Generalizations of the p-median problem

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Advances on logistics and transportation problems on complex networks: Evaluation and conclusions

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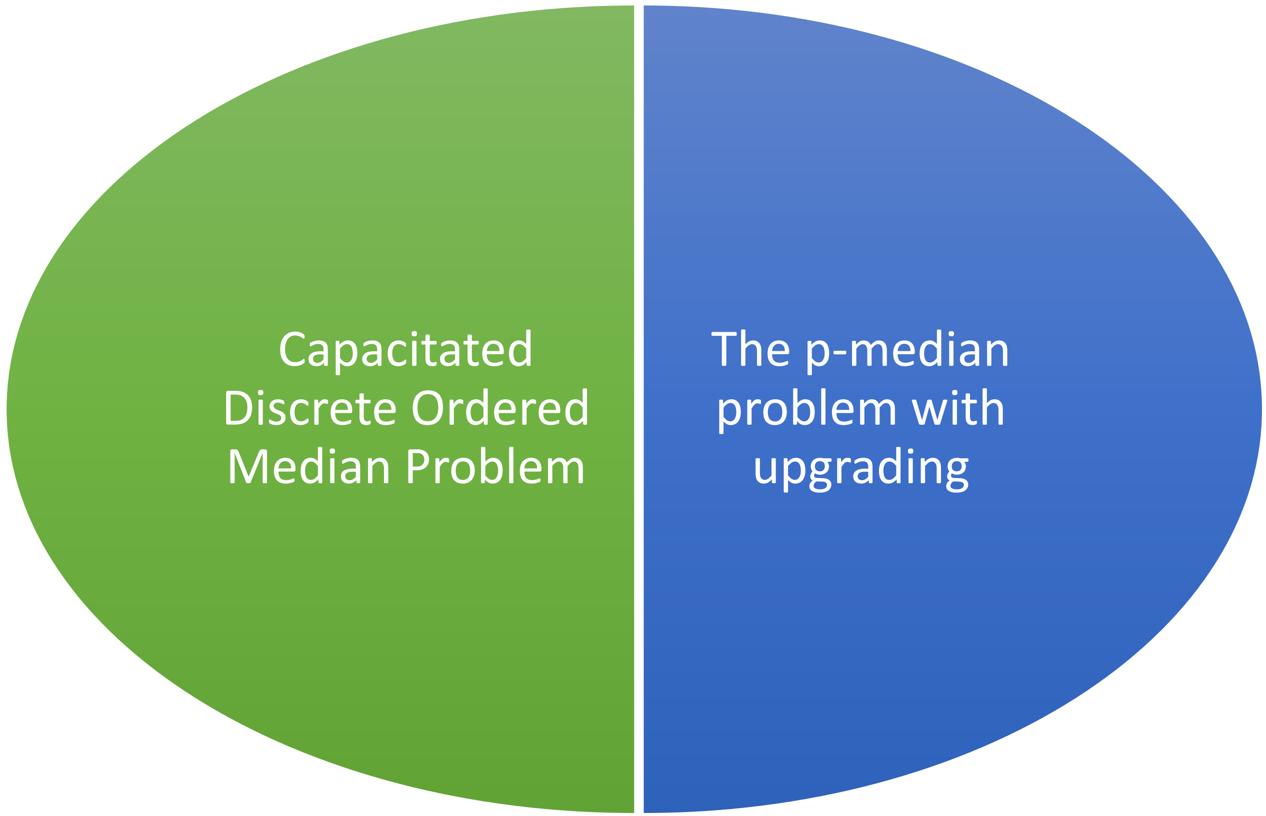
Generalizations of the p-median problem



Generalizations of the p-median problem

Capacitated Discrete Ordered Median Problem

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A comparative study of different formulations for the capacitated discrete ordered median problem

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A R T I C L E I N F O

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A B S T R A C T

This paper deals with the capacitated version of discrete ordered median problems. We present different formulations considering three-index variables or covering variables to address the order requirements in this problem. Some preprocessing phases for fixing variables and some valid inequalities are developed to enhance the initial formulations. Finally, an extensive computational analysis is addressed with data taken from the OR-library and AP-library showing the efficiency of the formulations and the improvements presented in the paper.

210 x 280 mm

Capacitated Discrete Ordered Median Problem

One formulation. Small instances.	(2008) J. Puerto. Operations Research Proceedings 2007
Number of facilities is not given. Demands can be Split.	(2010) Kalcsics,Nickel,Puerto,Rodríguez-Chía. TOP 18: 203-222.
Extension of DOMP (logistic system). Demands can be Split.	(2010) Kalcsics,Nickel,Puerto,Rodríguez-Chía. EJOR 202: 491-501.
Hubs	(2016) Puerto,Ramos,Rodríguez-Chía,Sánchez-Gil. Transportation Research Part C 70:142-156.

