The Maximal Covering Location Problem under different perspectives: Upgrading and Uncertainty

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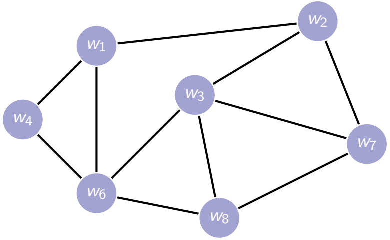
Advances on logistics and transportation problems on complex networks: Evaluation and conclusions

Outline

Maximal Covering Location Problem

MCLP aims to maximise the covered demand locating p facilities.

R. Church and C. ReVelle.
The maximal covering location problem.
Papers of the Regional Science Association, 3:101–118, 1974.



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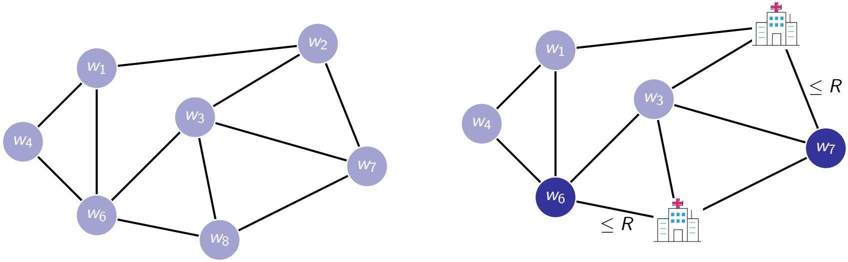
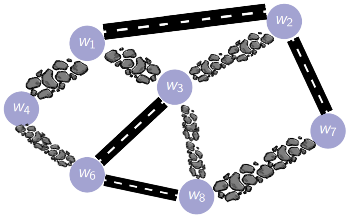


Upgrading edges MCLP

Upgrading MCLP aims to maximise the covered demand locating p facilities and reducing the length of some edges within a budget.



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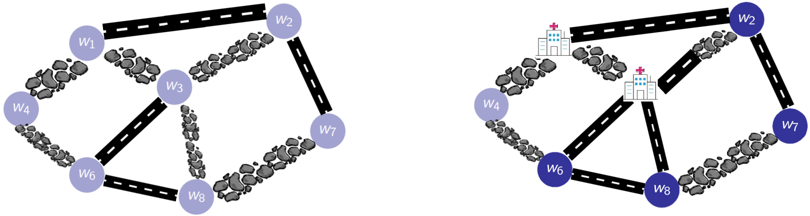

Applications

- Public government: Locate p public services at the same time that some roads are improved reducing travel times.

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- In shopping centres, airports, train stations: Locate services as defibrillators, information posts while installing passenger conveyors or escalators to reduce travel times.

Formulations

- (Flow-Cov)
  - 3-index var.: if a path of length $\leq R$ from i to j traverses arc a.
  - 2-index var.: if node i is assigned to a facility at node j (assignment variables).

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- (Path)
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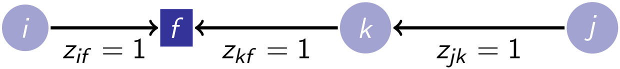

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- (Path-Cov)
  - 2-index var.: if node j is the next node on a path of length $\leq R$ from i to a facility.
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Preprocessing

Formulations: (Flow-Cov) and (Path-Cov).

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Set the assignment variable to zero (remove) if:

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Several procedures to find a balance between accuracy and computational time.

Valid inequalities

- An adaptation of closest assignment constraints.

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Valid inequalities

- An adaptation of closest assignment constraints.
- Strengthening several families of constraints.
    - Separation procedure.

Computational Experiments

- C++ using CPLEX 20.1.0 with Concert Technology.
  - Time limit 1800 seconds.
- Intel(R) Xeon(R) W-2135 CPU 3.70 GHz 32 GB RAM.

