

Mathematical modeling and multicriteria optimization in location problems and complex networks design.

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*Advances on logistics and transportation problems on complex networks:
Evaluation and conclusions.*

Nuevos Desafíos Matemáticos en Problemas Logísticos y de Transporte Integrado
sobre Redes Complejas: Diseño y Optimización (MTM2016-74983-C2-1-R)
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Introducción

Una **red** (o grafo dirigido) es un conjunto de nodos conectados a través de arcos que representan ciertas relaciones entre los nodos.

Las **redes complejas** son conjuntos de muchos nodos conectados que interactúan de alguna forma. Topológicamente son redes a las que se agregan algunas características dinámicas que hacen necesario estudiarlos desde varios puntos de vista. El uso y explotación eficiente de estas redes se estudia matemáticamente en el marco de la teoría de grafos, que está íntimamente ligada a los problemas de **optimización combinatoria**.

Abordar el diseño de una red compleja requiere definir **modelos matemáticos** que relacionen los elementos que intervienen en los nodos y arcos, **asignar variables de decisión** que permitan actuar sobre los mismos y definir medidas cuantitativas que permitan **evaluar las políticas** de actuación que se implementen. Además, muchas aplicaciones de las redes complejas buscan encontrar una **localización** óptima para una o más instalaciones en la red, minimizando una función de inconveniencia entre estas instalaciones y la demanda.

El proceso de optimización de una red compleja puede involucrar **múltiples criterios** y decisores con posibles objetivos diferentes y en conflicto.

Introducción

Algunos modelos de redes complejas de interés son:

- ▶ **Redes logísticas:** Desde el punto de vista empresarial, tienen como misión poner en manos del consumidor determinados productos (bienes y servicios) optimizando cuatro tareas fundamentales: producción, transporte, almacenaje, y distribución.
- ▶ **Redes de transporte:** Tienen como misión el transporte de pasajeros optimizando cuatro tareas fundamentales: diseño de la red, determinación de horarios, gestión de vehículos, y gestión de conductores.
- ▶ **Redes complejas de telecomunicación:** Un operador de telecomunicaciones que posea varias conexiones entre diferentes nodos de una red, puede tomar decisiones para ampliar la red, que suele contemplar una estructura de árbol e incluir hubs.
- ▶ **Redes complejas de infraestructuras industriales:** requieren de reglas de diseño de redes complejas que involucran diversos servicios independientes compitiendo por espacios de trazado escasos, respetando restricciones de trazado de alto nivel, compatibilidad y fabricabilidad.

Antecedentes

Avances en la investigación matemática y nuevas tecnologías han hecho que cuestiones recientemente inabordables puedan ser tratadas en la actualidad (Mesa, Ortega, Piedra and Pozo, 2021):

- ▶ Los problemas de **diseño de redes** surgen en el campo de las redes físicas: eléctricas, informáticas, comunicaciones, transporte, etc. (Ahuja et al., 1995; Medhi, 2004; Magnanti and Wolsey, 1995; Hinojosa et al., 2008; Blanco et al., 2011; Chow, 2018).
- ▶ Los **modelos de localización** son básicos en la fase inicial de análisis de una red logística o de transporte (Laporte et al., 2015). Consideran una región donde las demandas de un conjunto de clientes deben ser satisfechas por los servicios a ubicar (servidores, horarios de transporte y líneas de comunicación). Son aspectos fundamentales la elección correcta de la función objetivo (Daskin, 1995) y la inclusión de las restricciones de capacidad (Kalcscs et al., 2010; Espejo et al., 2012).
- ▶ La **planificación del transporte** se encuentran en plena evolución. En este campo destacan **los modelos de rutas de vehículos** (Toth and Vigo, 2002; Dror, 2000; Letchford and Lodi, 2007; Barrena, Laporte, Ortega and Pozo, 2016; Corberán et al., 2021) y los de **diseño de sistemas de transporte público** (see Mesa, Ortega and Pozo, 2014; Ortega, Pozo and Puerto, 2018).
- ▶ Los **modelos de localización de hubs** (Campbell et al., 2002; Alumur and Kara, 2008; Campbell and O'Kelly, 2012; Contreras and O'Kelly, 2019) están presentes en muchos de los sistemas de redes complejas (envíos postales, telecomunicaciones, aerolíneas, transporte). La función objetivo **mediana ordenada** se ha incorporado a estos problemas (Puerto et al., 2011, 2013, 2016; Pozo, Puerto and Rodríguez-Chía, 2021) induciendo nuevos patrones de distribución.
- ▶ A menudo el coste de la toma de decisiones es relativamente alto (Laporte, Mesa, Ortega and Pozo, 2009). Diferentes decisores pueden tener objetivos diferentes y en conflicto (Laporte, Ortega, Pozo and Puerto, 2017). El problema de agregar **múltiples criterios** es de especial interés cuando el número de soluciones de Pareto (Ehrgott, 2005) es demasiado grande o se requiere una única solución significativa. Los operadores **promedio ponderado ordenado** (Fernández, Pozo and Puerto, 2014; Fernández, Pozo, Puerto and Scozzari, 2017) son muy útiles en este contexto.

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MULTI-OBJECTIVE INTEGRATION OF TIMETABLES, VEHICLE SCHEDULES AND USER ROUTINGS IN A TRANSIT NETWORK

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Multi-objective integration of timetables, vehicle schedules and user routings in a transit network

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ABSTRACT

The Transit Network Timetabling and Scheduling Problem (TNTSP) aims at determining an optimal timetable for each line of a transit network by establishing departure and arrival times at each station and allocating a vehicle to each timetable. The current models for the planning of timetables and vehicle schedules use the *a priori* knowledge of users' routings. However, the actual route choice of a user depends on the timetable. This paper solves the TNTSP in a public transit network by integrating users' routings in the model. The proposed formulation guarantees that each user is allocated to the best possible timetable, while satisfying capacity constraints. In addition, we perform a trade-off analysis by means of a multi-objective formulation which jointly optimizes the operator's and the users' criteria.



Laporte, Ortega, Pozo and Puerto (2017) "Multi-objective integration of timetables, vehicle schedules and user routings in a transit network".

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- ② Multi-objective integration of timetables, vehicle schedules and user routings in a transit network

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The global transit planning process

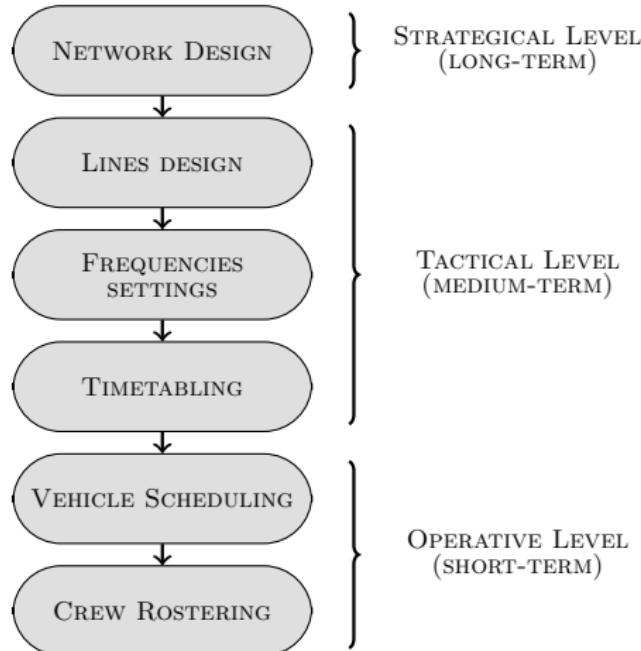


Figure: Global planning process

The (split) Timetabling and Vehicle Scheduling Problem

Timetabling Problem: Consists in determining arrival/departure times at each station of each line of a network.

Main features:

- ▶ Periodicity (timetables easy-to-remember vs time-dependent demand adjustment)
 - ▶ Infrastructure (single corridor vs complete network)
 - ▶ Objectives (transfer synchronization vs/with schedule delay minimization)
- ✉ Blanco, Conde, Hinojosa and Puerto (2020) *An optimization model for line planning and timetabling in automated urban metro subway networks. A case study.*

Vehicle Scheduling Problem: Consists in allocating a set of vehicles to a set of timetables.

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The scheduled delay in a transit network

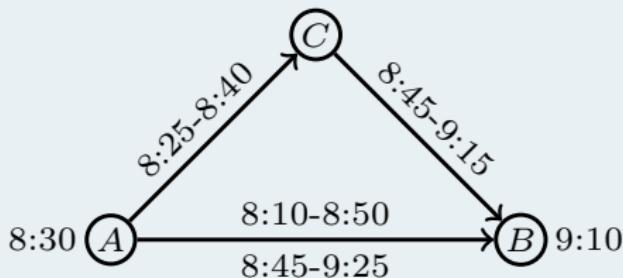
Example: Consider a user who aims to travel from station A to station B departing from A at 8:30 and reaching B at 9:10. There are two itineraries $A \rightarrow B$ and $A \rightarrow C \rightarrow B$, but if we consider the timetable this user may choose between three strategies:

- (1) depart from A at 8:10 to arrive at B at 8:50,
- (2) depart from A at 8:45 to arrive at B at 9:25, and
- (3) depart from A at 8:25 to arrive at C at 8:40 and from C at 8:45 to arrive at B at 9:15.

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Passenger routings and capacities

User-oriented optimization of public transport requires data about the users in order to develop realistic models. Current models take user data into account by using the following two-phase approach:

- (1) user routes are determined.
- (2) the actual planning of timetables takes place using the knowledge of which routes users wish to travel given the results of the first phase.

Example (The Braess's paradox): Suppose that 4000 users want to go from START to END in the minimum time possible. The numbers on the edges of the figure (45) indicate a fixed travel time of 45 minutes, while the labels $T/100$ express that the time in minutes required for traversing depends on the number of users using the arc divided by 100.

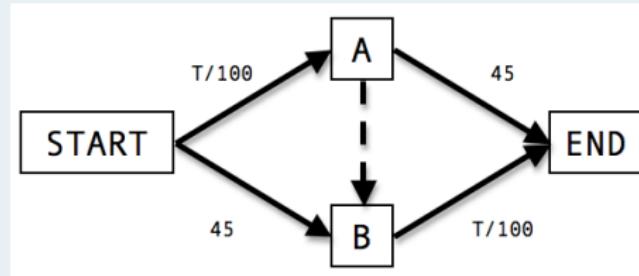
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Complete integration

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- ✖ van den Heuvel, van den Akker and van Kooten (2008) "Integrating timetabling and vehicle scheduling in public bus ..."
- ✖ Guihaire and Hao (2008) *Transit network re-timetabling and vehicle scheduling*
- ✖ Ceder (2011) "*Optimal multi-vehicle type transit timetabling and vehicle scheduling*"
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- ✖ Castelli, Pesenti and Ukovich (2004) "*Scheduling multimodal transportation systems*"
- ✖ Liu and Shen (2007) "*Regional bus operation bi-level programming model integrating timetabling and vehicle ...*"
- ✖ Fleurent and Lessard (2009) "*Integrated timetabling and vehicle scheduling in practice*"
- ✖ Guihaire and Hao (2010) "*Transit network timetabling and vehicle assignment for regulating authorities*"
- ✖ Ibarra-Rojas, Giesen and Rios-Solis (2014) "*An integrated approach for timetabling and vehicle scheduling ...*"

Integration of timetabling and passenger routings

- ✖ Siebert and Goerigk (2013) "*An experimental comparison of periodic timetabling models*"
- ✖ Schmidt and Schöbel (2015b) "*The complexity of integrating passenger routing decisions in public transportation ...*"
- ✖ Schmidt and Schöbel (2015a) "*Timetabling with passenger routing*"

TNTSP in a transit line

- ✖ Mesa, Ortega and Pozo (2014) "*Locating optimal timetables and vehicle schedules in a transit line*"

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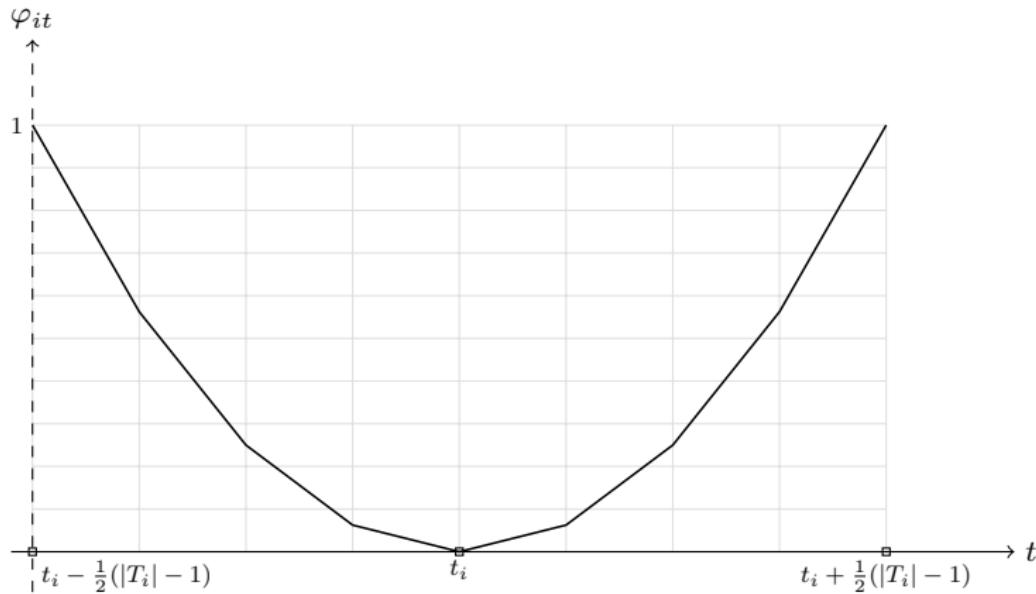
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Cost structure



$$\varphi_i(t) \equiv \varphi_{it} = \min \left\{ 1, \left(\frac{|t_i - t|^+}{\frac{1}{2}(|T_i| - 1)} \right)^2 + \left(\frac{|t + t_l(o_i, d_i) - t_{i+n}|^+}{\frac{1}{2}(|T_i| - 1)} \right)^2 \right\}$$

✉ Mesa, Ortega and Pozo (2014) "Locating optimal timetables and vehicle schedules in a transit line"

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② Multi-objective integration of timetables, vehicle schedules and user routings in a transit network

Introduction

Background

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Definition (PTN):

- ▶ A *Public Transportation Network* (PTN) is a graph $G = (S, A)$ with a set of nodes S representing stations and a set of arcs A , where each arc represents a direct connection between two stations of S .

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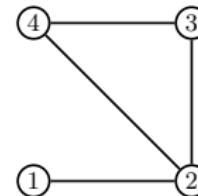


Figure: PTN example

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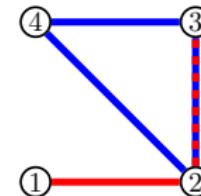


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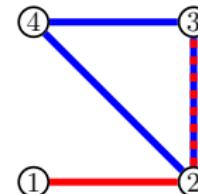


Figure: PTN example

Definitions:

- The associated *Change&Go Network* (CGN) of a PTN is a graph \mathcal{G} defined in order to include **transfer activities** between lines of the PTN.
- A *hyperpath* is the set of all possible **itineraries** connecting an origin and a destination. Each itinerary offers different travel **options** for traveling according to each combination of the potential timetables from the different lines that can be used for completing a trip.

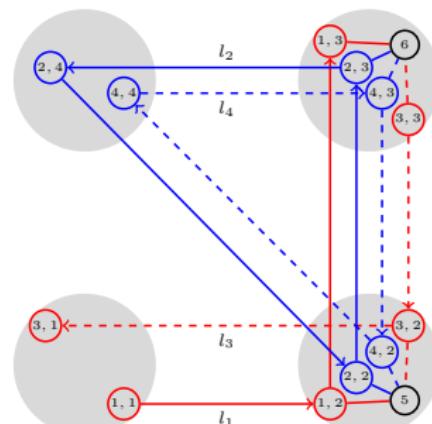


Figure: CGN example

Timetable characterization

Definition: Given the set of line runs $r \in R$, a timetable in T is defined as the set of arrival/departure times at each station for each line run: $\Theta = \{(\theta_{rls}^+, \theta_{rls}^-), r \in R, l \in \mathcal{L}, s \in S_l\}$

Lemma: Assuming that:

- (1) $\theta_{rls}^- - \theta_{rls}^+ = \lambda_{ls}, r \in R, l \in \mathcal{L}, s \in S_l$
- (2) $\theta_{rls+1}^+ - \theta_{rls}^- = \mu_{ls}, r \in R, l \in \mathcal{L}, s \in S_l$

the following properties can be stated:

- ▶ $\theta_{rl|S_l|}^+ - \theta_{rl1}^- = \tau_l, r \in R, l \in \mathcal{L}$
- ▶ $\Theta \equiv X = \{x_{lt}, l \in \mathcal{L}, t \in T\}$ where:
 $x_{lt} \in \{0, 1\}$ is a binary variable equal to 1 \Leftrightarrow a line run is allocated in line l at time t .