Capacitated Discrete Ordered Median Problem

- ***A***: set of clients and potential facility locations. **|A|=n**
- ***C=(cij)***: cost of satisfying demand client i from facility j.
- $\mathbf{1 \leq p \leq n-1}$: number of new facilities.
- $J \subset A$, **|J|=p**

q_i: *demand client i*, $\forall i \in A$
Q_j: *capacity facility j*, $\forall j \in$

$\mathbf{c_i(J)}$ the cost of satisfying demand client i from some facility in J.

$$\mathbf{c(J) := \left(c_1(J), \ldots, c_n(J)\right)}$$

Sort c(J): $c_{(1)}(J) \leq c_{(2)}(J) \leq \cdots \leq c_{(n)}(J)$

$c_{ii} = 0, \forall i \in A$
free self-service

CDOMP

$$\min_{\substack{J \subset A \\ |J|=p}} \sum_{i=1}^{n} \lambda_i c_{(i)}(J)$$

$\lambda = (\lambda_1, \ldots, \lambda_n), \lambda_i \geq 0$

Capacitated Discrete Ordered Median Problem

FORMULATIONS

Three-index

Two-index

CDOMP1

CDOMP2
Covering

CDOMP3
Blocks

Capacitated Discrete Ordered Median Problem

Three-index

$x_{ij}^{k} = 1$ **client *i* served by *j***
and *cij* *k*-th smallest cost

Boland N, Domínguez-Marín P, Nickel S, Puerto J (2006) COR 33: 3270-3300.

$$\sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{k=1}^{n} q_i x_{ij}^{k} \leq (Q_j - q_j)x_{jj}, \forall j \in A$$

Labbé M, Ponce D, Puerto J (2017) COR 78: 230-242.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}^{k-1} \leq \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}^{k}$$

$$\boldsymbol{\sum_{\substack{\hat{\imath}=1\\ \hat{\imath}\hat{\jmath}\geqslant ij}}^{n}\sum_{\hat{\jmath}=1}^{n} x_{\hat{\imath}\hat{\jmath}}^{k-1} + \sum_{\substack{\hat{\imath}=1\\ \hat{\imath}\hat{\jmath}\leqslant ij}}^{n}\sum_{\hat{\jmath}=1}^{n} x_{\hat{\imath}\hat{\jmath}}^{k} \leq 1, \forall i, j (i \neq j), k = 2 \dots n}$$

$$\boldsymbol{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\sum_{\substack{\hat{\imath}=1\\ \hat{\imath}\hat{\jmath}\geqslant ij}}^{n}\sum_{\hat{\jmath}=1}^{n} x_{\hat{\imath}\hat{\jmath}}^{k-1} + \sum_{\substack{\hat{\imath}=1\\ \hat{\imath}\hat{\jmath}\leqslant ij}}^{n}\sum_{\hat{\jmath}=1}^{n} x_{\hat{\imath}\hat{\jmath}}^{k}\right) \leq n^2, k = 2 \dots n}$$

Fernández E, Pozo MA, Puerto J (2014) Discrete Applied Mathematics 169(31): 97-118.

$$x_{ij} = \sum_{k=1}^{n} x_{ij}^{k} \qquad \omega_{ik} = \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^{k} \qquad \theta_{ik}$$

Capacitated Discrete Ordered Median Problem

Three-index

CDOMP1

Labbé M, Ponce D, Puerto J
(2017) COR 78: 230-242.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}^{k-1} \leq \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}^{k}$$

$$\boxed{\sum_{\hat{\imath}=1}^{n}\sum_{\substack{\hat{\jmath}=1 \\ \hat{\imath}\hat{\jmath} \geqslant ij}}^{n} x_{\hat{\imath}\hat{\jmath}}^{k-1} + \sum_{\hat{\imath}=1}^{n}\sum_{\substack{\hat{\jmath}=1 \\ \hat{\imath}\hat{\jmath} \leqslant ij}}^{n} x_{\hat{\imath}\hat{\jmath}}^{k} \leq 1, \forall i, j (i \neq j), k = 2 \ldots n}$$

Capacitated Discrete Ordered Median Problem

FORMULATIONS

Two-index

CDOMP2

Covering

Capacitated Discrete Ordered Median Problem

Two-index-covering

Puerto J (2008) Operations Research Proceedings 2007, 165-170 (Springer).