Data

- Complete graphs
- OR-Library: pmed
  - $p \in \{1, n/10, n/20\}$,
  - $R \in \{50\% DT_{\text{MCLP}}, 60\% DT_{\text{MCLP}}, 70\% DT_{\text{MCLP}}\}$,
  - $u_e \in (0, 30\% \ell_e)$,
  - $B \in \{5\% B_{max}, 1\% B_{max}, 0.5\% B_{max}\}$.

$$B_{max} = \sum_{t=1}^{\min(n-p,m)} u_{e_{\sigma(t)}} c_{e_{\sigma(t)}},$$

where ρ is a permutation of set E such that

$$c_{e_{\rho(1)}} u_{e_{\rho(1)}} \geq c_{e_{\rho(2)}} u_{e_{\rho(2)}} \geq \ldots \geq c_{e_{\rho(m)}} u_{e_{\rho(m)}} .$$

Performance of the preprocessing phase

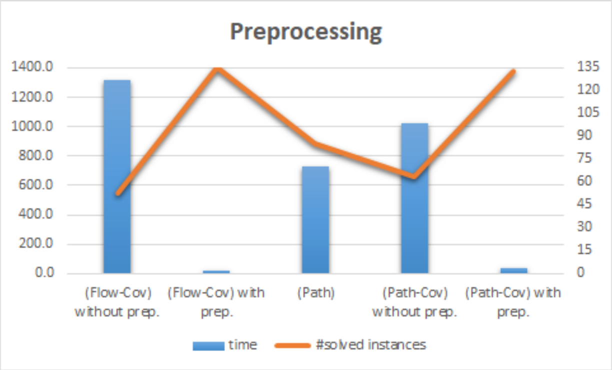

Figure: Performance on graph40

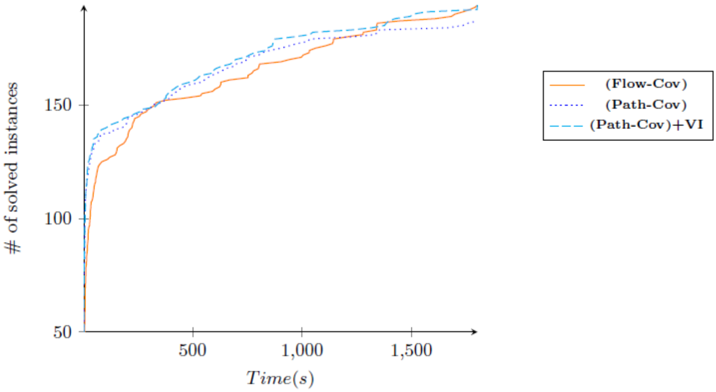

Figure: Performance profile graph of #solved instances using (Flow-Cov), (Path-Cov), (Path-Cov) + VI formulations on graph100 and graph120.

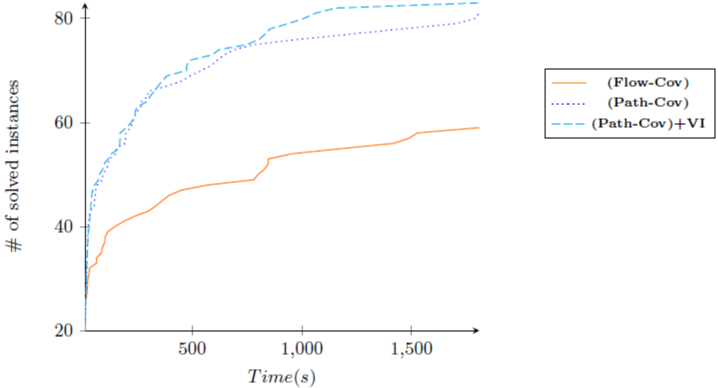

Figure: Performance profile graph of #solved instances using (Flow-Cov), (Path-Cov), (Path-Cov) + VI formulations on pmedb ($|V| = 200, |E| = 777.8$).

Conclusions and future work

- We present three MIP formulations for the upgrading maximal covering location problem.

Future work:

- Covering criteria: gradual covering, cooperative covering, etc.
- Location criteria: p-median, p-center, etc.

M. Baldomero-Naranjo, J. Kalcsics, A. Marín, and A. M. Rodríguez-Chía.
Upgrading edges in the Maximal Covering Location Problem.
Submitted

Uncertainty

A. Marín, L. I. Martínez-Merino, A. M. Rodríguez-Chía, and F. Saldanha-da-Gama.
Multi-period stochastic covering location problems: Modeling framework and solution approach.
European Journal of Operational Research, 268(2):432–449, 2018.

Introduction

Uncertainty

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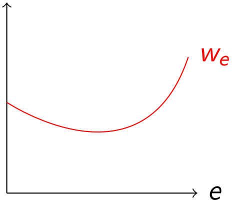

Introduction

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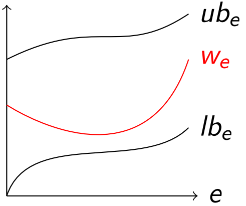

Introduction

Uncertainty

- Single-facility location problem on a network.
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- The demand is uncertain with only a known interval estimation.
- Aim: Minimise the *worst-case* of coverage loss.

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  - $z \in G$ is *covered* by x, if $d(x, z) \leq R$.
  - $C(x) := \{z \in G \mid d(x, z) \leq R\}$ is the *coverage area* of x.

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  The covered demand on an edge $e \in E$ by x for a specific demand realisation w:

$$g_e(x, w) = \int_{y=(e,t)\in C_e(x)} w_e(t)\, dt. \tag{1}$$

  The total amount of covered demand on the network:

$$g(x, w) = \sum_{e \in E} g_e(x, w). \tag{2}$$

Criteria

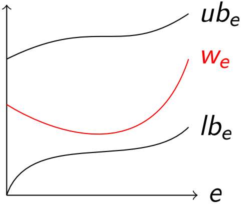
- Coverage criterion: Maximal Covering.

$$\max_{x \in G} g(x) = \sum_{e \in E} \int_{y=(e,t) \in C_e(x)} w_e(t)\, dt. \qquad (1)$$

O. Berman, J. Kalcsics, and D. Krass.
On covering location problems on networks with edge demand.
Computers & Operations Research, 74:214–227, 2016.

Criteria

- Coverage criterion: Maximal Covering.
- Uncertainty Demand: MinMax Regret. Minimise the *worst-case* of coverage loss.



$$r^* = \min_{x \in X} \max_{lb \leq w \leq ub} \left(\max_{y \in G} g(y, w) - g(x, w) \right).$$

Applications

- Locating AED, ATM, bus stops, automated parcel lockers, or bicycle parking racks in cities.
- Locating an aerosol dispenser in the air ducts of a building to disinfect the conduits.

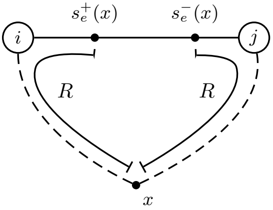
Edge coverage functions

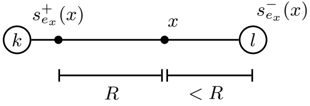
If $x \notin e = [i, j]$

$$s_e^+(x) = \min\left\{1, \max\left\{0, \frac{R - d(x,i)}{\ell_e}\right\}\right\}$$
$$s_e^-(x) = \max\left\{0, \min\left\{1, 1 - \frac{R - d(x,j)}{\ell_e}\right\}\right\}$$

If $x \in e_x = [k, l]$

$$s_{e_x}^+(x) = \max\left\{0, \frac{d(x,k) - R}{\ell_{e_x}}\right\}$$
$$s_{e_x}^-(x) = \min\left\{1, 1 - \frac{d(x,l) - R}{\ell_{e_x}}\right\}$$





Coverage area

The total coverage can now be written as

$$\begin{aligned} g(x,w) \;&= \int_{s_{e_x}^{+}(x)}^{s_{e_x}^{-}(x)} w_{e_x}(u)\,du + \sum_{e \in E^c(x)} \int_0^1 w_e(u)\,du \\ &\quad + \sum_{e \in E^p(x)} \left(\int_0^{s_e^{+}(x)} w_e(u)\,du + \int_{s_e^{-}(x)}^1 w_e(u)\,du \right) . \end{aligned}$$

Singularity points (PP)

- $PP_e \;:=\; \{i,j\} \cup NP_e \cup BP_e \cup EP_e$

  There are at most $\mathcal{O}(m)$ partition points on each $e \in E$.

- $PP \;:=\; \bigcup_{e\in E} PP_e$

  There are $\mathcal{O}(m^2)$ on the whole network.

Lemma

$s_e^+(x)$ and $s_e^-(x)$, $e \in E$, are continuous and piecewise linear functions over $x \in e_x$ with a constant number of pieces.

Edge coverage functions and singularity points

Theorem

Let $e_x \in E, x \in [z^1, z^2]$ *such that* $z^1, z^2 \in PP_{e_x}$.

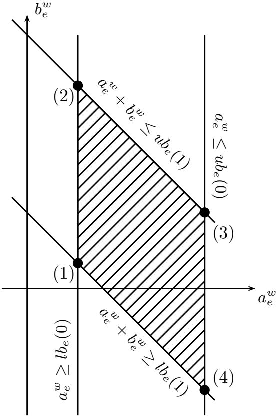
1. *The sets* $E^c(x)$, $E^u(x)$, *and* $E^p(x)$ *are identical for* $x \in [z^1, z^2]$.
2. $s_e^+(x)$ *and* $s_e^-(x)$ *have a unique linear representation for* $x \in [z^1, z^2]$.
3. $g_e(x, w)$, $e \in E$, *have a unique representation, for* $x \in [z^1, z^2]$ *and* w *a non-negative continuous demand function.*

Linear Demand realisation

Let $lb_e(t) = a_e^{lb} + b_e^{lb} \cdot t$, $ub_e(t) = a_e^{ub} + b_e^{ub} \cdot t$, and $w_e(t) = a_e^{w} + b_e^{w} \cdot t$.

Linear Demand realisation

Let $lb_e(t) = a_e^{lb} + b_e^{lb} \cdot t$, $ub_e(t) = a_e^{ub} + b_e^{ub} \cdot t$, and $w_e(t) = a_e^w + b_e^w \cdot t$.



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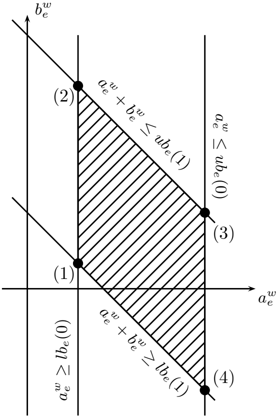


1. (a_e^{lb}, b_e^{lb}),
2. $(a_e^{lb}, a_e^{ub} + b_e^{ub} - a_e^{lb})$,
3. (a_e^{ub}, b_e^{ub}),
4. $(a_e^{ub}, a_e^{lb} + b_e^{lb} - a_e^{ub})$.

Worst-case demand realisation

Theorem

The worst-case demand realisation for a fixed x, y, and e such that $x, y \in G$ and $e \in E$ can be obtained by solving the following linear program:

$$\max_{lb_e \leq w_e \leq ub_e} a_e^w \left(c_e(y) - c_e(x)\right) + \frac{1}{2} b_e^w \left(\bar{c}_e^2(y) - \bar{c}_e^2(x)\right), \qquad (1)$$

Worst-case demand realisation

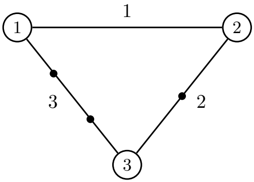
Theorem

An optimal solution of (1), (a_e^{w*}, b_e^{w*}), is given by the first column of the following table whenever the conditions of columns 2-4 are fulfilled.

(a_e^{w*}, b_e^{w*})	Conditions		
	$c_e(y) - c_e(x)$	$\bar{c}_e^2(y) - \bar{c}_e^2(x)$	$(\bar{c}_e^2(y) - \bar{c}_e^2(x)) - 2(c_e(y) - c_e(x))$
(a_e^{lb}, b_e^{lb})	≤ 0	≤ 0	≥ 0
$(a_e^{lb}, a_e^{ub} + b_e^{ub} - a_e^{lb})$	≥ 0	≥ 0	≥ 0
	≤ 0	≥ 0	$-$
(a_e^{ub}, b_e^{ub})	≥ 0	≥ 0	≤ 0
$(a_e^{ub}, a_e^{lb} + b_e^{lb} - a_e^{ub})$	≤ 0	≤ 0	≤ 0
	≥ 0	≤ 0	$-$

Algorithm

- Determine the set PP of partition points.



Let $R = 1$,

$$\begin{aligned} lb_{[1,2]}(t) &= 3 - 3t, \quad & ub_{[1,2]}(t) &= 15 + 7t, \\ lb_{[2,3]}(t) &= 3t, & ub_{[2,3]}(t) &= 7 + 3t, \\ lb_{[1,3]}(t) &= 2 + 3t, & ub_{[1,3]}(t) &= 8 + 10t. \end{aligned}$$

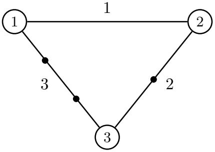
Algorithm

For $e = [i, j] \in E$:

- Sort the partition $PP_e = \{z^1, \ldots, z^{|PP_e|-1}\}$ in non-decreasing distance from node i.
- Derive the representation of the edge coverage functions over each sub-edge $[z^q, z^{q+1}]$, for $q \in \mathcal{I}_e := \{1, \ldots, |PP_e|-1\}$.

For $x_1 \in ([1,2], t_1)$ the edge coverage functions are given by

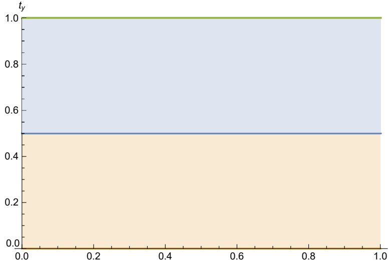
$$
\begin{aligned}
s^+_{[1,2]}(x_1) &= 0, & s^-_{[1,2]}(x_1) &= 1,\\
s^+_{[2,3]}(x_1) &= \frac{t_1}{2}, & s^-_{[2,3]}(x_1) &= 1,\\
s^+_{[1,3]}(x_1) &= \frac{1-t_1}{3}, & s^-_{[1,3]}(x_1) &= 1.
\end{aligned}
$$



Algorithm

1: **for** $e_x \in E$ **do**
2: **for** $i \in \mathcal{I}_{e_x}$ **do**
3: **for** $e_y \in E$ **do**
4: **for** $j \in \mathcal{I}_{e_y}$ **do**
5: Generate the subdivision in the rectangle $[z^i, z^{i+1}] \times [z^j, z^{j+1}]$ by the arcs defining the conditions of the worst case demand realisation for any $e \in E$.

For $(x, y) \in [1, 2] \times [2, 3]$.
Cells for $r_{[1,2]}(x, y)$:



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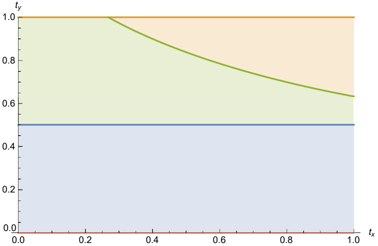

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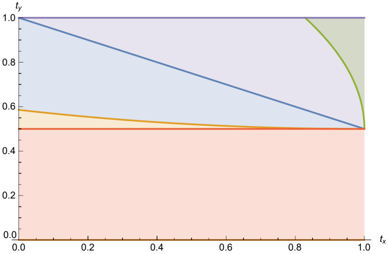

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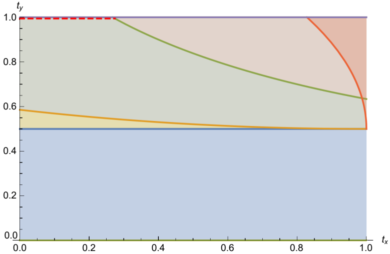
    for $i \in \mathcal{I}_{e_x}$ **do**
        for $e_y \in E$ **do**
            for $j \in \mathcal{I}_{e_y}$ **do**
                Let $\mathcal{C}^{ij}_{e_x e_y}$ be the family of arcs defined by:
                -boundaries of the cells previously obtained.
                -For any cell where $r(x,y)$ is concave, the intersection of the curve $\frac{\partial r}{\partial y}(x,y) = 0$ with that cell.

For $(x,y) \in [1,2] \times [2,3]$.
Cells for $r(x,y)$



Algorithm

```
for e_x ∈ E do
    for i ∈ I_{e_x} do
        Obtain the upper envelope, h^i_{e_x}(x), of r(x, y(x)) of the arcs
        contained in ⋃_{e_y ∈ E, j ∈ I_{e_y}} C^{ij}_{e_x e_y}.
        Find the minimum x_i^* of h^i_{e_x}(x) over [z^i, z^{i+1}].
        if h^i_{e_x}(x_i^*) < r(x^*) then
            set x^* := x_i^*, r(x^*) = h^i_{e_x}(x_i^*).
```

for $e_x \in E$ **do**

  for $i \in \mathcal{I}_{e_x}$ **do**

    Obtain the upper envelope, $h^i_{e_x}(x)$, of $r(x, y(x))$ of the arcs contained in $\bigcup_{e_y \in E, j \in \mathcal{I}_{e_y}} \mathcal{C}^{ij}_{e_x e_y}$.

    Find the minimum x_i^* of $h^i_{e_x}(x)$ over $[z^i, z^{i+1}]$.

    if $h^i_{e_x}(x_i^*) < r(x^*)$ **then**

      set $x^* := x_i^*$, $r(x^*) = h^i_{e_x}(x_i^*)$.

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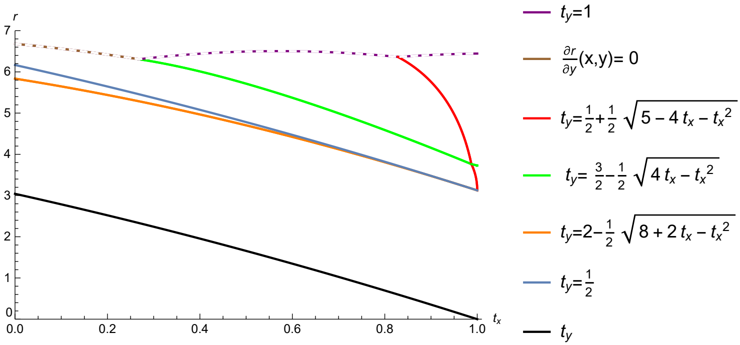

Figure: $r(x, y(x))$, $x \in [1, 2]$, and $y \in [2, 3]$.

Algorithm

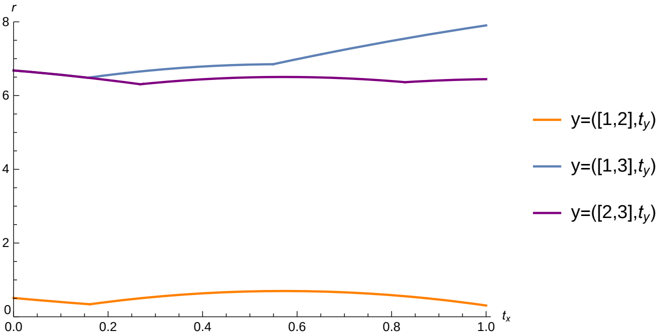

Figure: Upper envelope of $r(x, y(x))$, $x \in [1, 2]$.

The minimum value of r is 6.3055, where $x^*_{[1,3]} = ([1, 3], 0.0533)$.
This should be repeated for each $x \in [z^i, z^{i+1}]$, where $i \in \mathcal{I}_{e_x}$ and $e_x \in E$.

Algorithm

Theorem

The single facility MinMax Regret Maximal Covering Location Problem on a network with edge linear demand realisations can be solved exactly in $\mathcal{O}(m^4 \log^* m)$ *time using the previous Algorithm.*

Conclusions

- Although the majority of problems become NP-hard in the minmax regret version, we propose a polynomial time algorithm for solving the single-facility MinMax Regret MCLP on a network where the demand is
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  - distributed along the edges,
  - constant or linear functions,
  - uncertain with only a known interval estimation.

Conclusions

- Although the majority of problems become NP-hard in the minmax regret version, we propose a polynomial time algorithm for solving the single-facility MinMax Regret MCLP on a network where the demand is
  - distributed along the edges,
  - constant or linear functions,
  - uncertain with only a known interval estimation.

M. Baldomero-Naranjo, J. Kalcsics, and A. M. Rodríguez-Chía.
Minmax regret maximal covering location problems with edge demands.
Computers & Operations Research, 130:105181, 2021.

Future work

Potential avenues for future research:

- Other kind of demand realisation functions.

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- Other kind of demand realisation functions.
- Multi-facility location version of the problem.

Future work

Potential avenues for future research:

- Other kind of demand realisation functions.
- Multi-facility location version of the problem.
- Apply a different criterion of coverage, e.g. the gradual covering.


thank you
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