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Schedule characterization

Proposition: (Vehicle Schedule) The timetable X is a vehicle schedule \Leftrightarrow the number of vehicles required to perform X is lower than $\kappa \Leftrightarrow$

$$\sum_{t'=1}^{\tau_l} x_{lt'} \leq \kappa_l \quad l \in \mathcal{L} \quad (1)$$

$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l+\lvert\vec{\mathcal{L}}\rvert,t'} \leq \kappa_l \quad l \in \vec{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (2)$$

$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l-\lvert\vec{\mathcal{L}}\rvert,t'} \leq \kappa_l \quad l \in \overleftarrow{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (3)$$

$$\sum_{l \in \mathcal{L}} \kappa_l \leq \kappa \quad (4)$$

$$\kappa_l \in \mathbb{Z}^+ \quad l \in \mathcal{L}. \quad (5)$$

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Decision variables and main constraints

 ρ_l

number of line runs located on each line l

 κ_l

number of vehicles initially available on each line l

 x_{lt}

equal to one if and only if a line run is located on line l at time slot t

 $y_{i\pi r}$

equal to one if and only if user i is allocated to itinerary $\pi \in \Pi_i$ and option $r \in \mathcal{R}_{i\pi}$

Subproblems of the TNTSP depending on the types of constraints imposed.

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Figure: Inclusion relationships between the solution spaces of the TNTSP subproblems.

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$$\text{TNTP}^S \subseteq \text{TNTP}^O \subseteq \text{TNTP}^U$$

$$\text{UI} \qquad \text{UI} \qquad \text{UI}$$

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Subproblems of the TNTSP depending on the types of constraints imposed.

$$\text{TNTPS} \subseteq \text{TNTPO} \subseteq \text{TNTPU}$$

UI	UI	UI
----	----	----

$$\text{TNTSPS} \subseteq \text{TNTSPO} \subseteq \text{TNTSPU}$$

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TNTSP^S

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

$$s.t. \sum_{t \in T_l} x_{lt} \leq \rho_l \quad l \in \mathcal{L} \quad (6b)$$

$$\sum_{l \in \mathcal{L}} c_l \rho_l \leq \rho \quad (6c)$$

$$\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r} \leq 1 \quad i \in I \quad (6d)$$

$$|\mathcal{L}_\pi| y_{i\pi r} \leq \sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6e)$$

$$\sum_{i \in I} \sum_{\pi \in \Pi_i} \sum_{l \in \mathcal{L}_\pi} \sum_{r \in \mathcal{R}_{i\pi}: t_{i\pi rl} = t} y_{i\pi r} m_{\pi a} \leq Q x_{lt} \quad l \in \mathcal{L}, a \in A_l, t \in T \quad (6f)$$

$$\sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} \leq (|\mathcal{L}_\pi| - 1) + \sum_{\pi' \in \Pi_i} \sum_{r' \in \mathcal{R}_{i\pi}} y_{i\pi' r'} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6g)$$

$$\sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} + \sum_{\pi' \in \Pi_i} \sum_{\substack{r' \in \mathcal{R}_{i\pi}: \\ \varphi_{i\pi' r'} > \varphi_{i\pi r}}} y_{i\pi' r'} \leq |\mathcal{L}_\pi| \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6h)$$

$$\sum_{\substack{t'=1 \\ t'=1}}^{\tau_l} x_{lt'} \leq \kappa_l \quad l \in \mathcal{L} \quad (6i)$$

$$\sum_{\substack{t'=1 \\ t'=1}}^{\tau_l} x_{lt'} - \sum_{\substack{t'=1 \\ t'=1}}^{\tau_l} x_{l+|\vec{\mathcal{L}}|, t'} \leq \kappa_l \quad l \in \vec{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6j)$$

$$\sum_{\substack{t'=1 \\ t'=1}}^{\tau_l} x_{lt'} - \sum_{\substack{t'=1 \\ t'=1}}^{\tau_l} x_{l-|\vec{\mathcal{L}}|, t'} \leq \kappa_l \quad l \in \overleftarrow{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6k)$$

$$\sum_{l \in \mathcal{L}} \kappa_l \leq \kappa \quad (6l)$$

$$x_{lt} \in \{0, 1\} \quad l \in \mathcal{L}, t \in T_l \quad (6m)$$

$$y_{i\pi r} \in \{0, 1\} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6n)$$

TNTP^U

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

$$s.t. \sum_{t \in T_l} x_{lt} \leq \rho_l \quad l \in \mathcal{L} \quad (6b)$$

$$\sum_{l \in \mathcal{L}} c_l \rho_l \leq \rho \quad (6c)$$

$$\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r} \leq 1 \quad i \in I \quad (6d)$$

$$|\mathcal{L}_\pi| y_{i\pi r} \leq \sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6e)$$

(6f)

(6g)

(6h)

(6i)

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TNTPO

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

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$$\sum_{i \in I} \sum_{\pi \in \Pi_i: l \in \mathcal{L}_\pi} \sum_{r \in \mathcal{R}_{i\pi}: t_{i\pi rl} = t} y_{i\pi r} m_{\pi a} \leq Q x_{lt} \quad l \in \mathcal{L}, a \in A_l, t \in T \quad (6f)$$

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$$x_{lt} \in \{0, 1\} \quad l \in \mathcal{L}, t \in T_l \quad (6m)$$

$$y_{i\pi r} \in \{0, 1\} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6n)$$

TNTPS

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

$$s.t. \sum_{t \in T_l} x_{lt} \leq \rho_l \quad l \in \mathcal{L} \quad (6b)$$

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$$x_{lt} \in \{0, 1\}$$

$$l \in \mathcal{L}, t \in T_l \quad (6m)$$

$$y_{i\pi r} \in \{0, 1\}$$

$$i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6n)$$

TNTSP^U

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

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$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l-|\mathcal{L}|, t'} \leq \kappa_l \quad l \in \overleftarrow{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6k)$$

$$\sum_{l \in \mathcal{L}} \kappa_l \leq \kappa \quad (6l)$$

$$x_{lt} \in \{0, 1\} \quad l \in \mathcal{L}, t \in T_l \quad (6m)$$

$$y_{i\pi r} \in \{0, 1\} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6n)$$

TNTSP^O

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

$$s.t. \sum_{t \in T_l} x_{lt} \leq \rho_l \quad l \in \mathcal{L} \quad (6b)$$

$$\sum_{l \in \mathcal{L}} c_l \rho_l \leq \rho \quad (6c)$$

$$\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r} \leq 1 \quad i \in I \quad (6d)$$

$$|\mathcal{L}_\pi| y_{i\pi r} \leq \sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6e)$$

$$\sum_{i \in I} \sum_{\pi \in \Pi_i} \sum_{l \in \mathcal{L}_\pi} \sum_{r \in \mathcal{R}_{i\pi}: t_{i\pi rl} = t} y_{i\pi r} m_{\pi a} \leq Q x_{lt} \quad l \in \mathcal{L}, a \in A_l, t \in T \quad (6f)$$

(6g)

(6h)

$$\sum_{t'=1}^{\tau_l} x_{lt'} \leq \kappa_l \quad l \in \mathcal{L} \quad (6i)$$

$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l+|\vec{\mathcal{L}}|, t'} \leq \kappa_l \quad l \in \vec{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6j)$$

$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l-|\vec{\mathcal{L}}|, t'} \leq \kappa_l \quad l \in \overleftarrow{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6k)$$

$$\sum_{l \in \mathcal{L}} \kappa_l \leq \kappa \quad (6l)$$

$$x_{lt} \in \{0, 1\} \quad l \in \mathcal{L}, t \in T_l \quad (6m)$$

$$y_{i\pi r} \in \{0, 1\} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6n)$$

TNTSP^S

$$\min \sum_{i \in I} [\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r})] \quad (6a)$$

$$s.t. \sum_{t \in T_l} x_{lt} \leq \rho_l \quad l \in \mathcal{L} \quad (6b)$$

$$\sum_{l \in \mathcal{L}} c_l \rho_l \leq \rho \quad (6c)$$

$$\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r} \leq 1 \quad i \in I \quad (6d)$$

$$|\mathcal{L}_\pi| y_{i\pi r} \leq \sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6e)$$

$$\sum_{i \in I} \sum_{\pi \in \Pi_i} \sum_{l \in \mathcal{L}_\pi} \sum_{r \in \mathcal{R}_{i\pi}} t_{i\pi rl} = t \quad y_{i\pi r} m_{\pi a} \leq Q x_{lt} \quad l \in \mathcal{L}, a \in A_l, t \in T \quad (6f)$$

$$\sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} \leq (|\mathcal{L}_\pi| - 1) + \sum_{\pi' \in \Pi_i} \sum_{r' \in \mathcal{R}_{i\pi}} y_{i\pi' r'} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6g)$$

$$\sum_{l \in \mathcal{L}_\pi} x_{lt} t_{i\pi rl} + \sum_{\pi' \in \Pi_i} \sum_{\substack{r' \in \mathcal{R}_{i\pi}: \\ \varphi_{i\pi' r'} > \varphi_{i\pi r}}} y_{i\pi' r'} \leq |\mathcal{L}_\pi| \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6h)$$

$$\sum_{t'=1}^{\tau_l} x_{lt'} \leq \kappa_l \quad l \in \mathcal{L} \quad (6i)$$

$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l+|\vec{\mathcal{L}}|, t'} \leq \kappa_l \quad l \in \vec{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6j)$$

$$\sum_{t'=1}^t x_{lt'} - \sum_{t'=1}^{t-\tau_l} x_{l-|\vec{\mathcal{L}}|, t'} \leq \kappa_l \quad l \in \overleftarrow{\mathcal{L}}, t \in T : t > \tau_l \wedge t < |T| - \tau_l \quad (6k)$$

$$\sum_{l \in \mathcal{L}} \kappa_l \leq \kappa \quad (6l)$$

$$x_{lt} \in \{0, 1\} \quad l \in \mathcal{L}, t \in T_l \quad (6m)$$

$$y_{i\pi r} \in \{0, 1\} \quad i \in I, \pi \in \Pi_i, r \in \mathcal{R}_{i\pi} \quad (6n)$$

The multi-objective TNTSP

$$\begin{aligned} \min (z_1, z_2, z_3) \equiv \min & \left(\sum_{i \in I} \left[\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r}) \right], \sum_{l \in \mathcal{L}} \sum_{t \in T_l} x_{lt} c_l, \sum_{l \in \mathcal{L}} \kappa_l \right) \\ s.t. \quad & (7b) - (7n) \end{aligned}$$

The multi-objective TNTSP

$$\min (z_1, z_2, z_3) \equiv \min \left(\sum_{i \in I} \left[\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r}) \right], \sum_{l \in \mathcal{L}} \sum_{t \in T_l} x_{lt} c_l, \sum_{l \in \mathcal{L}} \kappa_l \right)$$

s.t. (7b) – (7n)

The multi-objective TNTSP

$$\begin{aligned} \min (z_1, z_2, z_3) \equiv \min & \left(\sum_{i \in I} \left[\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r}) \right], \sum_{l \in \mathcal{L}} \sum_{t \in T_l} \textcolor{red}{x_{lt} c_l}, \sum_{l \in \mathcal{L}} \kappa_l \right) \\ s.t. \quad & (7b) - (7n) \end{aligned}$$

The multi-objective TNTSP

$$\begin{aligned} \min (z_1, z_2, z_3) \equiv \min & \left(\sum_{i \in I} \left[\sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} \varphi_{i\pi r} y_{i\pi r} + (1 - \sum_{\pi \in \Pi_i} \sum_{\mathcal{R}_{i\pi}} y_{i\pi r}) \right] , \sum_{l \in \mathcal{L}} \sum_{t \in T_l} x_{lt} c_l , \sum_{l \in \mathcal{L}} \kappa_l \right) \\ s.t. \quad & (7b) - (7n) \end{aligned}$$

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② Multi-objective integration of timetables, vehicle schedules and user routings in a transit network

Introduction

Background

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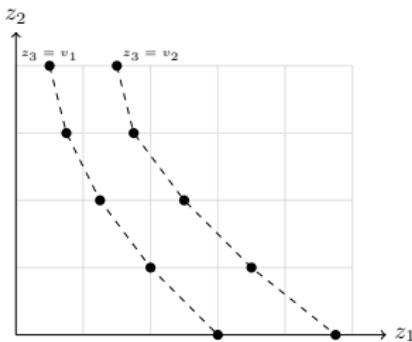
An ϵ -constraint algorithm

Computational experiments

Conclusions

3-dimensional non-dominated set of solutions

We are interested in computing Pareto optimal solutions, i.e., an undominated set of solutions with respect to the values of the different objective functions. Since we are minimizing three different objectives, a special solution representation is required to avoid plotting of a 3-dimensional non-dominated set of solutions (\mathcal{P}), which would be difficult to analyze by the decision maker. To this end, we project the solutions onto the $z_1 \times z_2$ plane, where z_1 is the users' inconvenience and z_2 is the line runs cost. In this plane the level curves plot points with the same z_3 value (fleet size). This kind of graphical representation is very informative since it shows the demand improvement obtained by increasing either the number of vehicles or the budget for line-runs.



An ϵ -constraint algorithm

Algorithm 1: Non-dominated points set \mathcal{P}

```
1 Select values  $\rho^{min}, \rho^{max}, \kappa^{min}, \kappa^{max}, \epsilon_1, \epsilon_2$ ;           // select input parameters
2  $\rho := \rho^{max}$ ;  $\kappa := \kappa^{max}$ ;  $\mathcal{P} = \{\emptyset\}$ ;                                // reset values
3 while  $\kappa \geq \kappa^{min}$  do          // while budget  $\kappa$  is above its lower bound...
4   while  $\rho \geq \rho^{min}$  do          // while budget  $\rho$  is above its lower bound...
5     solve:  $(z_1^*, z_2^*, z_3^*) := \arg \text{lex min}(z_1, z_2, z_3)$  s.t. (??)-(??); // compute a
       new non-dominated point
6      $\mathcal{P} := \mathcal{P} \cup \{(z_1^*, z_2^*, z_3^*)\}$ ;           // Add the new point to the set on
       non-dominated points
7      $\rho := z_2^* - \epsilon_1$ ;                           // decrease  $\rho$ 
8      $\kappa := \kappa - \epsilon_2$ ;                         // set  $\kappa$  for obtaining a new level curve
9      $\rho := \rho^{max}$ ;                                // reset  $\rho$ 
```

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Computational experience

Computational experience:

- ▶ We have considered six networks inspired from some already existing in the literature (see Laporte et al. 1994; Laporte et al. 1997)
- ▶ For each O/D pair we have precomputed the different itineraries by using a *k*-shortest path algorithm (Shier, 1979).
- ▶ The different travel options for traveling were calculated for each user, according to the available itineraries, time windows, and travel times in the network.
- ▶ We perform a **trade off** analysis between users' and operator's cost (see Ibarra et al., 2014)

All instances were solved with the MIP Xpress 7.5 optimizer, under a Windows 7 environment in an Intel(R) Core(TM)i7 CPU 2.93 GHz processor and 8 GB RAM.

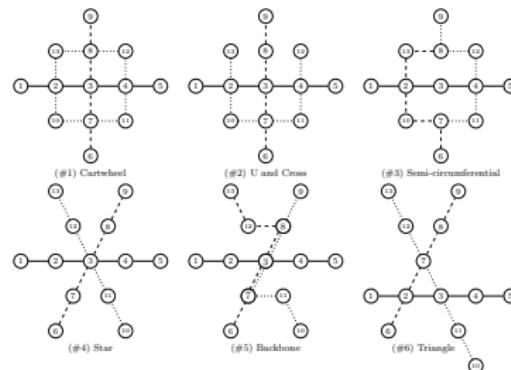


Figure: Basic configurations obtained from 3 lines.

Computational experience

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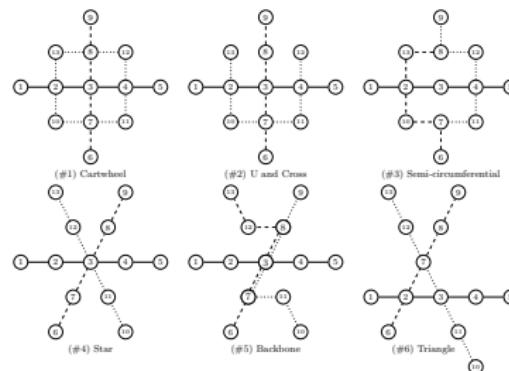
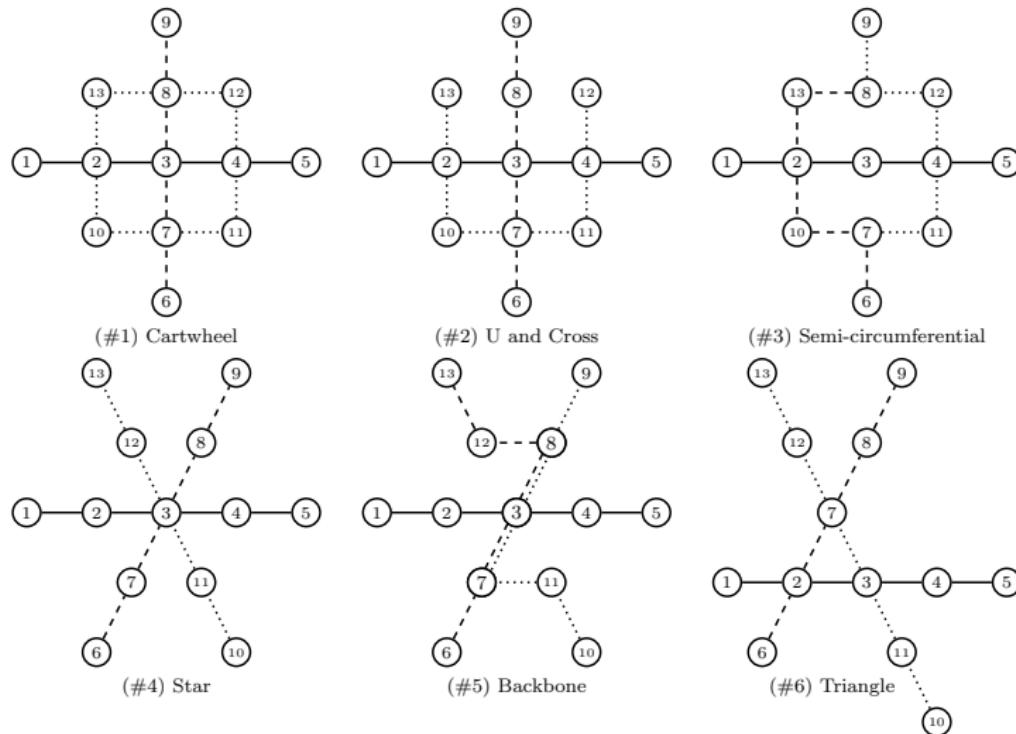


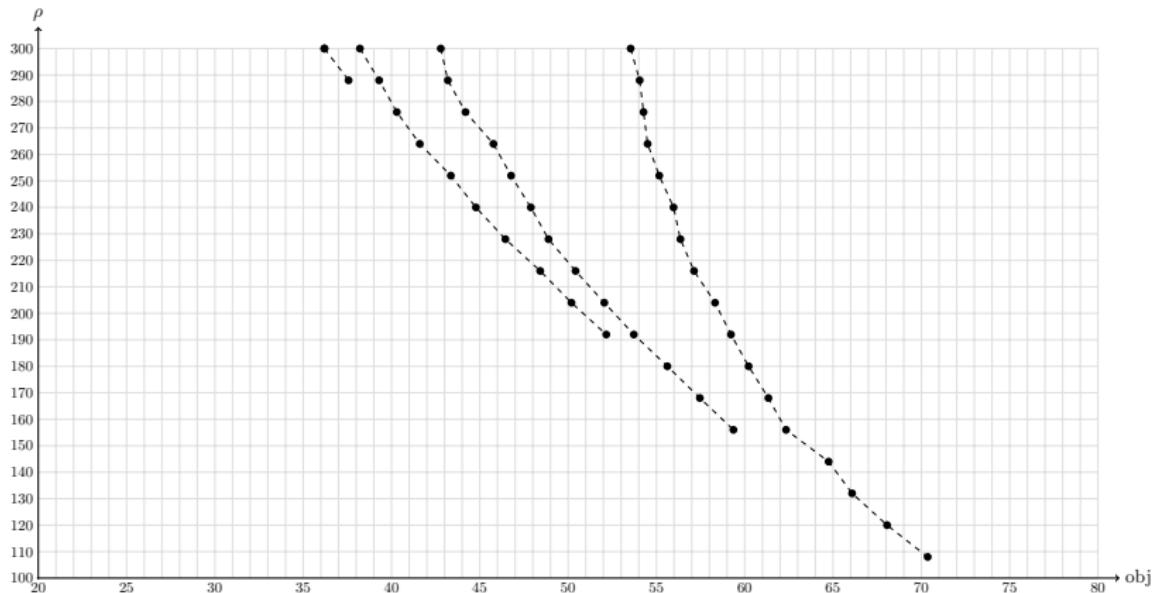
Figure: Basic configurations obtained from 3 lines.

Network configurations



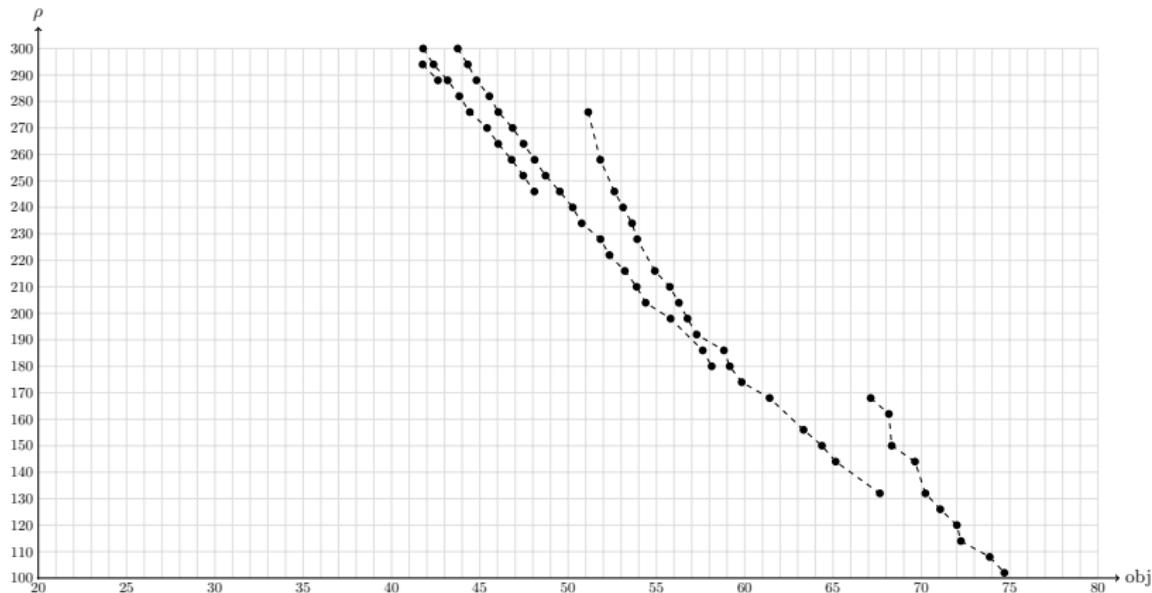
Basic configurations obtained from 3 lines.

Non-dominated solutions: Cartwheel



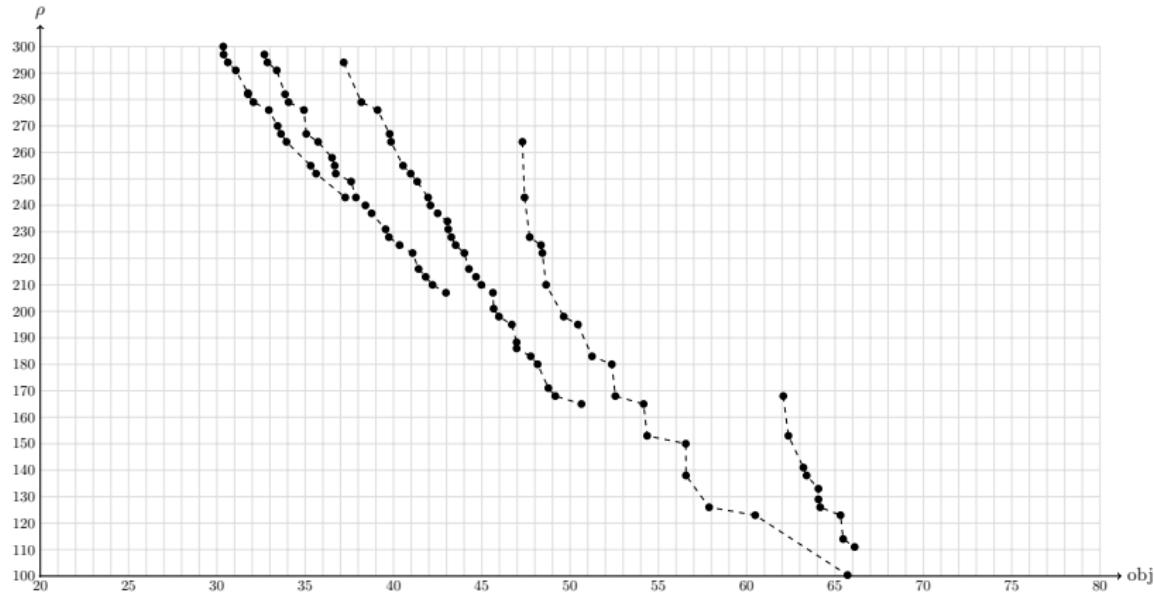
Level curves ($\kappa \in \{18, 15, 12, 9, 6\}$) of the Pareto fronts for the different graph configurations.

Non-dominated solutions: U and Cross



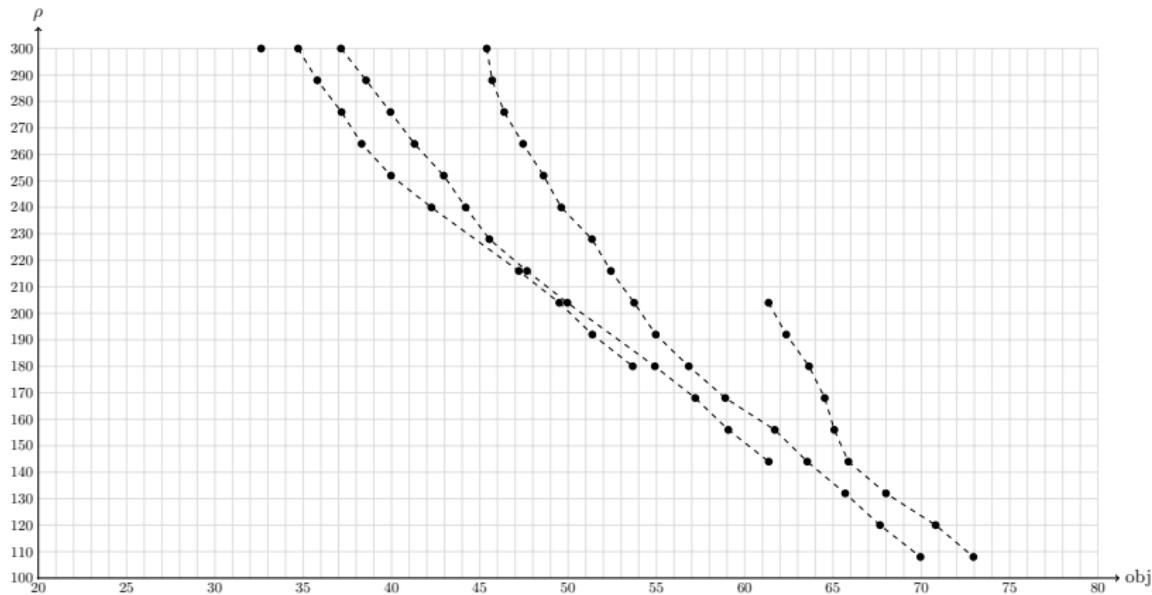
Level curves ($\kappa \in \{18, 15, 12, 9, 6\}$) of the Pareto fronts for the different graph configurations.

Non-dominated solutions: Semi-circumferential



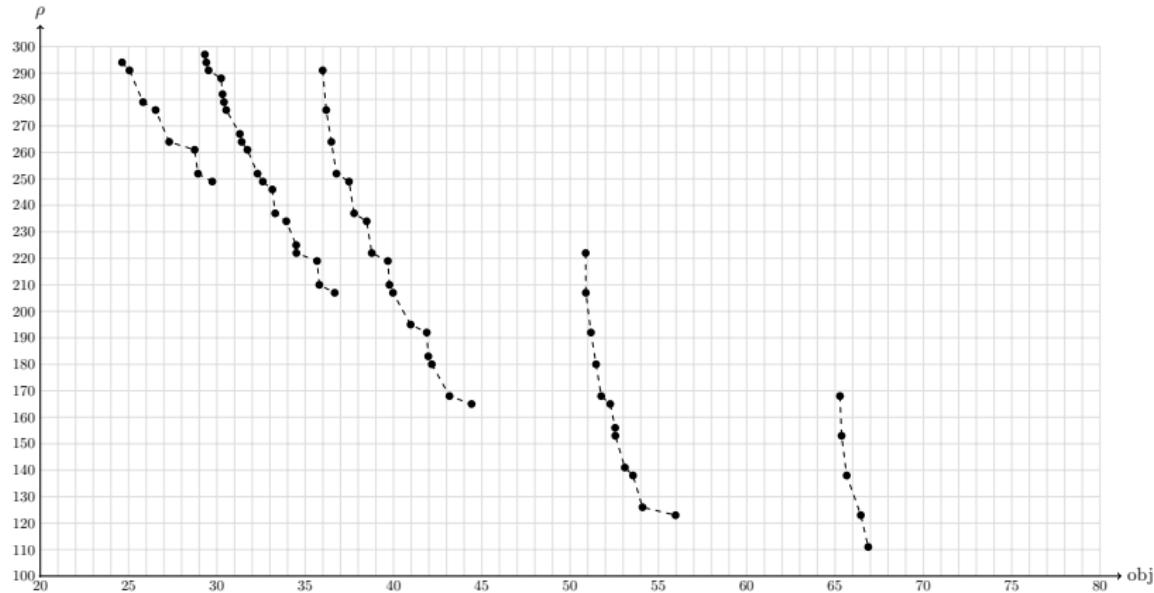
Level curves ($\kappa \in \{18, 15, 12, 9, 6\}$) of the Pareto fronts for the different graph configurations.

Non-dominated solutions: Star



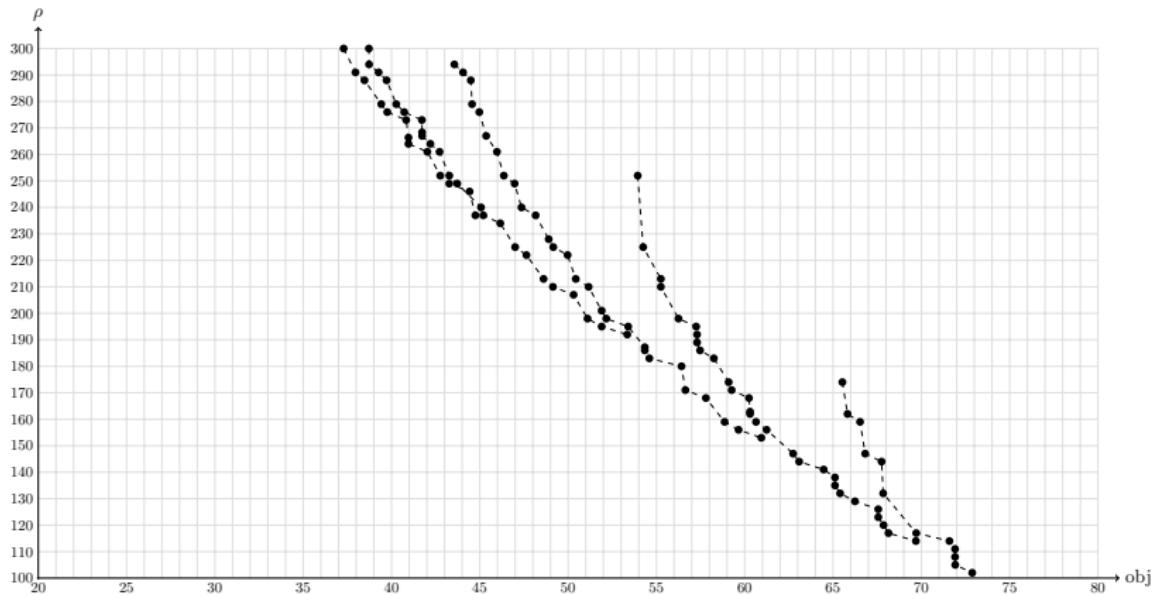
Level curves ($\kappa \in \{18, 15, 12, 9, 6\}$) of the Pareto fronts for the different graph configurations.

Non-dominated solutions: Backbone



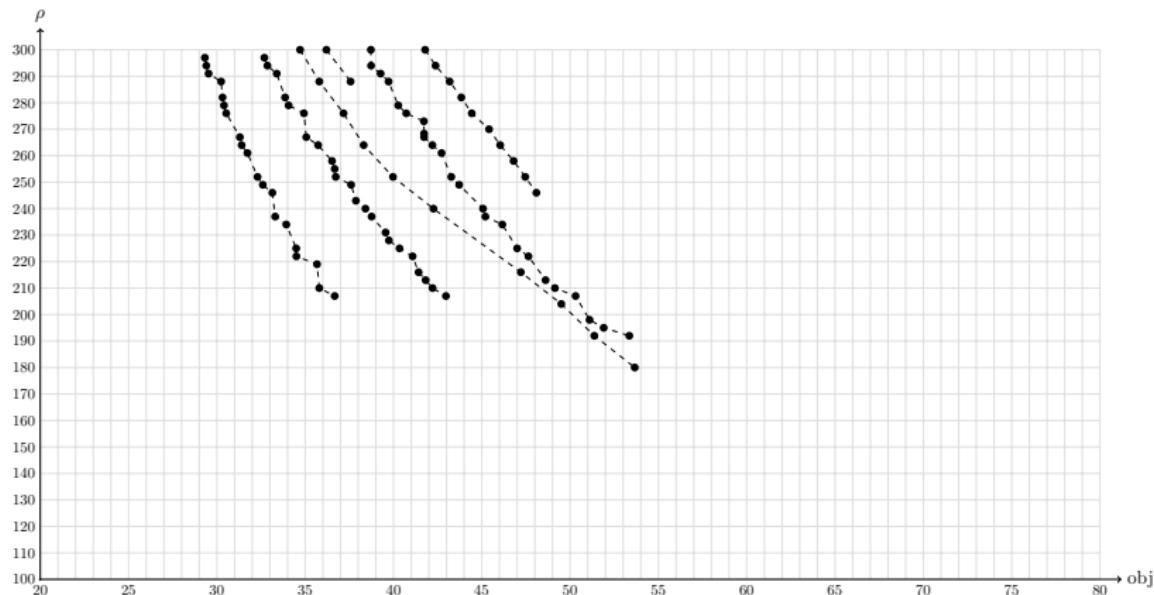
Level curves ($\kappa \in \{18, 15, 12, 9, 6\}$) of the Pareto fronts for the different graph configurations.

Non-dominated solutions: Triangle



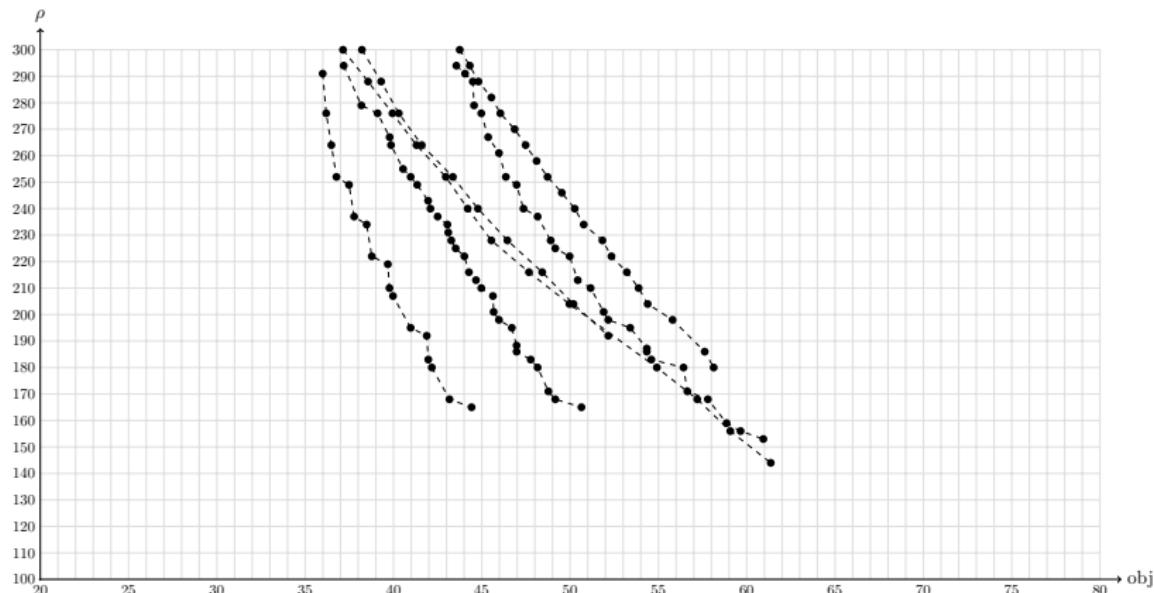
Level curves ($\kappa \in \{18, 15, 12, 9, 6\}$) of the Pareto fronts for the different graph configurations.

Non-dominated solutions: $\kappa = 15$



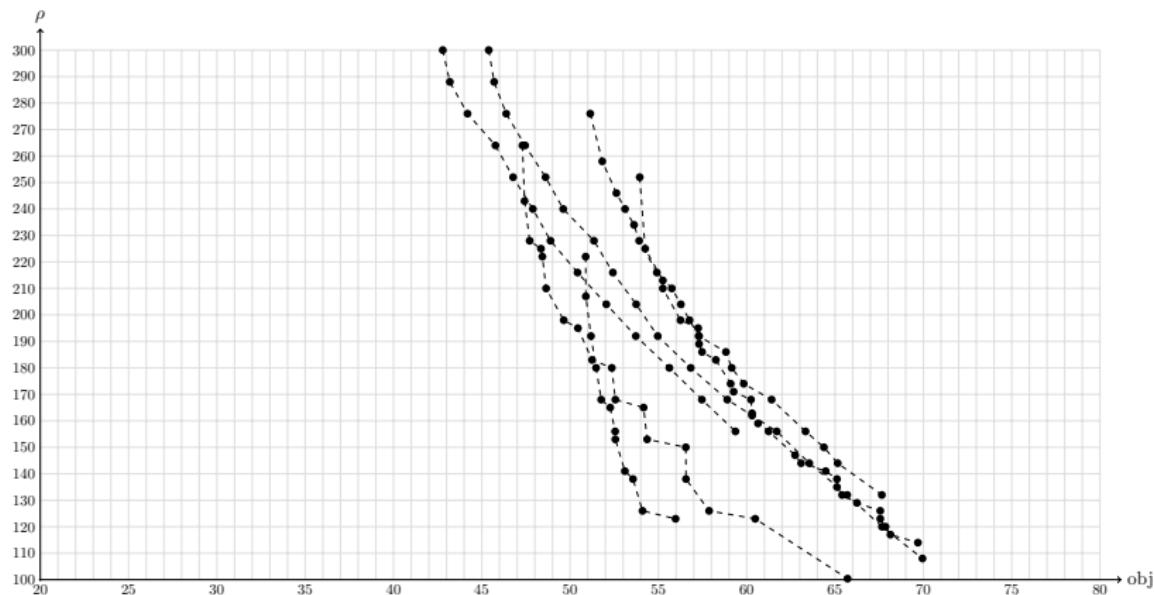
Configurations #5, #3, #4, #1, #6, #2, from left to right

Non-dominated solutions: $\kappa = 12$



Configurations #5, #3, #4, #1, #6, #2, from left to right

Non-dominated solutions: $\kappa = 9$



$$\kappa = 9$$

Parametric analysis of solutions

#	G	p	Q	t	gapLR	nod	obj	t	gapLR	nod	obj	t	gapLR	nod	obj
1	6	1	7.7	37.86	15	77.2	0.5	0	1	86.6	13.1	2.36	15	90.1	
1	6	2	7.6	37.86	15	77.2	0.7	0.29	1	81.4	17	1.05	10	82.3	
1	6	3	7.6	37.86	15	77.2	5.1	1.56	1	79.1	20.4	2.13	26	79.7	
1	12	1	20.7	49.74	158	61	0.5	0.02	1	76.8	12.8	3.77	19	81.5	
1	12	2	20.6	49.74	158	61	5.3	1.58	1	68.6	29.2	3.38	165	69.8	
1	12	3	20.5	49.74	158	61	12.6	5.3	27	63.6	95.9	6.16	1084	64.3	
1	24	1	18.4	61.99	70	37.6	1.1	0.04	1	62.3	19.3	5.24	71	67.1	
1	24	2	18	61.99	70	37.6	10.9	5.44	34	46.2	114.4	8.18	4023	48.1	
1	24	3	18.2	61.99	70	37.6	24.8	21.51	308	40.6	158.2	22.88	1747	41.3	
2	6	1	2	20.67	1	80.4	0.3	0	1	88.2	2.2	0.64	1	89.7	
2	6	2	1.9	20.67	1	80.4	0.3	0.09	1	83.7	6.1	0.94	1	84.7	
2	6	3	1.9	20.67	1	80.4	0.5	0.9	1	81.4	3.6	1.65	1	82.1	
2	12	1	3.4	33.13	1	66.3	0.3	0.08	1	79.9	2.5	1.92	1	82.3	
2	12	2	3.4	33.13	1	66.3	0.7	0.79	1	72.2	4.7	1.53	1	73.1	
2	12	3	3.5	33.13	1	66.3	4.1	3.56	1	68.5	6.8	4.35	1	69.1	
2	24	1	3.2	47.48	1	45.2	0.3	0	1	67.3	1.1	2.51	1	69.6	
2	24	2	3.3	47.48	1	45.2	6.3	4.22	154	53.2	17.5	6.01	91	54.4	
2	24	3	3.2	47.48	1	45.2	8	14.56	112	48.2	26.8	15.97	566	49	
3	6	1	0	0	1	83.4	0	0	1	90	0	1.14	1	91.7	
3	6	2	0	0	1	83.4	0	0	1	86.1	0	0.27	1	86.5	
3	6	3	0	0	1	83.4	0	0.04	1	84.1	0.1	0.34	1	84.4	
3	12	1	0	0	1	72	0	0	1	83.6	0	1.83	1	85.6	
3	12	2	0	0	1	72	0	0.05	1	75.1	0.2	0.9	1	75.8	
3	12	3	0	0	1	72	0	0.03	1	72.4	0.1	0.19	1	72.5	
3	24	1	0	0	1	57.2	0	0	1	71.6	0	2.14	1	73.6	
3	24	2	0	0	1	57.2	0	1	1	59.1	0	0.97	1	59.1	
3	24	3	0	0	1	57.2	0	0.2	1	57.4	0	0.31	1	57.5	
4	6	1	3.4	22.19	10	85.4	0.2	0	1	90.9	0.6	0.56	1	91.9	
4	6	2	3.4	22.19	10	85.4	0.6	0.66	1	87.7	0.8	0.89	1	87.9	
4	6	3	3.5	22.19	10	85.4	1.6	2.34	1	85.9	3	2.34	1	85.9	
4	12	1	0.9	35.17	1	71.9	0.2	0	1	84.1	1.5	0.77	1	85.4	
4	12	2	0.9	35.17	1	71.9	1.4	1.58	1	77	8.3	2.19	1	77.6	
4	12	3	0.9	35.17	1	71.9	0.9	5.5	1	73.7	1.4	5.53	1	73.7	
4	24	1	1.1	50.92	1	49	0.2	0	1	72.1	0.7	1.1	1	73.3	
4	24	2	1.1	50.92	1	49	4.6	5.24	1	59	15.4	6.78	893	60.1	
4	24	3	1.1	50.92	1	49	4.8	13.74	9	52.2	12.7	14.85	51	53	
5	6	1	0.1	4.19	1	86	0	0.05	1	91.2	0.1	0.57	1	91.9	
5	6	2	0.1	4.19	1	86	0	1	87	0.2	0.02	1	87.1		
5	6	3	0.1	4.19	1	86	0.1	0.24	1	86	0.2	0.24	1	86	
5	12	1	0.1	6.28	1	75.6	0	0	1	84.6	0.1	0.44	1	85	
5	12	2	0.1	6.28	1	75.6	0.1	0.1	1	76.8	0.2	0.48	1	77.2	
5	12	3	0.1	6.28	1	75.6	0.1	0.93	1	75.6	0.2	0.93	1	75.6	
5	24	1	0.1	9.23	1	61.4	0	0	1	72.6	0.1	0.41	1	72.9	
5	24	2	0.1	9.23	1	61.4	0.1	0.82	1	62.2	0.3	1.1	1	62.5	
5	24	3	0.1	9.23	1	61.4	0.1	2.71	1	61.4	0.2	2.7	1	61.4	
6	6	1	2.5	20.8	1	83.7	0.2	0.02	1	89.2	1.8	1.66	1	91.2	

 $TNTSP^U$ $TNTSP^O$ $TNTSP^S$

#	G	p	Q	t	gapLR	nod	obj	t	gapLR	nod	obj	t	gapLR	nod	obj
1	12	6	1	32.9	50.16	487	61.5	1.3	0.16	1	77.1	24.5	4.99	129	82.7
1	12	6	2	32.9	50.16	487	61.5	6.3	1.54	4	68.2	45.9	3.49	415	70
1	12	6	3	32.8	50.16	487	61.5	12.6	5.3	16	63.6	92.3	6.16	567	64.3
1	24	6	1	428.1	62.08	25223	40.2	5.4	0.71	67	64	176.4	10.07	7102	72
1	24	6	2	428.7	62.08	25223	40.2	33	7.27	251	48.2	2147.1	11.65	24412	50.9
1	24	6	3	428.8	62.08	25223	40.2	636.4	24.32	16521	42.8	2281.5	26.45	17749	44
1	24	12	1	26.8	61.98	164	37.6	1	0.04	1	62.3	20.1	5.27	178	67.1
1	24	12	2	26.6	61.98	164	37.6	14.5	5.44	57	46.2	131.1	8.18	3166	48.1
1	24	12	3	26.7	61.98	164	37.6	44.9	21.51	827	40.6	170.9	22.88	4545	41.3
2	12	6	1	4.5	33.39	1	66.7	0.5	0.1	1	80.1	3.8	1.62	1	82.5
2	12	6	2	4.4	33.39	1	66.7	2.6	3.31	1	68.6	11.2	4.58	45	69.6
2	12	6	3	4.4	33.39	1	66.7	9	4.51	166	55	118.2	7.94	3378	51
2	24	6	1	7.9	47.11	15	46.3	1.9	0.21	1	68	15.3	4.05	1556	72
2	24	6	2	7.9	47.11	15	46.3	9	4.51	166	55	118.2	7.94	3378	51
2	24	6	3	7.9	47.11	15	46.3	20	14.87	158	49.4	132.5	17.41	3378	51
2	24	12	2	3.7	47.48	1	45.2	7.3	4.22	24	53.2	29.2	6.35	1001	54.6
2	24	12	3	3.7	47.48	1	45.2	8.7	14.56	82	48.2	29.2	15.97	566	49
3	12	6	1	0.1	0	1	72	0	0.1	1	83.6	0.1	1.83	1	85.6
3	12	6	2	0.1	0	1	72	0.1	0.05	1	75.1	1.4	1.25	1	76.1
3	12	6	3	0.1	0	1	72	0.1	0.03	1	72.4	0.1	0.19	1	72.5
3	24	6	1	0.1	0	1	57.8	0.2	0.05	1	71.6	1.7	4.08	59	75.2
3	24	6	2	0.1	0	1	57.8	0.1	0.27	1	60.2	3.1	20.6	36	61.5
3	24	6	3	0.1	0	1	57.8	0.1	0.16	1	58.2	0.2	0.52	1	58.4
3	24	12	1	0.1	0	1	57.2	0.1	0	1	71.6	0.1	2.14	1	73.6
3	24	12	2	0.1	0	1	57.2	0.2	1	1	59.1	0.1	0.97	1	59.1
3	24	12	3	0.1	0	1	57.2	0.1	0.2	1	57.4	0.1	0.31	1	57.5
4	12	6	1	3.3	35.24	2	72	0.3	0	1	84.1	2.5	0.89	1	85.5
4	12	6	2	3.3	35.24	2	72	1.3	1.56	1	77	9.9	2.17	1	77.6
4	12	6	3	3.3	35.24	2	72	1.6	5.5	1	73.7	2.1	5.52	1	73.7
4	24	6	1	2.9	50.4	1	49.5	1.1	0.13	1	72.3	7.5	2.5	329	74.8
4	24	6	2	3	50.4	1	49.5	7.2	5.82	20	251.6	8.21	19754	62.1	
4	24	6	3	3	50.4	1	49.5	11.7	15.91	189	54.2	86.5	17.74	5034	55.5
4	24	12	1	1.8	50.92	1	49	0.3	0	1	72.1	2.3	1.19	1	73.3
4	24	12	2	1.8	50.92	1	49	4.4	5.24	18	59	14.1	6.78	407	60.1
4	24	12	3	1.8	50.92	1	49	6.1	13.74	33	52.2	12	14.85	58	53
5	12	6	1	0.5	6.4	1	75.7	0.3	0.1	1	84.7	0.7	0.62	1	85.3
5	12	6	2	0.5	6.4	1	75.7	0.2	0.19	1	76.9	0.4	0.57	1	77.3
5	12	6	3	0.5	6.4	1	75.7	0.3	1.06	1	75.7	0.4	1.05	1	75.7
5	24	6	1	0.9	9.01	1	61.8	0.1	0	1	72.6	0.8	0.58	1	73.1
5	24	6	2	0.9	9.01	1	61.8	0.3	0.79	1	63	1	1.51	1	63.5
5	24	6	3	0.9	9.01	1	61.8	0.7	2.96	1	62	1.2	2.93	1	62
5	24	12	1	0.5	9.23	1	61.4	0.1	0	1	72.6	0.3	0.41	1	72.9
5	24	12	2	0.5	9.23	1	61.4	0.2	0.82	1	62.2	0.5	1.1	1	62.5
5	24	12	3	0.5	9.23	1	61.4	0.3	2.71	1	61.4	0.5	2.7	1	61.4
6	12	6	1	5.7	33.94	1	70.4	0.3	0	1	82.5	4.7	2.29	1	84.9
6</td															

Índice

② Multi-objective integration of timetables, vehicle schedules and user routings in a transit network

Introduction

Background

Problem description

Formulation

An ϵ -constraint algorithm

Computational experiments

Conclusions

Conclusions

- ▶ We develop a framework for integrating the TNTP, the VSP and passenger routings.
- ▶ We provide a TNTSP formulation starting from the TNTP and adding constraints regarding to capacities, optimal passenger assignment and fleet size.
- ▶ We compute the set of itineraries and options available for each transportation request considering hard time windows, constraints for trip duration, departure and arrival times, as well as inconvenience costs related to deviations from these times.
- ▶ Formulations are tested and compared on a testbed of random instances and on different networks as similarly proposed in previous studies in the literature.

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- ➊ Introducción
- ➋ Multi-objective integration of timetables, vehicle schedules and user routings in a transit network
- ➌ New improvements for the Stackelberg Minimum Spanning Tree Game

New improvements for the Stackelberg Minimum Spanning Tree Game

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Reference

Main details available at:

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Índice

① New improvements for the Stackelberg Minimum Spanning Tree Game

Preliminary

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StackMST formulations

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Spanning Tree Problems (STPs) and formulations

- ▶ STPs are among the core problems in combinatorial optimization.
- ▶ The combinatorial object that represents spanning trees has important structural properties and applications in many fields. Furthermore, they often appear as subproblems of other more complex optimization problems.
- ▶ There exist several representations of the STP polytope.

Spanning Tree Problems (STPs) and formulations

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- ▶ There exist several representations of the STP polytope.

Formulation	notation	main constraints	root	# vars	# const.	int
Subtour Edmonds (1970)	\mathcal{T}^{sub}	$\sum_{e \in E(S)} x_e \leq S - 1, \emptyset \neq S \subset V$		$O(E)$	$Exp(n)$	Yes
Kipp Martin Martin (1991)	\mathcal{T}^{km}	$\sum_{(u,v) \in \delta^+(u)} q_{kuv} \leq \begin{cases} 1, & k \in V, u \in V : u \neq k \\ 0, & k \in V, u = k \end{cases}$	$\forall k$	$O(n E)$	$O(n E)$	Yes
Miller-Tucker-Zemlin Miller et al. (1960)	\mathcal{T}^{mtz}	$l_v \geq l_u + 1 - n(1 - y_{uv}), (u, v) \in A$	r	$O(E)$	$O(E)$	No
Flow Gavish (1983)	\mathcal{T}^{flow}	$\sum_{(u,v) \in \delta^+(u)} \varphi_{uv} - \sum_{(v,u) \in \delta^-(u)} \varphi_{vu} = \begin{cases} n-1, & u = r \\ -1, & u \in V \setminus \{r\} \end{cases}$	r	$O(E)$	$O(E)$	No
KM extended Fernández et al. (2017)	\mathcal{T}^{km2}	$\sum_{(u,v) \in \delta^+(u)} q_{uv} \leq \begin{cases} 1, & u \in V : u \neq r \\ 0, & u = r \end{cases}$	r	$O(E)$	$Exp(n)$	Yes

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Property: Let $P(\mathcal{T}^{(\cdot)})$ denote the polyhedron associated with the linear programming relaxation of formulation $\mathcal{T}^{(\cdot)}$ and $P_x(\mathcal{T}^{(\cdot)})$ the projected polyhedron associated with formulation $\mathcal{T}^{(\cdot)}$. Then

$$P_x(\mathcal{T}^{sub}) = P_x(\mathcal{T}^{km}) = P_x(\mathcal{T}^{km2}) \subseteq \begin{cases} P_x(\mathcal{T}^{mtz}) \\ \neq P_x(\mathcal{T}^{flow}) \end{cases}$$

Spanning Tree Problems (STPs) and formulations

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$$\min \sum_{e \in E} c_e x_e \quad \text{Kipp Martin MSTP formulation (Martin, 1991)} \quad (7a)$$

$$\sum_{e \in E} x_e = n - 1 \quad (7b)$$

$$\sum_{s \in V : (k,s) \in A} q_{kks} \leq 0 \quad k \in V \quad (7c)$$

$$\sum_{v \in V : (u,v) \in A} q_{kuv} \leq 1 \quad k, u \in V : u \neq k \quad (7d)$$

$$q_{kuv} + q_{kvu} = x_{uv} \quad k \in V, (u, v) \in E \quad (7e)$$

$$x_{uv} \geq 0 \quad (u, v) \in E \quad (7f)$$

$$q_{kuv} \geq 0 \quad k \in V, (u, v) \in A \quad (7g)$$

STP as a subproblem: OWA and OWASTP

Definition (*Ordered Weighted Average (OWA) operator*): Given

- ▶ $Q \subseteq \mathbb{Z}^n$: a combinatorial object (feasible set),
- ▶ p linear objective functions. ($P = \{1, \dots, p\}$)
- ▶ C^i : coefficients of i -th objective function. $C \in \mathbb{R}^{p \times n}$.
- ▶ $y = Cx \in \mathbb{R}^p$: obj. funct. values for $x \in Q$. $y = (y_1, \dots, y_p) \in \mathbb{R}^p$.
- ▶ σ : permutation of indices of P such that $y_{\sigma_1} \geq \dots \geq y_{\sigma_p}$.
- ▶ $\omega \in \mathbb{R}^p$ weights vector.

the **OWA operator** is defined as

$$OWA_{(C, \omega)}(x) = \omega' y_\sigma$$

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the **OWA operator** is defined as

$$OWA_{(C, \omega)}(x) = \omega' y_\sigma$$

Definition (OWA Problem (OWAP)): The **OWA optimization problem (OWAP)** is to find

$$\min_{x \in Q} OWA_{(C, \omega)}(x).$$

See: Fernández, E.; Pozo, M.A. & Puerto, J. (2014). A modeling framework for Ordered Weighted Average Combinatorial Optimization. Discrete Applied Mathematics

STP as a subproblem: OWA and OWASTP

Definition (*Ordered Weighted Average (OWA) operator*): Given

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- ▶ $\omega \in \mathbb{R}^p$ weights vector.

the **OWA operator** is defined as

$$OWA_{(C,\omega)}(x) = \omega' y_\sigma$$

Definition (*Ordered Weighted Average Spanning Tree Problem (OWASTP)*): Let \mathcal{T} denote the set of spanning trees defined on G . Then,

$$\text{OWASTP: } \min_{x \in \mathcal{T}} OWA_{(C,\omega)}(x).$$

Galand, L. & Spanjaard, O. (2012). *Exact algorithms for OWA-optimization in multiobjective spanning tree problems*. Computers & OR

Fernández, E.; Pozo, M.A. & Puerto, J. (2017). *Ordered Weighted Average Optimization in multiobjective spanning tree problems*. EJOR

STP as a subproblem: Ordered Median Tree of Hubs Location Problem

$$|V| = 10 \quad p = 5$$

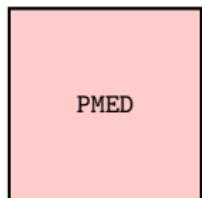
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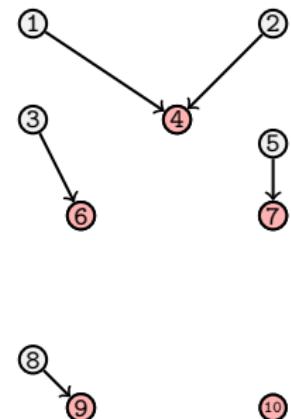
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STP as a subproblem: Ordered Median Tree of Hubs Location Problem



$$|V| = 10$$

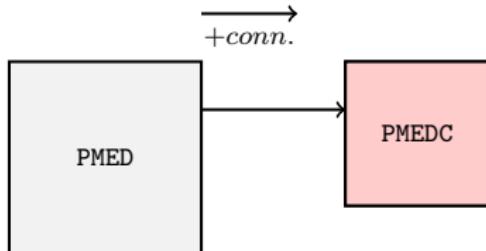
$$p = 5$$



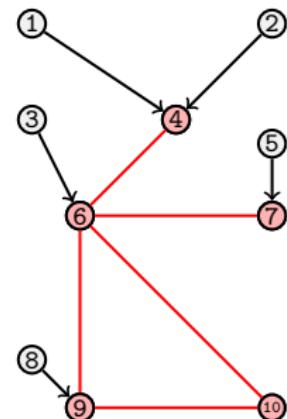
PMED: p -median location problem: Find the location of p facilities and allocations that minimizes the sum of the weighted access cost of each node to a facility.

See: [Hakimi \(1964\)](#)

STP as a subproblem: Ordered Median Tree of Hubs Location Problem

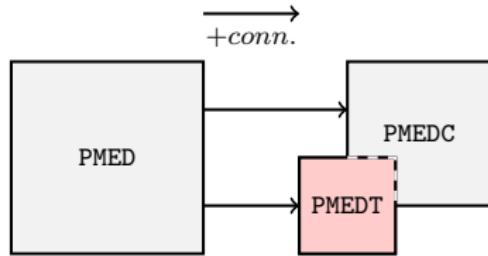


$$|V| = 10 \quad p = 5$$

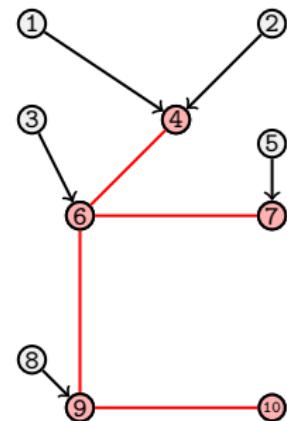


PMEDC: p-median location problem with inner Connected structure: Solve the p-median problem connecting facilities with a network.

STP as a subproblem: Ordered Median Tree of Hubs Location Problem

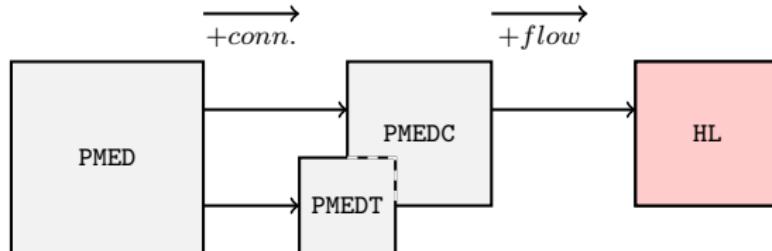


$$|V| = 10 \quad p = 5$$

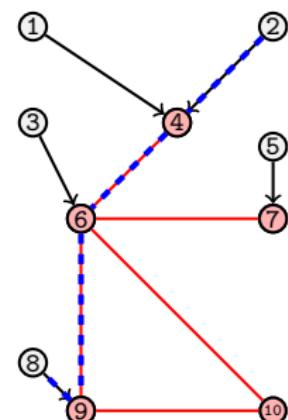


PMEDT: p-median location problem with inner Tree structure: Solve the p-median problem connecting facilities with a tree.

STP as a subproblem: Ordered Median Tree of Hubs Location Problem



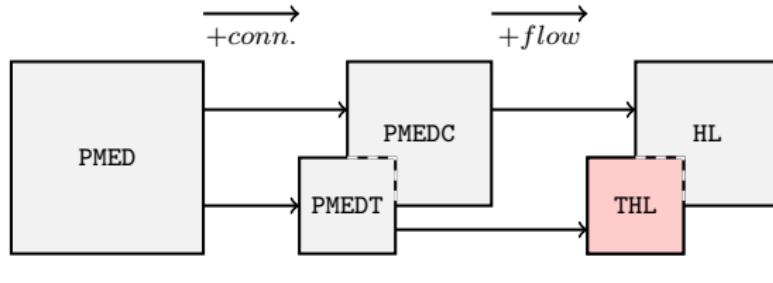
$$|V| = 10 \quad p = 5$$



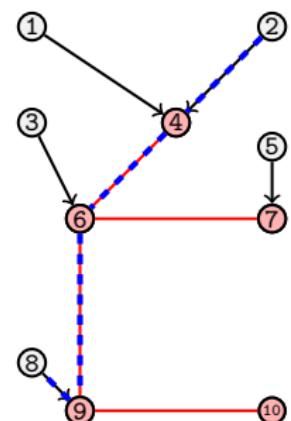
HL: Network Hub Location Problem: Find the location of hub facilities and spokes allocations that minimizes the sum of the weighted shortest paths between all vertex pairs, subject to a budget constraint for the inter-hub connected structure.

See: [O'Kelly \(1986\)](#)

STP as a subproblem: Ordered Median Tree of Hubs Location Problem



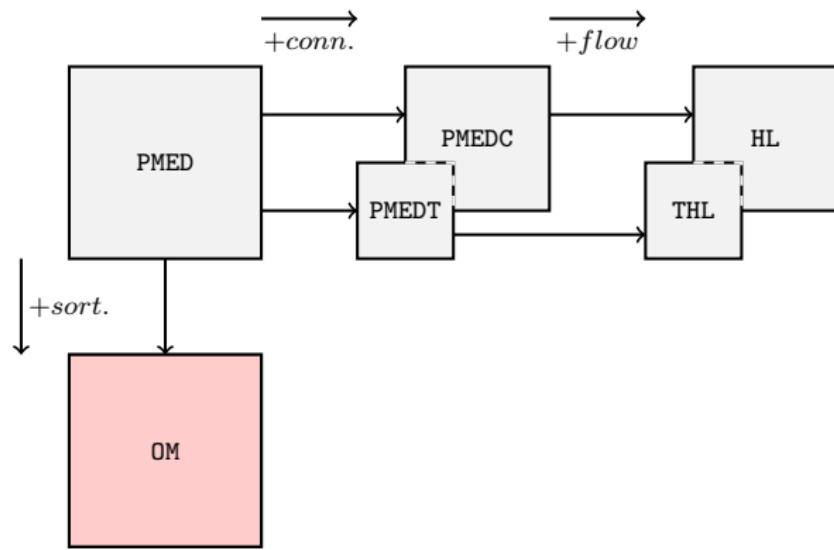
$$|V| = 10 \quad p = 5$$



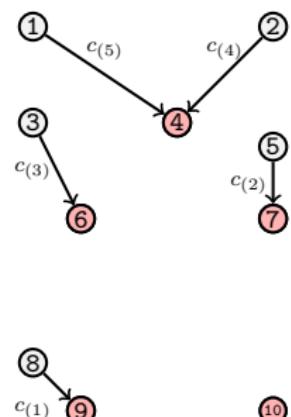
THL: Tree of Hubs Location problem: Find the location of hub facilities and spokes allocations that minimizes the sum of the weighted shortest paths between all vertex pairs, imposing that the inter-hub structure is a tree.

See: [Contreras, I., Fernández, E., Marín, A. \(2010\)](#); [Contreras, I., Fernández, E., Marín, A. \(2009\)](#); [Martins, E., Saraiva, R., Miranda, G. \(2013\)](#)

STP as a subproblem: Ordered Median Tree of Hubs Location Problem



$$|V| = 10 \quad p = 5$$

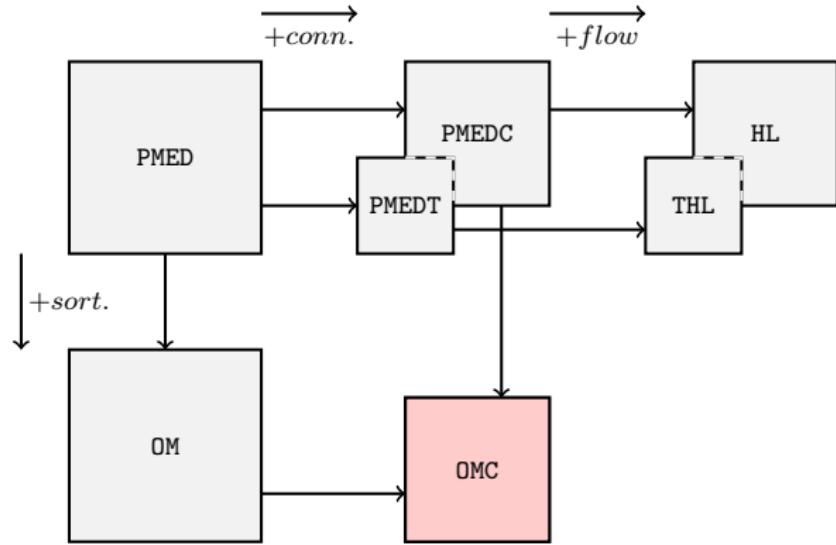


For $c_{(1)} \leq c_{(2)} \leq \dots \leq c_{(|V|)}$ and $\lambda_1 \geq \dots \geq \lambda_{|V|}$: $c_{(1)} \leftarrow \lambda_1, \dots, c_{(|V|)} \leftarrow \lambda_{|V|}$.

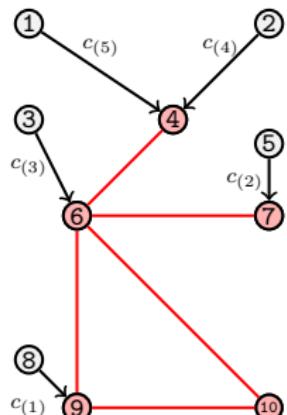
OM: Ordered Median location problem: Find the location of p facilities and allocations that minimizes the sum of the **sorted weighted access cost** of each node to a facility.

See: [Nickel, S. and Puerto, J.\(2005\)](#)

STP as a subproblem: Ordered Median Tree of Hubs Location Problem

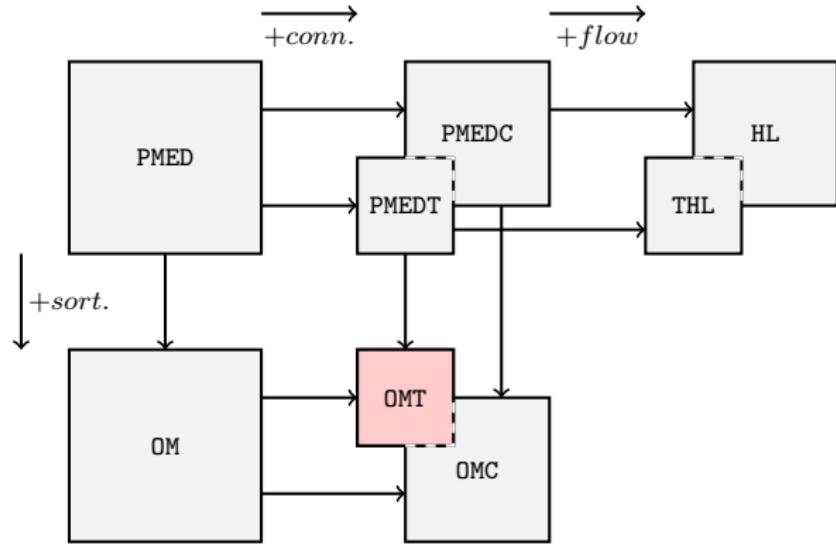


$$|V| = 10 \quad p = 5$$

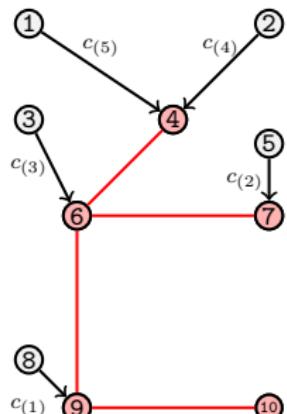


OMC: Ordered Median Location Problem with inner Connected structure: Solve the OM problem connecting facilities with a network.

STP as a subproblem: Ordered Median Tree of Hubs Location Problem

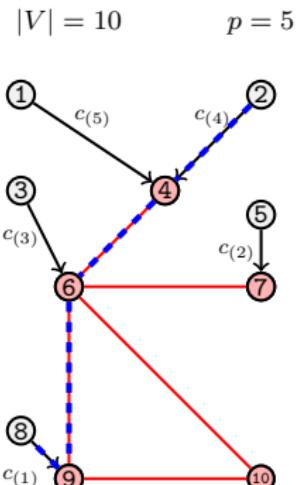
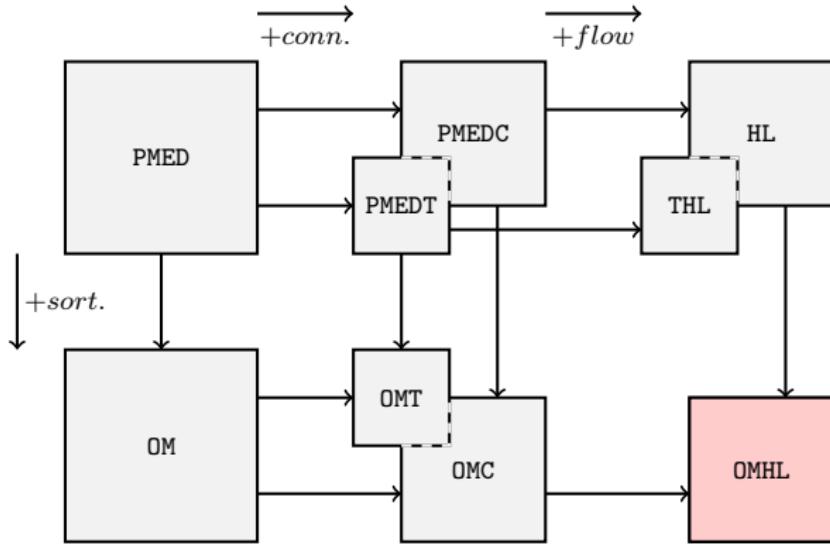


$$|V| = 10 \quad p = 5$$



OMT: Ordered Median Location Problem with inner Tree structure: Solve the OM problem connecting facilities with a tree.

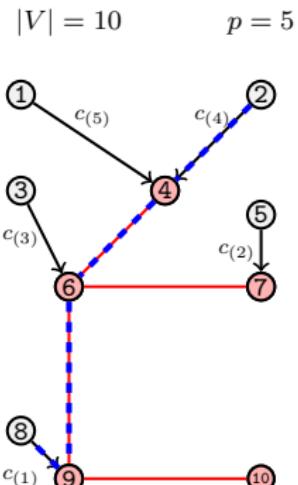
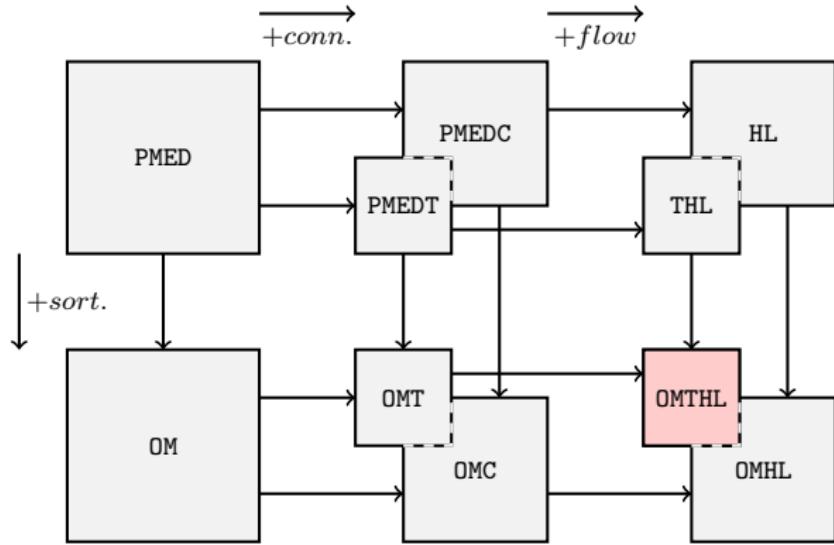
STP as a subproblem: Ordered Median Tree of Hubs Location Problem



OMHL: Ordered Median Hub Location problem: Network hub location problem with single assignment where a fixed number of hubs have to be located and connected.

See: Puerto, J., Ramos, A. B., Rodriguez-Chia, A.M. (2011); Puerto, J., Ramos, A. B., Rodriguez-Chia, A.M.(2013); Puerto, J., Ramos, A. B., Rodriguez-Chia, A.M., Sánchez-Gil, M.C. (2013)

STP as a subproblem: Ordered Median Tree of Hubs Location Problem



OMTHL: Ordered Median Tree of Hubs Location problem: Network hub location problem with single assignment where a fixed number of hubs have to be located and connected by means of a non-directed tree and costs are ordered weighted averaged.

See: *Pozo, M.A., Puerto, J., Rodriguez-Chia, A.M. (2021)*

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① New improvements for the Stackelberg Minimum Spanning Tree Game

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Bilevel Optimization and Stackelberg game

Bilevel optimization problem:

$$\begin{aligned} \text{opt} \quad & F_u(x_u, x_\ell) \\ (x_u, x_\ell) \in X_u \quad & \\ \text{s.t.} \quad & x_\ell \in \arg \underset{x_\ell}{\text{opt}} \{F_\ell(x_u, x_\ell) : (x_u, x_\ell) \in X_\ell\} \end{aligned}$$

A bilevel optimization problem is that involving two types of decision vectors x_u, x_ℓ called upper and lower level variables where the lower level variables belong to the set of optimal solutions of another optimization problem that is parametric in the upper-level variables. Here we denote F_u and F_ℓ the upper and lower level objective functions respectively and we denote X_u and X_ℓ the upper and lower level feasible solution sets respectively.

Stackelberg game: In game theory, a bilevel optimization problem is known under the name of Stackelberg game (von Stackelberg, 1934) and it consists in a leader and a follower who play sequentially. Those players compete with each other: the leader makes the first move, and then the follower reacts optimally to the leader's action. The leader knows ex ante that the follower observes its actions before responding in an optimal manner. Therefore, to optimize its objective, the leader anticipates the optimal response of the follower.

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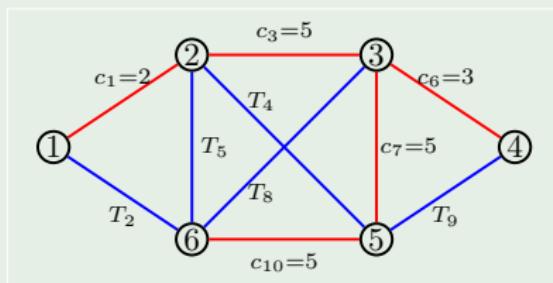
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Introduction: StackMST example

Example

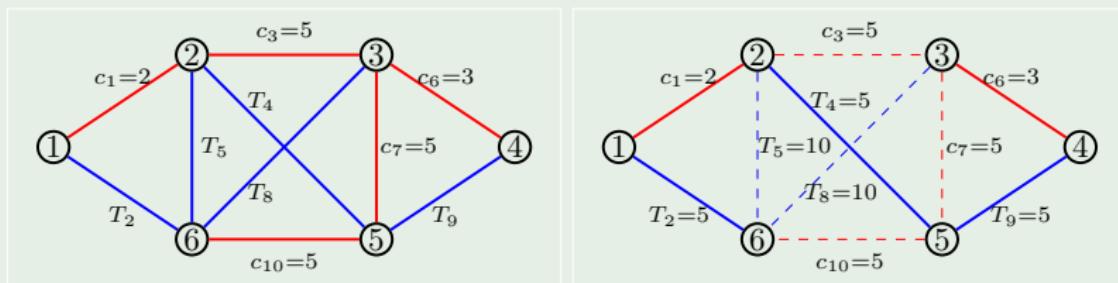
Let G be the graph depicted in Figure (left) where red edges provide a spanning tree of total cost 20.



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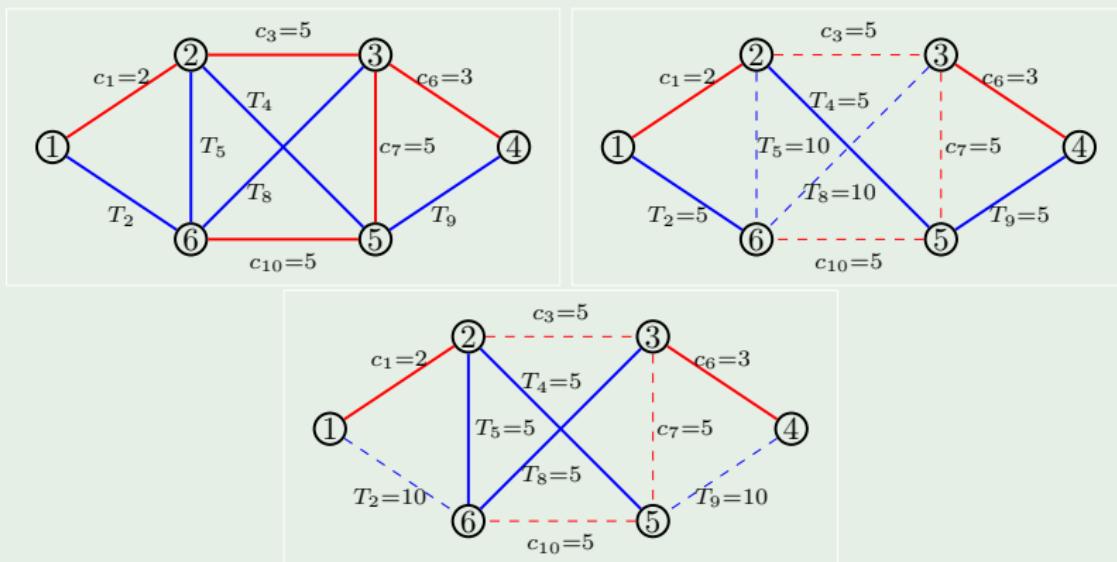


Blue edges can be priced in order to provide a StackMST solution of value 20 and a revenue of 15.

Introduction: StackMST example

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Let G be the graph depicted in Figure (left) where red edges provide a spanning tree of total cost 20.



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Problem definition

- ▶ Let be given a graph $G = (V, E)$ whose edge set is partitioned into a set B of blue edges (controlled by the leader) and a set R of red edges, and assume that red edges are weighted and contain at least one spanning tree of G .
- ▶ A positive cost c_e is associated to each red edge $e \in R$ and a positive price T_e has to be determined for each blue edge $e \in B$
- ▶ We denote by x the design variables used to describe the STP polytope \mathcal{T} .

$$\text{(StackMST)} \quad F^0 : \max_{T \geq 0} \sum_{e \in B} T_e x_e \tag{8a}$$

$$s.t. \quad x = \operatorname{argmin}_{x \in \mathcal{T}} \left\{ \sum_{e \in B} T_e x_e + \sum_{e \in R} c_e x_e \right\}. \tag{8b}$$

Basic results

Property: In every optimal StackMST solution $T_e \in C^R = \{c_{e'} : e' \in R\}$ for each $e \in B$.

Let $\mathcal{C}(e, S)$ be the set of cycles of G that include edge e and edges of the set $S \subseteq E \setminus \{e\}$

Property (MSTP optimality condition): A spanning tree \mathcal{T}^* is an optimal MSTP solution if for each edge $e \notin \mathcal{T}^*$, each edge $e' \in \mathcal{T}^*$ and in the cycle that contains e has a cost less or equal than c_e :

\mathcal{T}^* is an opt. MSTP solution $\Leftrightarrow c_{e'} \leq c_e, \forall e \in E : e \notin \mathcal{T}^*, e' \in \mathcal{C}(e, \mathcal{T}^*) : e' \neq e$

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Second, we observe (see Cardinal et al., 2011) that the cost of each blue edge belonging to an optimal solution is bounded from above by the minimum among the maximum of the red costs of each cycle that contain the blue edge.

Property (Strong necessary condition for an optimal StackMST solution): If T^* is an optimal StackMST solution and \mathcal{T}^* the associated tree, then

$$T_e \leq \min_{\Theta \in \mathcal{C}(e, E)} \max_{e' \in \Theta \cap R} c_{e'}, \quad e \in B \cap \mathcal{T}^*$$

Property (Weak necessary condition for an optimal StackMST solution): If T is an optimal StackMST solution then

$$m_e = \min_{\Theta \in \mathcal{C}(e, R), e' \in \Theta} c_{e'} \leq T_e \leq \max_{\Theta \in \mathcal{C}(e, R), e' \in \Theta} c_{e'} = M_e \quad e \in B$$

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Basic results (cont.)

Second, we observe (see [Cardinal et al., 2011](#)) that the cost of each blue edge belonging to an optimal solution is bounded from above by the minimum among the maximum of the red costs of each cycle that contain the blue edge.

Property (Strong necessary condition for an optimal StackMST solution): If T^* is an optimal StackMST solution and \mathcal{T}^* the associated tree, then

$$T_e \leq \min_{\Theta \in \mathcal{C}(e, E)} \max_{e' \in \Theta \cap R} c_{e'}, \quad e \in B \cap \mathcal{T}^*$$

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Basic results (cont.): The Best-Out-Of- k algorithm

Property (The Best-Out-Of- k algorithm): Let $c^1 < \dots < c^{|K|}$ be the $|K|$ different edge costs that appear in the initial red set of edges, where $k \in K$ is the index of the $k - th$ cost and $C^R = \{c^1 < \dots < c^{|K|}\}$ is the set of costs. The Best-Out-Of- k algorithm consists in choosing the best cost c^k to be assigned to all T values. Note that the T values returned by this algorithm can be expressed in the following way:

$$T = (c^k)_{1 \times |B|} / k = \arg \max_{k \in K} \left\{ \sum_{e \in B} c^k x_e : x = \operatorname{argmin}_{x \in \mathcal{T}} \left\{ \sum_{e \in B} c^k x_e + \sum_{e \in R} c_e x_e \right\} \right\}$$

Theorem: (Cardinal et al., 2011)

The Best-out-of- k is a $\min\{k, 1 + \ln b, 1 + \ln W\}$ -approximation algorithm, where b denotes the number of blue edges, and $W = c^k/c^1$ is the maximum ratio between red costs.

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A general framework for providing StackMST feasible solutions

Algorithm 2: StackMST-H algorithm

input :

- ▶ sol_{best} : Current best solution (by default $T_e = c|K|$, $\forall e \in B$).
- ▶ b : Number of blue edges to modify (by default $b = |B|$)
- ▶ S : Set of edges that has been chosen in previous iterations (by default $S = \emptyset$)
- ▶ p_1 : probability of choosing edges from B or from $B \setminus S$ (by default $p_1 = 0$)
- ▶ p_2 : probability of choosing a direction of movement where "moving up" is chosen with probability p_2 and "moving down" with probability $1 - p_2$ (by default $p_2 = 0$)
- ▶ $STOP_c$: stopping condition (by default "repeat $|K|$ times")

output: sol_{best} : Current best solution.

```

1 while  $STOP_c = \text{false}$  do
2   According to  $p_1$ , a subset  $B_S$  of  $b$  blue edges is chosen from  $B \cup S$  or from  $B \setminus S$ .
3    $S \leftarrow S \cup B_S$ .
4   Edges  $e \in B_S$  verifying  $T_e > M_e$  or  $T_e < m_e$  are removed from  $B_S$ .
5   According to  $p_2$ , for each  $e \in B_S$  increase/decrease by one unit  $k$  in  $c_e^k$ .
6   Evaluate the StackMST solution updating  $c_e^k$  for all  $e \in B_S$ .
7   if  $sol_{best}$  is outperformed then
8     Update  $sol_{best}$ 
9   else
10    reset values  $c_e^k$  for all  $e \in B_S$ 
```

Property (StackMST-H algorithm and the Best-Out-Of-k algorithm): The Best-Out-Of- k algorithm is equivalent to StackMST-H for the following set of input parameters: $b = |B|$, $p_1 = 1$, $p_2 = 0$, $sol_{best} = \emptyset$, $STOP_c$ = "repeat $|K|$ times".

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Non-linear formulation

$$\text{(StackMST)} \quad F^0 : \quad \max_{T \geq 0} \sum_{e \in B} T_e x_e \quad (10a)$$

$$s.t. \quad x = \operatorname{argmin}_{x \in \mathcal{T}} \left\{ \sum_{e \in B} T_e x_e + \sum_{e \in R} c_e x_e \right\} \quad (10b)$$

Primal-dual non-linear formulation

Let $\min_{x \geq 0} \{cx : x \in \mathcal{T}\}$ be a continuous linear MSTP formulation and $\max_{\mu \geq 0} \{d\mu : \mu \in \mathcal{T}^D(c)\}$ its dual form. We also denote by $\mathcal{T}^D(c, T)$ the polytope resulting by replacing c_e by T_e for each $e \in B$ of \mathcal{T}^D .

Therefore, we can express the StackMST as

$$F^0 : \quad \max_{T \geq 0} \sum_{e \in B} T_e x_e \quad (11a)$$

$$s.t. \quad x \in \mathcal{T} \quad (11b)$$

$$\mu \in \mathcal{T}^D(c, T) \quad (11c)$$

$$\sum_{e \in B} T_e x_e + \sum_{e \in R} c_e x_e = d\mu. \quad (11d)$$

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Primal-dual linear formulation

$$F_p : \max_{T \geq 0} \sum_{e \in B} p_e \quad (12a)$$

$$x \in \mathcal{T} \quad (12b)$$

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$$m_e x_e \leq p_e \leq M_e x_e \quad e \in B \quad (12e)$$

$$p_e \leq T_e \quad e \in B \quad (12f)$$

$$T_e \leq p_e + M_e(1 - x_e) \quad e \in B \quad (12g)$$

$$m_e \leq T_e \leq M_e \quad e \in B \quad (12h)$$

$$x_e \in \{0, 1\} \quad e \in E \quad (12i)$$

P-d linear formulation with discrete values of T and p

Let $\{c^1, \dots, c^{|K|}\}$ be the set made up of the $|K|$ different edge costs that appear in the initial red tree. If $k \in K$ is the index of the $k - th$ cost and $K = \{1, \dots, |K|\}$ we can also specify the set K for each edge as $K_e = \{k \in K : m_e \leq c^k \leq M_e\}$.

Let z_e^k be a binary variable equal to one \Leftrightarrow edge e is priced with the $k - th$ cost.

$$T_e = \sum_{k \in K_e} z_e^k c^k \quad e \in B, \quad (13)$$

assuming that each edge is priced with just one of the costs, that is

$$\sum_{k \in K_e} z_e^k = 1 \quad e \in B. \quad (14)$$

Analogously, the values of p can be also discretized by means of using the binary variable \bar{z}_e^k equal to one \Leftrightarrow edge e gives the benefit of the $k - th$ cost.

$$p_e = \sum_{k \in K_e} \bar{z}_e^k c^k, \quad e \in B. \quad (15)$$

Additionally we would need to impose that

$$\sum_{k \in K_e} \bar{z}_e^k = x_e, \quad e \in B \quad (16)$$

$$\bar{z}_e^k \leq z_e^k, \quad e \in B, k \in K_e \quad (17)$$

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P-d linear formulation with discrete values of T and p

$$F_z : \max \sum_{e \in B} \sum_{k \in K_e} \bar{z}_e^k c^k \quad (18a)$$

$$x \in \mathcal{T} \quad (18b)$$

$$\mu \in \mathcal{T}^D(c, z) \quad (18c)$$

$$\sum_{e \in B} \sum_{k \in K_e} \bar{z}_e^k c^k + \sum_{e \in R} c_e x_e = d\mu \quad (18d)$$

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$$\sum_{k \in K_e} \bar{z}_e^k = x_e, \quad e \in B \quad (18f)$$

$$\bar{z}_e^k \leq z_e^k, \quad e \in B, k \in K_e \quad (18g)$$

$$z_e^k \leq \bar{z}_e^k + (1 - x_e), \quad e \in B, k \in K_e \quad (18h)$$

$$z_e^k, \bar{z}_e^k \in \{0, 1\} \quad e \in E, k \in K_e \quad (18i)$$

$$x_e \in \{0, 1\} \quad e \in E \quad (18j)$$

F_z and F_p comparison

Property: Let $\Omega_{LR}^{p,T;x}$ be the projection of the polytope defined by constraints (12b)–(12h) over the x variables and $\Omega_{LR}^{z,\bar{z};x}$ the projection of the polytope given by (18b)–(18h) over the x variables. Then $\Omega_{LR}^{z,\bar{z};x} \subseteq \Omega_{LR}^{p,T;x}$.

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Computational experience

- ▶ Instances $G = (V, E)$ are generated according to those described in [Morais et al., 2016](#). We choose $|V| \in \{20, 30, 50, 70\}$, $c_{max} = 150$, $d \in \{10\%, 20\%, 30\%, 50\%\}$ and $|C| = \{3, 5, 7\}$.
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- ▶ All instances were solved with the MIP Xpress 7.7 optimizer, under a Windows 10 environment in an Intel(R) Core(TM)i7 CPU 2.93 GHz processor and 16 GB RAM. Default values were initially used for all parameters of Xpress solver and a CPU time limit of 1800 seconds was set. We have also tested different combinations of parameters for the solver cut strategy and intensity of heuristics but, unless it is specified, the best results were obtained with the parameters of the solver set to the default values.
- ▶ The caption just below each block gives the formulation the block refers to. Throughout the section we denote by $F_p^{(\cdot)}$ the combination of the BMSTP F_p formulation together with a STP $\mathcal{T}^{(\cdot)}$ (idem with $F_z^{(\cdot)}$).
- ▶ The separation of the cutset inequalities in formulation \mathcal{T}^{km2} was implemented using a max-flow based algorithm ([Gusfield, 1990](#)). Heuristics in Xpress solver were configured with intensity 2 (out of 3) and an initial solution was given to the problem.

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Table: StackMST results for the $F_p^{(.)}$ formulations.

$ V $	d	$ C $	gRL	gUL	$g\bar{U}L$	gUL	gUL^*	#	nod	$gR\bar{L}$	$g\bar{U}\bar{L}$	$g\bar{U}L$	gUL	gUL^*	#	nod	$g\bar{R}\bar{L}$	$g\bar{U}\bar{L}$	$g\bar{U}L$	gUL	gUL^*	#	nod							
20	30	3	13.5	-	-	-	-	10	1e3	3.9	-	-	-	-	10	76	4.8	-	-	-	-	10	1e2							
20	30	5	18.2	-	-	-	-	10	4e4	9.5	-	-	-	-	10	8e3	9.5	-	-	-	-	10	2e4							
20	30	7	17.4	-	-	-	-	10	1e4	8.5	-	-	-	-	10	5e3	9.2	-	-	-	-	10	1e4							
20	50	3	4.8	1.6	-	1.6	8.8	7	2e5	3.4	1.4	-	1.4	8.6	8	5e4	3.4	1.8	-	1.8	8.1	6	1e5	3.4	0.8	-	0.8	7.8	9	1e5
20	50	5	7.1	5.8	-	5.8	12	0	4e5	7.1	5.2	-	5.2	10.6	1	2e5	7.1	5.2	-	5.2	11.1	1	3e5	7.1	5.2	-	5.2	11.4	1	4e5
20	50	7	8.9	7.8	1.7	7.9	16	1	3e5	8.9	7.9	1.6	7.9	16	1	2e5	8.9	8	1.7	8	16	1	3e5	8.9	8	1.7	8	16	1	4e5
30	30	3	6.9	3.4	0.3	3.4	14.5	6	1e4	4.4	3.3	0.3	3.3	14.8	5	1e4	4.4	3.2	0.3	3.2	15	5	2e4	4.4	2.8	0.3	2.8	13.3	6	2e4
30	30	5	9.8	6	3.1	6.1	14.1	2	3e4	7.2	6	2.9	6	14	2	2e4	7.2	6	2.9	6	14	1	5e4	7.2	6.4	2.9	6.4	14	1	6e4
30	30	7	13.4	9.4	6.8	9.5	16.2	0	6e4	10.1	9.4	6.8	9.4	15.2	0	2e4	10.1	9.4	6.9	9.5	15.2	0	5e4	10.1	9	6.8	9	13.3	0	7e4
30	50	3	0.4	0.2	-	0.2	1.6	9	3e3	0.2	0.2	-	0.2	1.6	9	1e3	0.2	0.2	-	0.2	1.6	9	4e3	0.2	0.2	-	0.2	1.6	9	4e3
30	50	5	4.1	3.8	0.8	4.1	10.3	1	3e4	3.8	3.8	0.5	3.8	7.4	1	1e4	3.8	3.8	0.5	3.8	7.6	1	3e4	3.8	3.8	0.5	3.8	7.4	1	5e4
30	50	7	5.9	5.7	4.4	6.5	21	0	3e4	5.7	5.7	4.3	6.3	19.5	0	1e4	5.7	5.7	4.5	6.5	21	0	3e4	5.7	5.7	4.2	6.2	18.5	0	6e4
50	10	3	16.2	4.1	1	4.4	13	2	1e4	3.7	1.9	0.6	1.9	10.9	7	2e3	4.4	2	0.6	2	10.4	7	2e3	4.8	1.8	0.6	1.8	9.6	6	5e3
50	10	5	19.3	5.3	1.5	6.3	9.7	1	1e4	5.5	2.4	0.5	2.5	7	5	3e3	6.3	2.9	0.5	2.9	7.2	4	7e3	6.8	2.3	0.6	2.4	7.3	5	1e4
50	10	7	22.5	6.3	2.8	7.4	18.2	2	1e4	8	3.7	1.9	4	14.9	5	2e3	8.6	4.7	1.7	4.8	13.2	4	7e3	8.6	3.4	1.6	3.4	10.5	5	9e3
50	20	3	4.1	2.6	0.6	2.6	6.6	1	5e3	2.8	2.4	0.6	2.4	6.6	2	1e3	2.8	2.5	0.6	2.5	6.6	1	4e3	3	2.7	0.6	2.7	6.6	0	8e3
50	20	5	9.4	8.2	14.4	14.8	25.1	0	5e3	8.2	8.2	9.7	10.2	17.3	0	1e3	8.2	8.2	9.9	10.3	16.6	0	5e3	8.2	8.2	8.7	9.1	14.5	0	8e3
50	20	7	13.8	12.1	20.1	20.4	30.8	0	5e3	12.1	12.1	15.7	16	25.9	0	1e3	12.1	12.1	15.5	15.8	24.1	0	5e3	12.1	12.1	14.3	14.7	26.6	0	9e3
70	10	3	16.3	7.6	7.3	11.2	21.3	0	3e3	7.3	6.8	6	9.2	19.2	1	5e2	7.5	6.5	6.4	9.3	19.1	1	9e2	7.5	6.7	6	9.1	19.5	1	1e3
70	10	5	18.7	13.1	15.7	16.4	26.6	0	9e2	13	12.4	15.8	16	26.7	0	1e2	13.1	12.8	15.4	15.9	27.3	0	8e2	13.1	12.5	14.4	14.6	23.7	0	9e2
70	10	7	21	14.8	22.7	23.4	33.2	0	9e2	15.1	14.4	22	22.3	33.2	0	1e2	15.2	14.5	22	22.3	30.4	0	7e2	15.2	14.6	19.9	20.4	32.7	0	1e3
70	20	3	1.1	1.1	4.7	5.1	18.6	0	1e3	1.1	1.1	5.7	6.1	24.6	0	25	1.1	1.1	1	1.4	3.8	0	7e2	1.1	1.1	0.9	1.3	3.8	0	5e2
70	20	5	4.7	4.6	7.7	7.7	23.9	0	2e2	4.6	4.6	7.7	7.8	24.3	0	3	4.6	4.6	5.4	5.4	13.2	0	4e2	4.6	4.6	5.2	5.3	13.8	0	6e2
70	20	7	8.1	8	11.9	11.9	23.7	0	2e2	8	8	11.9	11.9	23.7	0	0	8	8	10.1	10.1	21.8	0	4e2	8	8	10.8	10.8	21.8	0	8e2

 F_p^{flow} F_p^{km} F_p^{mtz} $F_p^{km^2}$

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Table: StackMST results for the $F_z^{(.)}$ formulations.