CDOMP2

$C = (c_{ij})$ → Different nonzero elements

$$\boxed{c_{(0)} := 0 < c_{(1)} < \cdots < c_{(G-1)} < c_{(G)} = max\{c_{ij}\}}$$

$x_{ij} = 1$ **client i is served by j**

$u_{kh} = 1$ **k − th smallest allocation cost is at least $c_{(h)}$**

Fix $u_{kh} - variables$ and some valid inequalities

Capacitated Discrete Ordered Median Problem

FORMULATIONS

Two-index

CDOMP3
Blocks

Capacitated Discrete Ordered Median Problem

Two-index-blocks

CDOMP3

Taking advantage of sequences of repetitions in the λ-vector…

$\boldsymbol{c_{ii} = 0, \mathrm{i} \in A}$ → $\hat{\lambda} = (\lambda_{p+1}, \dots, \lambda_n)$

I: number of blocks of consecutive equal non-null elements in $\hat{\lambda}$ →

New set of variables:

$\overline{u}$: u variables in each blocks of non-null λ values

v: assignments in each block

Fix $\overline{u_{kh}} - variables$ and some valid inequalities

For hubs:

Puerto J, Ramos AB, Rodríguez-Chía AM (2013) Discrete Applied Mathematics

Puerto J, Ramos AB, Rodríguez-Chía AM, Sánchez-Gil MC (2016) Transportation Research Part C

Capacitated Discrete Ordered Median Problem

- 4.00Ghz PC with 32GB RAM
- Xpress IVE 8.5
- All cuts from Xpress disabled.

Type	λ
1	(1,…,1,1)
2	(0,…,0,1)
3	(1,…,0,1,…,1)
4	(0,…,0,1,…,1,0,…,0)
5	(1,…,1,0,…,0,1,…,1)
6	(0,…,0,1,…,1,0,…,0,1,…,1,0…,0,1…1)

Capacitated Discrete Ordered Median Problem

different blocks

DATA SET I

- APData set. ORLIB (Erns)
- Capacitated Hubs
- Matrix Cost
  - non-symmetrical
  - Number of different nonzero elements high.
- Capacities: randomly generated from the demand

DATA SET II

- ORLIB (Beasly)
- Capacitated p-Median
- Matrix Cost
  - symmetrical
  - Number of different nonzero elements smaller than DATA SET I.
- Capacities: all equal

Capacitated Discrete Ordered Median Problem

DATA SET I n=15 p=3

Type		CDOMP1	CDOMP2	CDOMP3
1	Time	6	2	0,2
	Gap	9	9	9
2	Time	83	12	0,5
	Gap	54	75	75
3	Time	66	3	0,7
	Gap	24	51	51
4	Time	6	3	0,2
	Gap	14	16	16
5	Time	24^3	2	0,8
	Gap	39	67	67
6	Time	7	2	0,7
	Gap	23	29	29

GAP: Gap(%) of the linear relaxation

Time: CPU total time

Superindex: number of unsolved instances within 2 h

Capacitated Discrete Ordered Median Problem

DATA SET I Y II n=50 p=5

Type	CDOMP2		CDOMP3	
	DATA SET I (G=2196)	DATA SET II (G=895)	DATA SET I (G=2196)	DATA SET II (G=895)
1	2612[2]	426	87	15
2	301	214	81	11
3	3912[3]	924	222	25
4	1111	141	84	10
5	3290[4]	661	157	15
6	1404	274	222	46

Capacitated Discrete Ordered Median Problem

Improving formulations

Fixing variables

Preprocessing procedures to fix u-variables ($\bar{u}$,v-variables) → Solving auxiliary problems

For hubs:

Puerto J, Ramos AB, Rodríguez-Chía AM (2013) Discrete Applied Mathematics

Puerto J, Ramos AB, Rodríguez-Chía AM, Sánchez-Gil MC (2016) Transportation Research Part C

Valid inequalities

Valid inequalities to strengthen the capacity constraints.

(variants of the ones obtained by minimal covers for the knapsack constraints)

Capacitated Discrete Ordered Median Problem

DATA SET I n=50 p=5

	Type=1		Type=2		Type=3		Type=4		Type=5		Type=6	
	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap
CDOMP2	2612^2	2	301	85	3912^3	50	1111	0	3289^4	73	1404	15
CDOMP3	87	2	81	85	222	50	84	0	156	73	222	15
CDOMP3 + pre+dv	**52**	**2**	**61**	**44**	**136**	**32**	**50**	**0**	**83**	**38**	**202**	**10**

Capacitated Discrete Ordered Median Problem

DATA SET I n=120 p=12

	Type=1		Type=2		Type=3		Type=4		Type=5		Type=6	
	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap
CDOMP3+pre	1441^3	0	**2273**	16	4313	19	1486^3	0,3	**2643**	12	4989^1	11
CDOMP3 + pre+dv	**2730^1**	0	2486	16	**3410**	19	**2809^1**	0,3	2711	12	**4370^1**	11

Concluding remarks

- ✓ Different formulations for the CDOMP

CDOMP3: THE BLOCK FORMULATION

- ