_{hij}, \qquad \forall i \in V \setminus \{s,t\}, (h,i), (i,j) \in A, \tag{9c}$$

$$\sum_{(i,j)\in A} \Phi_{hije} = \lambda_{hie}, \qquad \forall i \in V \setminus \{s,t\}, (h,i) \in A, e \in \textsf{Ext}(F_{hi}), \tag{9d}$$

$$\sum_{e\in \textsf{Ext}(F_{ij})} \Psi_{hije} = \rho_{hij}, \qquad \forall i \in V \setminus \{s,t\}, (h,i), (i,j) \in A, \tag{9e}$$

$$\sum_{(h,i)\in A} \Psi_{hije} = \lambda_{ije}, \qquad \forall i \in V \setminus \{s,t\}, (i,j) \in A, e \in \textsf{Ext}(F_{ij}), \tag{9f}$$

$$d_i \geq 0, \qquad \forall i \in V, \tag{9g}$$

$$z_{ij} \in \{0,1\}, \qquad \forall (i,j) \in A, \tag{9h}$$

$$\lambda_{ije} \geq 0, \qquad \forall (i,j) \in A, e \in \textsf{Ext}(F_{ij}), \tag{9i}$$

$$\rho_{hij} \geq 0, \qquad \forall i \in V \setminus \{s,t\}, (h,i), (i,j) \in A, \tag{9j}$$

$$\Phi_{hije} \geq 0, \qquad \forall i \in V \setminus \{s,t\}, (h,i), (i,j) \in A, e \in \textsf{Ext}(F_{hi}), \tag{9k}$$

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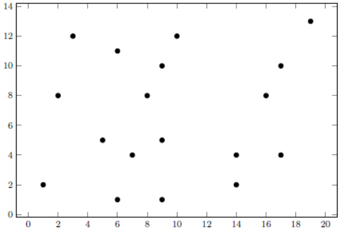
Comparison between Formulation 1 and Formulation 2

Preposition

Let $\zeta_1, \zeta_2 \geq 0$ be the objective values of the continuous relaxations of Formulation 1 and Formulation 2, respectively. Then, $\zeta_1 \leq \zeta_2$.

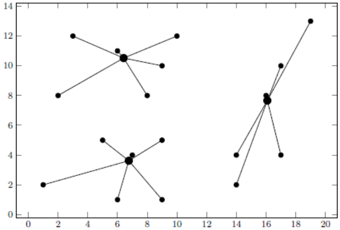
Multisource Weber problem

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} \sum_{j=1}^{p} w_{ij} \|x_j - a_i\|_2 \\ \text{s.t.} \quad & \sum_{j=1}^{p} w_{ij} = w_i, \quad i = 1, \ldots, m \\ & x_j \in \mathbb{R}^n, \quad w_{ij} \geq 0, \quad \forall i, j \end{aligned}$$



Multisource Weber problem

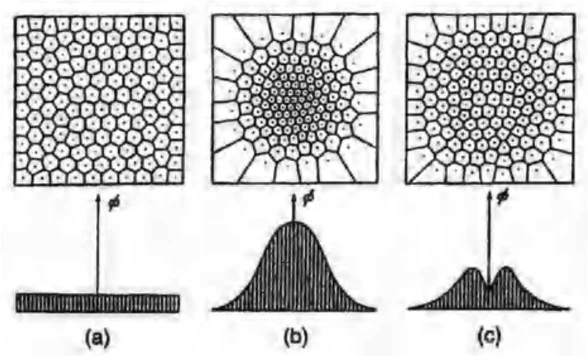
$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m} \sum_{j=1}^{p} w_{ij} \|x_j - a_i\|_2 \\
\text{s.t.} \quad & \sum_{j=1}^{p} w_{ij} = w_i, \quad i = 1, \dots, m \\
& x_j \in \mathbb{R}^n, \quad w_{ij} \geq 0, \quad \forall i, j
\end{aligned}
$$



Iri, Murota and Ohya (1984)

$$\min \quad \sum_{j=1}^{p} \int_{V_j} \|x - x_j\|^2 \phi(x) dx$$

$$\text{s.t.} \quad x_j \in \mathbb{R}^2, \quad j = 1, \ldots, p$$

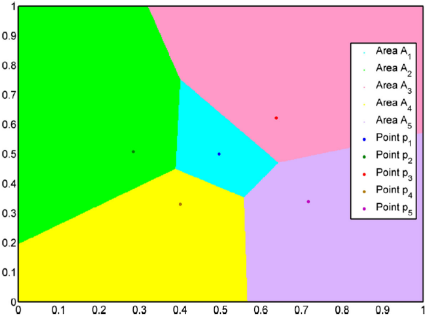


Mallozzi and Passarelli di Napoli (2017)

$$\min \quad \sum_{j=1}^{p} C\left(\int_{V_j} \phi(x)dx\right)$$

$$\text{s.t.} \quad (V_1,\dots,V_p) \in \arg\min \sum_{j=1}^{p} \int_{V_j} \|x - x_j\|^2 \phi(x)dx$$

$$x_j \in \mathbb{R}^2, \quad j = 1,\dots,p$$

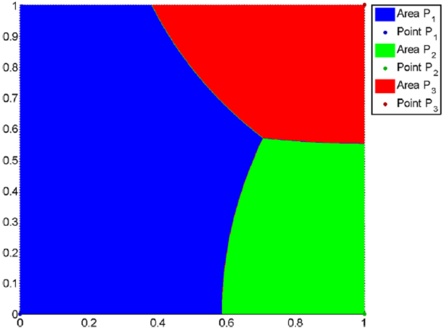


Mallozzi and Puerto (2017)

$$\min \quad \sum_{j=1}^{p} C\left(\int_{V_j} \phi(x)dx\right)$$

$$\text{s.t.} \quad (V_1,\ldots,V_p) \in \arg\min \sum_{j=1}^{p} \int_{V_j} F(x-x_j)^{r_j}\phi(x)dx$$

$$x_j \in \mathbb{R}^2, \quad j=1,\ldots,p$$



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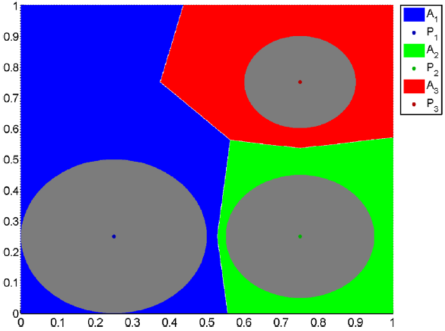
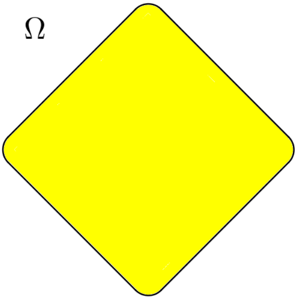


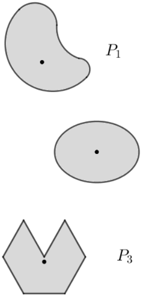
Mallozzi, Puerto and Rodríguez-Madrena (2019)

$$\min \quad \sum_{j=1}^{p} C\left(\int_{V_j} \phi(x)dx\right) + \dots$$

$$\text{s.t.} \quad (V_1, \dots, V_p) \in \arg\min \sum_{j=1}^{p} \int_{V_j} f(x, x_j + P_j)\phi(x)dx$$

$$x_1 + P_1, \dots, x_p + P_p \subseteq \Omega$$





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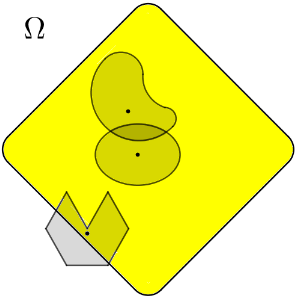

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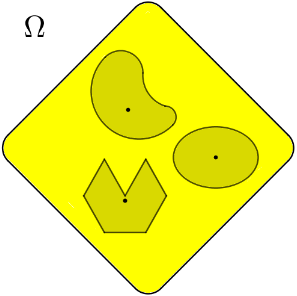

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On Location-Allocation Problems for Dimensional Facilities

Lina Mallozzi[1] · Justo Puerto[2] · Moisés Rodríguez-Madrena[2]

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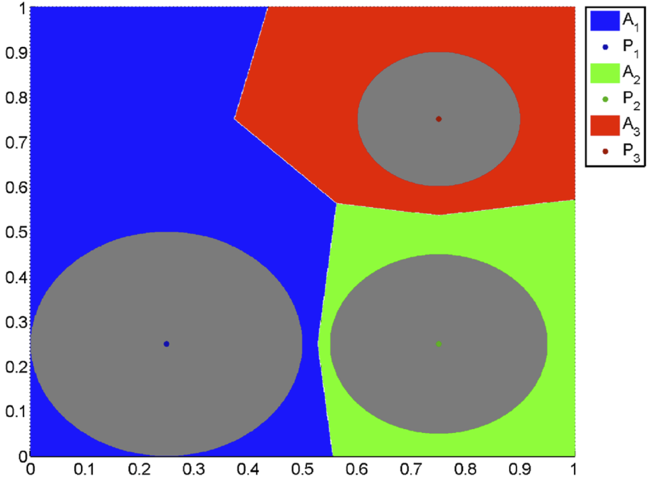
Abstract
This paper deals with a bilevel approach of the location-allocation problem with dimensional facilities. We present a general model that allows us to consider very general shapes of domains for the dimensional facilities, and we prove the existence of optimal solutions under mild assumptions. To achieve these results, we borrow tools from optimal transport mass theory that allow us to give explicit solution structure of the considered lower level problem. We also provide a discretization approach that can approximate, up to any degree of accuracy, the optimal solution of the original problem. This discrete approximation can be optimally solved via a mixed-integer linear program. To address very large instance sizes, we also provide a GRASP heuristic that performs rather well according to our experimental results. The paper also reports some experiments run on test data.

Keywords Bilevel optimization · Dimensional facilities · Optimal transport mass · Mixed-integer programming · Heuristics

Mathematics Subject Classification 90B85 · 49M25 · 90B80 · 90C30

Discrete approximation scheme



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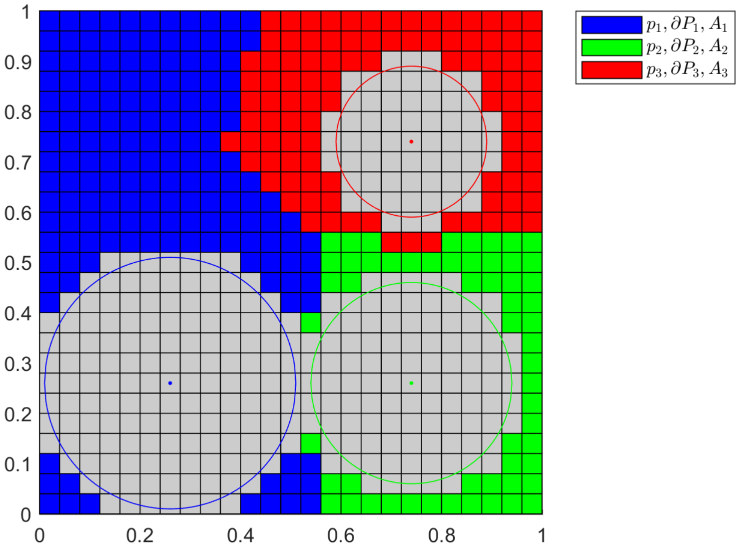


MILP formulation

$$\min \quad \sum_{i=1}^{\rho} \overline{I_i^{\mathbf{PL}}} \left(\sum_{(r,s)\in\mathbf{\Omega}} \sum_{(k,l)\in\boldsymbol{E}_{rs}^i} w_{rs}^B \theta_{kl}^i \right) + \sum_{i=1}^{\rho} \overline{C_i^{\mathbf{PL}}} \left(\sum_{(r,s)\in\mathbf{\Omega}} w_{rs}^D \tau_{rs}^i \right) + \overline{L^{\mathbf{PL}}} \left(\sum_{(r,s)\in\mathbf{\Omega}} w_{rs}^D \left[1 - \sum_{i=1}^{\rho} \tau_{rs}^i \right] \right) \tag{10}$$

s.t. $SCDVPL$ (see Fourer (1985)), (11)

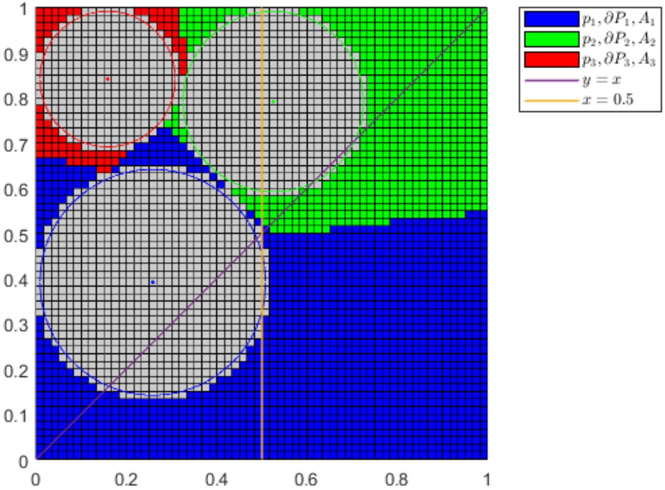
$$\sum_{(k,l)\in\mathbf{\Omega}_i} \theta_{kl}^i = 1, \quad \forall i \in \{1, \dots, \rho\}, \tag{12}$$

$$\sum_{i=1}^{\rho} \tau_{rs}^i + \sum_{i=1}^{\rho} \sum_{(k,l)\in\boldsymbol{E}_{rs}^i} \theta_{kl}^i = 1, \quad \forall (r,s) \in \mathbf{\Omega}, \tag{13}$$

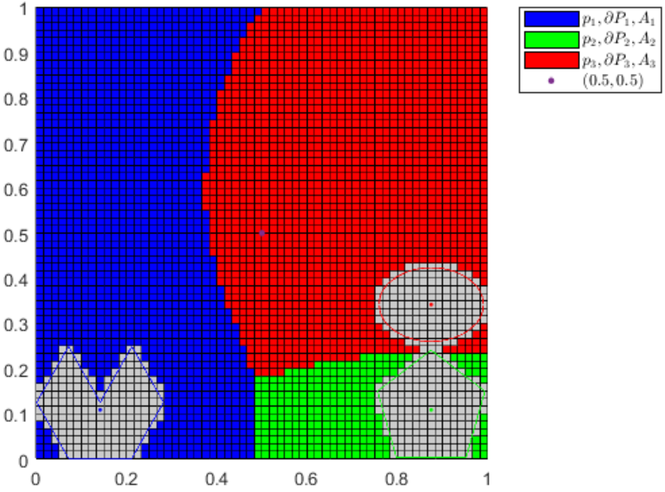
$$\sum_{j=1}^{\rho} a_j w_{rs}^D \tau_{rs}^j + w_{rs}^D \varphi_{rs} \le a_i w_{rs}^D + \sum_{(k,l)\in\mathbf{\Omega}_i} w_{rs}^D u_{rs,kl}^i \theta_{kl}^i, \qquad \forall (r,s) \in \mathbf{\Omega},\ i \in \{1, \dots, \rho\}, \tag{14}$$

$$\varphi_{rs} \gtreqless \sum_{(k,l)\in\mathbf{\Omega}_i} u_{rs,kl}^i \theta_{kl}^i \mp M(1 - \tau_{rs}^i), \qquad \forall (r,s) \in \mathbf{\Omega},\ i \in \{1, \dots, \rho\}. \tag{15}$$

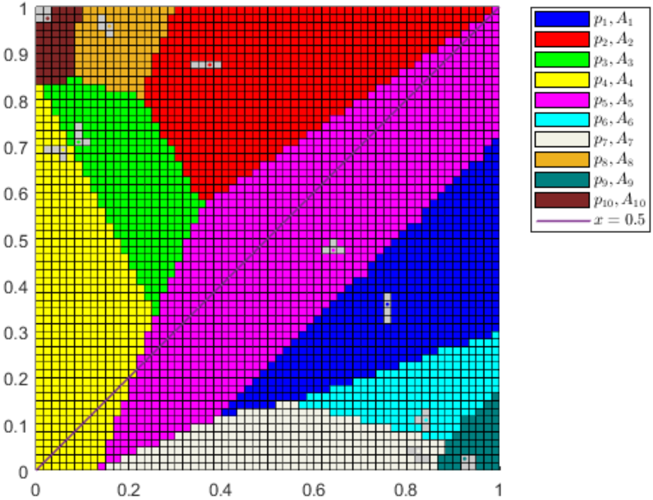
Test experiments



Test experiments



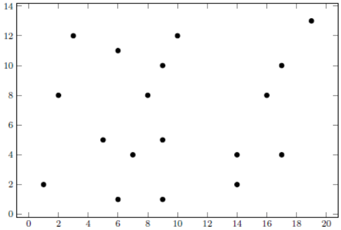
Test experiments



p-median problem

$$\min \quad \sum_{i=1}^{m} w_i \min_{j=1,\dots,p} \{d_G(x_j, a_i)\}$$

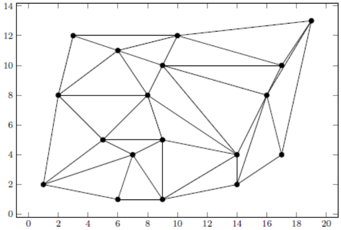
$$\text{s.t.} \quad x_j \in G, \quad j = 1, \dots, p$$



p-median problem

$$\min \quad \sum_{i=1}^{m} w_i \min_{j=1,\ldots,p} \{d_G(x_j, a_i)\}$$

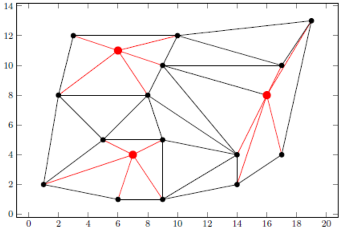
$$\text{s.t.} \quad x_j \in G, \quad j = 1, \ldots, p$$



p-median problem

$$\min \quad \sum_{i=1}^{m} w_i \min_{j=1,\dots,p} \{d_G(x_j, a_i)\}$$

$$\text{s.t.} \quad x_j \in V, \quad j = 1, \dots, p$$



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Clustering and portfolio selection problems: A unified framework

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Justo Puerto[a,b], Moisés Rodríguez-Madrena[a,b], Andrea Scozzari[c,*]

[a] *Institute of Mathematics University of Seville (IMUS), Seville, Spain*
[b] *Department of Statistics and OR, Universidad de Sevilla, 41012 Seville, Spain*
[c] *Faculty of Economics, Università degli Studi Niccolò Cusano Roma, Italy*

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ABSTRACT

Given a set of assets and an investment capital, the classical portfolio selection problem consists in determining the amount of capital to be invested in each asset in order to build the most profitable portfolio. The portfolio optimization problem is naturally modeled as a mean-risk bi-criteria optimization problem where the mean rate of return of the portfolio must be maximized whereas a given risk measure must be minimized. Several mathematical programming models and techniques have been presented in the literature in order to efficiently solve the portfolio problem. A relatively recent promising line of research is to exploit clustering information of an assets network in order to develop new portfolio optimization paradigms. In this paper we endow the assets network with a metric based on correlation coefficients between assets' returns, and show how classical location problems on networks can be used for clustering assets. In particular, by adding a new criterion to the portfolio selection problem based on an objective function of a classical location problem, we are able to measure the effect of clustering on the selected assets with respect to the non-selected ones. Most papers dealing with clustering and portfolio selection models solve these problems in two distinct steps: cluster first and then selection. The innovative contribution of this paper is that we propose a Mixed-Integer Linear Programming formulation for dealing with this problem in a unified phase. The effectiveness of our approach is validated reporting some computational experiments on some real financial datasets.

Cardinality constrained mean-CVaR model

$$\max \quad \eta - \frac{1}{\beta}\sum_{t=1}^{T}\frac{1}{T}d_t^- \tag{16}$$

$$\text{s.t.} \quad d_t^- \geq \eta - \sum_{j=1}^{n} r_{jt}x_j \quad t = 1,\dots,T \tag{17}$$

$$d_t^- \geq 0 \quad t = 1,\dots,T \tag{18}$$

$$\mu(x) \geq \mu_0 \tag{19}$$

$$x \in \Delta \tag{20}$$

$$\sum_{j=1}^{n} z_j = p \tag{21}$$

$$\ell_j z_j \leq x_j \leq u_j z_j \qquad j = 1,\dots,n \tag{22}$$

$$z_j \in \{0,1\} \qquad j = 1,\dots,n. \tag{23}$$

The network of asset correlations

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The network of asset correlations

A distance between asset i and j can be defined as

$$d_{ij} = \sqrt{2(1 - \rho_{ij})}$$

where ρ_{ij} is the Pearson correlation coefficient between assets' logarithmic returns.

Clustering and portfolio selection model

$$\max \quad \eta - \frac{1}{\beta} \sum_{t=1}^{T} \frac{1}{T} d_t^- \tag{24}$$

$$\text{s.t.} \quad (\mathbf{d}, \eta) \in \mathcal{S} \tag{25}$$

$$\mu(\mathbf{x}) \geq \mu_0 \tag{26}$$

$$\mathbf{x} \in \Delta \tag{27}$$

$$F_p(\mathbf{x}) \leq F_p^0 \tag{28}$$

$$\sum_{j=1}^{n} z_{jj} = p \tag{29}$$

$$\sum_{j=1}^{n} z_{ij} = 1 \qquad i = 1, \ldots, n \tag{30}$$

$$z_{ij} \leq z_{jj} \qquad i, j = 1, \ldots, n \tag{31}$$

$$\ell_j z_{jj} \leq x_j \leq u_j z_{jj} \qquad j = 1, \ldots, n \tag{32}$$

$$z_{ij} \in \{0, 1\} \qquad i, j = 1, \ldots, n. \tag{33}$$

Selection of F_p^0

Proposition

Consider problem (24)-(33) with all the paremeters fixed except F_p^0.

Let F_p^ℓ be the optimal value of the problem

$$\begin{array}{ll} \min & F_p(\mathbf{x}) \\ \text{s.t.} & (25)-(26) \text{ and } (29)-(33) \end{array}$$

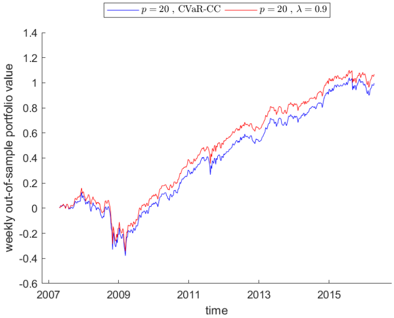
and let $F_p^u = F_p(\mathbf{x}^*)$ being $\mathbf{x}^*$ the optimal solution of the problem (16)-(23).

Then, the valid and meaningful values for F_p^0 in problem (24)-(33) are

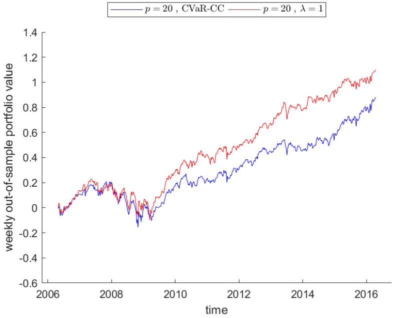
$$F_p^0(\lambda) = \lambda F_p^\ell + (1-\lambda) F_p^u, \qquad \forall \lambda \in [0,1].$$

Out-of-sample performance evaluation

S&P 500



FTSE 100



Many thanks for your attention.