$ V $	d	$ C $	gRL	gUL	$g\overline{UL}$	gUL	gUL^*	#	nod	gRL	gUL	$g\overline{UL}$	gUL	gUL^*	#	nod	$g\overline{RL}$	$g\overline{UL}$	$g\overline{UL}$	gUL	gUL^*	#	nod							
20	30	3	13.5	-	-	-	-	10	1e3	3.9	-	-	-	-	10	52	4.8	-	-	-	-	10	2e2	4.9	-	-	-	10	5e2	
20	30	5	18.2	-	-	-	-	10	1e4	9.5	-	-	-	-	10	5e3	9.5	-	-	-	-	10	7e3	9.5	-	-	-	10	1e4	
20	30	7	17.4	-	-	-	-	10	8e4	8.5	-	-	-	-	10	1e4	9.2	-	-	-	-	10	2e4	9.2	-	-	-	10	1e5	
20	50	3	4.8	0.7	-	0.7	6.8	9	5e4	3.4	0.8	-	0.8	8.1	9	1e4	3.4	0.7	-	0.7	6.7	9	3e4	3.4	-	-	-	-	10	2e4
20	50	5	7.1	2.5	0.1	2.6	8	1	5e5	7.1	0.8	0.6	1.4	8.1	5	7e4	7.1	0.7	-	0.7	6.2	7	2e5	7.1	0.2	0.4	0.6	4.2	6	2e5
20	50	7	8.9	4.2	2.6	5.1	10.5	0	5e5	8.9	3.9	2	4.2	10.1	3	1e5	8.9	2.3	1.7	2.4	5.8	3	2e5	8.9	2.4	1.8	2.5	5.5	3	3e5
30	30	3	6.9	2.1	1.1	2.9	12	6	5e4	4.4	1	0.3	1	7.2	8	3e3	4.4	1.1	0.3	1.1	5.7	7	5e4	4.4	0.7	0.4	0.8	4.8	8	2e4
30	30	5	9.8	5	4.2	6.3	21.8	1	8e4	7.2	5.7	4	6.7	22.1	2	1e4	7.2	4.7	3.5	5.3	14	2	4e4	7.2	3.6	3.1	3.8	11.8	3	6e4
30	30	7	13.4	8.2	11.9	13.3	24.9	0	9e4	10.1	9.4	10.3	12.9	24.9	0	1e4	10.1	8.5	9.1	10.8	19	0	6e4	10.1	7.5	10.5	11.2	23.9	0	1e5
30	50	3	0.4	0.2	0.2	0.3	3.1	9	1e4	0.2	-	-	-	-	10	1e3	0.2	-	-	-	-	10	2e2	0.2	-	-	-	10	2e2	
30	50	5	4.1	3.8	6.1	9.1	29.5	1	7e4	3.8	3.6	5.4	8.3	29.5	2	1e4	3.8	2.4	2.3	4.1	18.5	5	2e4	3.8	1.5	0.5	1.6	4.6	4	8e4
30	50	7	5.9	5.7	9.1	11	21	0	8e4	5.7	5.7	6.4	8.4	21	0	1e4	5.7	5.7	7.2	9	21	0	6e4	5.7	4.7	7.3	8.3	20	0	1e5
50	10	3	16.2	2.7	1.1	3.2	13.5	3	1e4	3.7	1.9	0.6	1.9	10	6	1e3	4.4	1	0.8	1.2	12.2	9	1e3	4.8	1.2	0.8	1.4	8.6	6	6e3
50	10	5	19.3	5.7	2.4	7.5	10.5	0	2e4	5.5	2.6	0.6	2.7	6.6	4	2e3	6.3	2.5	1.1	3.1	12.8	3	9e3	6.8	3	1.2	3.7	12.2	2	2e4
50	10	7	22.5	7.1	3.7	9	24.2	1	1e4	8	4.8	2	5.2	13.3	3	3e3	8.6	5.2	2.4	5.9	13.5	3	8e3	8.6	4.5	2.3	5.2	14	3	1e4
50	20	3	4.1	2.4	6.5	8	25.2	2	1e4	2.8	2.3	4	5.5	25.2	3	7e2	2.8	2.4	2.3	4	13.9	2	4e3	3	1.6	1.3	2.3	6.6	2	2e4
50	20	5	9.4	8.2	16.1	16.5	26.4	0	1e4	8.2	8.2	16	16.4	26.4	0	1e3	8.2	8.2	15.1	15.6	26.4	0	8e3	8.2	8.2	15.3	15.7	26.4	0	1e4
50	20	7	13.8	12.1	20.8	21.1	30.8	0	1e4	12.1	12.1	20.6	20.9	30.8	0	9e2	12.1	12.1	20.7	21	30.8	0	8e3	12.1	12.1	21	21.3	30.8	0	1e4
70	10	3	16.3	5.8	7.7	9.9	21.2	1	1e3	7.3	6.1	7.8	10.3	19.7	1	1e2	7.5	6.4	7.8	10.5	19.7	1	6e2	7.5	6.4	7.1	10	19.3	1	8e2
70	10	5	18.7	13.2	16.6	17.5	28.1	0	1e3	12.6	12.6	16.3	16.6	25.5	0	1e2	13.1	12.5	16.5	16.6	26	0	6e2	13.1	12.7	16.6	17	28.1	0	1e3
70	10	7	21	14.9	22.8	23.6	33.2	0	1e3	15.1	14.2	22.7	22.8	33.2	0	1e2	15.2	14.7	22.8	23.3	33.2	0	7e2	15.2	14.6	22.8	23.3	33.2	0	1e3
70	20	3	1.1	1.1	9.3	9.7	28.4	0	2e3	1.1	1.1	9.2	9.6	28.4	0	3	1.1	1.1	4.3	4.6	28.4	0	1e3	1.1	0.9	8.3	8.5	28.4	2	2e3
70	20	5	4.7	4.6	7.7	7.8	24.3	0	3e3	4.6	4.6	7.7	7.8	24.3	0	3	4.6	4.6	7.7	7.8	24.3	0	2e3	4.6	4.6	7.7	7.8	24.3	0	5e3
70	20	7	8.1	8	12.1	12.1	24.8	0	2e3	8	8	12.1	12.1	24.8	0	3	8	8	12.1	12.1	24.8	0	1e3	8	8	12.1	12.1	24.8	0	4e3

 F_z^{flow} F_z^{km} F_z^{mtz} F_z^{km2}

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Table: StackMST results comparison for the best formulations with time limits of 0.5h and 5h.

$ V $	d	$ C $	$g\bar{RL}$	$g\bar{UL}$	$g\bar{UL}$	gUL	gUL^*	#	nod	$g\bar{RL}$	$g\bar{UL}$	$g\bar{UL}$	gUL	gUL^*	#	nod	$g\bar{RL}$	$g\bar{UL}$	$g\bar{UL}$	gUL	gUL^*	#	nod	$g\bar{RL}$	$g\bar{UL}$	$g\bar{UL}$	gUL	gUL^*	#	nod	
20	30	7	9.2	-	-	-	-	10	1e4	9.2	-	-	-	-	10	1e5	9.2	-	-	-	-	10	1e4	9.2	-	-	-	-	10	1e5	
20	50	3	3.4	0.8	-	0.8	7.8	9	1e5	3.4	-	-	-	-	10	2e4	3.4	0.3	-	0.3	3.4	9	3e5	3.4	-	-	-	-	10	2e4	
20	50	5	7.1	5.2	-	5.2	11.4	1	4e5	7.1	0.2	0.4	0.6	4.2	6	2e5	7.1	4.2	-	4.2	10.2	3	3e6	7.1	-	-	-	-	0.2	9	3e6
30	30	3	4.4	2.8	0.3	2.8	13.3	6	2e4	4.4	0.7	0.4	0.8	4.8	8	2e4	4.4	2.2	0.3	2.2	9.7	6	2e5	4.4	0.3	0.3	0.3	2.8	9	1e5	
30	50	3	0.2	0.2	-	0.2	1.6	9	4e3	0.2	-	-	-	-	10	2e2	0.2	-	-	-	-	10	1e4	0.2	-	-	-	-	10	2e2	
30	50	5	3.8	3.8	0.6	3.8	7.4	1	5e4	3.8	1.5	0.7	1.6	4.6	4	8e4	3.8	3.5	0.6	3.5	7.4	2	5e5	3.8	0.6	0.7	0.7	3.4	7	7e5	
30	50	7	5.7	5.7	4.2	6.2	18.5	0	6e4	5.7	4.7	7.3	8.3	20	0	1e5	5.7	5.7	3.7	5.7	16.1	0	6e5	5.7	3.7	6.2	6.2	16.8	0	1e6	
50	10	5	6.8	2.3	1.1	2.4	7.3	5	1e4	6.8	3	1.7	3.7	12.2	2	2e4	6.8	1.3	1	1.3	5.3	7	6e4	6.8	2.1	1.6	2.7	12.2	6	1e5	
50	10	7	8.6	3.4	2.2	3.4	10.5	5	9e3	8.6	4.5	2.9	5.2	14	3	1e4	8.6	2.2	2.2	2.2	9	6	8e4	8.6	3.2	2.3	3.3	11	4	1e5	
50	20	3	3	2.7	0.8	2.7	6.6	0	8e3	3	1.6	1.5	2.3	6.6	2	2e4	3	2.5	0.8	2.5	6.6	2	6e4	3	0.9	1	1.1	3.4	4	2e5	
50	20	7	12.1	12.1	14.3	14.7	26.6	0	9e3	12.1	12.1	21	21.3	30.8	0	1e4	12.1	12.1	11.9	12.2	21.3	0	9e4	12.1	11.8	18.1	18.1	30.3	0	9e4	
70	10	3	7.5	6.7	7.9	9.1	19.5	1	1e3	7.5	6.4	9.1	10	19.3	1	8e2	7.5	6.6	6	7.1	19.3	1	1e4	7.5	5.5	7.1	7.1	18.8	2	7e3	
70	10	7	15.2	14.6	20.1	20.4	32.7	0	1e3	15.2	14.6	23	23.3	33.2	0	1e3	15.2	14.5	14.4	14.5	26.9	0	1e4	15.2	14.5	21.2	21.3	33.2	0	1e4	
70	20	3	1.1	1.1	0.9	1.3	3.8	0	5e2	1.1	0.9	8.3	8.5	28.4	2	2e3	1.1	1	0.7	1	2.6	1	5e3	1.1	0.7	6.3	6.3	28.4	4	3e4	
70	20	5	4.6	4.6	5.2	5.3	13.8	0	6e2	4.6	4.6	7.7	7.8	24.3	0	5e3	4.6	4.6	4.6	4.6	10.7	0	7e3	4.6	4.6	6.5	6.5	24.3	0	5e4	
F_p^{km2} 0.5h				F_z^{km2} 0.5h				F_p^{km2} 5h				F_z^{km2} 5h				F_p^{km2} 5h				F_z^{km2} 5h				F_p^{km2} 5h							

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Table: StackMST results comparison for the best formulations.

$ V $	d	$ C $	$objL$	$objU$	gUL	t	$objL$	$objU$	gUL	t	$objL$	$objU$	gUL	t
20	30	7	541	597	9.38	-	541	541	-	101.6	541	556	2.7	-
20	50	3	190	190	-	0	190	190	-	0	190	190	-	0
20	50	5	395	467	15.42	-	407	467	12.85	-	407	414	1.69	-
30	30	3	413	425	2.82	-	413	413	-	947.1	413	413	-	150.1
30	50	3	1830	1862	1.72	-	1830	1830	-	19.7	1830	1830	-	9.9
30	50	5	1254	1320	5	-	1320	1320	-	31.3	1188	1320	10	-
30	50	7	497	524	5.15	-	506	524	3.44	-	506	506	-	1511.9
50	10	5	1470	1588	7.43	-	1470	1528.6	3.83	-	1470	1470	-	3686.1
50	10	7	732	828	11.59	-	734	769.2	4.58	-	734	778.6	5.73	-
50	20	3	2239	2301	2.69	-	2239	2301	2.69	-	2239	2239	-	2068.3
50	20	7	582	760	23.42	-	683	799	14.52	-	641	795	19.37	-
70	10	3	4599	4694	2.02	-	4641	4641	-	6409.9	4641	4641	-	5878.6
70	10	7	1604	2002	19.88	-	1787	2023	11.67	-	1646	2023	18.64	-
70	20	3	759	763	0.52	-	763	763	-	245.6	763	763	-	388.6
70	20	5	934	1173	20.38	-	1086	1173	7.42	-	1019	1173	13.13	-
70	30	5	1167	1227	4.89	-	1227	1227	-	500.7	1083	1227	11.74	-

 F_{mor} 5h $F_p^{km^2}$ 5h $F_z^{km^2}$ 5h

Índice

❶ New improvements for the Stackelberg Minimum Spanning Tree Game

Preliminary

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Conclusions

A path-based StackMST formulation

Let P denote the set of pairs of nodes such that $i < j$. We define now φ_{uv}^{ij} as the flow through edge (u, v) going from origin i to destination j with $(i, j) \in P$. The following set of constraints define a polyhedral description of the spanning trees of G .

$$\mathcal{T}^{path} : \sum_{v \in V : (i, v) \in A} \varphi_{iv}^{ij} = 1 \quad (i, j) \in P \quad (19a)$$

$$\sum_{(u, v) \in A} \varphi_{uv}^{ij} - \sum_{(v, u) \in A} \varphi_{vu}^{ij} = 0 \quad (i, j) \in P, v \in V : v \neq i, j \quad (19b)$$

$$\sum_{(u, j) \in A} \varphi_{uj}^{ij} = 1 \quad (i, j) \in P \quad (19c)$$

$$\varphi_{uv}^{ij} + \varphi_{vu}^{ij'} \leq x_{uv} \quad (i, j) \in P, (i, j') \in P, (u, v) \in E : u, v \neq i, j \quad (19d)$$

$$\sum_{(u, v) \in E} x_{uv} = n - 1 \quad (19e)$$

$$\varphi_{uv}^{ij} \geq 0 \quad (i, j) \in P, (u, v) \in A : v \neq i, u \neq j \quad (19f)$$

$$0 \leq x_e \leq 1 \quad e \in E \quad (19g)$$

Property (Optimality cuts (minimal cost) for \mathcal{T}^{path}):

$$(\varphi_{uv}^{ij} + \varphi_{vu}^{ij}) c_{uv} \leq c_{ij}(1 - x_{ij}) \quad (i, j) \in P, (u, v) \in E : (u, v) \neq (i, j)$$

A path-based StackMST formulation

Let P denote the set of pairs of nodes such that $i < j$. We define now φ_{uv}^{ij} as the flow through edge (u, v) going from origin i to destination j with $(i, j) \in P$. The following set of constraints define a polyhedral description of the spanning trees of G .

$$\mathcal{T}^{path} : \sum_{v \in V : (i, v) \in A} \varphi_{iv}^{ij} = 1 \quad (i, j) \in P \quad (19a)$$

$$\sum_{(u, v) \in A} \varphi_{uv}^{ij} - \sum_{(v, u) \in A} \varphi_{vu}^{ij} = 0 \quad (i, j) \in P, v \in V : v \neq i, j \quad (19b)$$

$$\sum_{(u, j) \in A} \varphi_{uj}^{ij} = 1 \quad (i, j) \in P \quad (19c)$$

$$\varphi_{uv}^{ij} + \varphi_{vu}^{ij'} \leq x_{uv} \quad (i, j) \in P, (i, j') \in P, (u, v) \in E : u, v \neq i, j \quad (19d)$$

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Property (*Optimality cuts (minimal cost) for \mathcal{T}^{path}*):

$$(\varphi_{uv}^{ij} + \varphi_{vu}^{ij})c_{uv} \leq c_{ij}(1 - x_{ij}) \quad (i, j) \in P, (u, v) \in E : (u, v) \neq (i, j)$$

A path-based StackMST formulation

$$F^{path} : \max \sum_{e \in B} p_e \quad (20a)$$

$$(x, \varphi) \in \mathcal{T}^{path} \quad (20b)$$

$$p_e \leq M_e x_e \quad e \in B \quad (20c)$$

$$p_e \leq T_e \quad e \in B \quad (20d)$$

$$T_e \leq p_e + M_e(1 - x_e) \quad e \in B \quad (20e)$$

$$t_{uv}^{ij} \leq (\varphi_{uv}^{ij} + \varphi_{vu}^{ij})M_e \quad (i, j) \in P, (u, v) \in B \quad (20f)$$

$$T_{uv} \leq t_{uv}^{ij} + M_e(1 - \varphi_{uv}^{ij} - \varphi_{vu}^{ij}) \quad (i, j) \in P, (u, v) \in B \quad (20g)$$

$$(\varphi_{uv}^{ij} + \varphi_{vu}^{ij})c_{uv} \leq c_{ij}(1 - x_{ij}) \quad (i, j) \in R, (u, v) \in R : (u, v) \neq (i, j) \quad (20h)$$

$$t_{uv}^{ij} \leq c_{ij}(1 - x_{ij}) \quad (i, j) \in R, (u, v) \in B : (u, v) \neq (i, j) \quad (20i)$$

$$(\varphi_{uv}^{ij} + \varphi_{vu}^{ij})c_{uv} \leq T_{ij} - p_{ij} \quad (i, j) \in B, (u, v) \in R : (u, v) \neq (i, j) \quad (20j)$$

$$t_{uv}^{ij} \leq T_{ij} - p_{ij} \quad (i, j) \in B, (u, v) \in B : (u, v) \neq (i, j) \quad (20k)$$

$$\varphi_{uv}^{ij} \geq 0 \quad (i, j) \in P, (u, v) \in A \quad (20l)$$

$$T_e \geq 0 \quad e \in B \quad (20m)$$

$$t_{uv}^{ij} \geq 0 \quad (i, j) \in P, (u, v) \in A \quad (20n)$$

$$p_e \geq 0 \quad e \in B \quad (20o)$$

Índice

❶ New improvements for the Stackelberg Minimum Spanning Tree Game

Preliminary

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- ▶ In this paper we have presented a catalog of new mathematical programming formulations for the StackMST based on the properties of the MSTP and the bilevel optimization paradigm.
- ▶ We have established theoretical and empirical comparisons between these new formulations that have shown to be effective for efficiently solving random instances of 20 to 70 nodes.
- ▶ In particular, formulations F_p^{km2} and F_z^{km2} outperform previous computational results in the literature based on a Branch-and-Cut-and-Price approach reported in Morais et al. (2016).

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Thanks for your attention

Questions, comments, suggestions... are welcome.

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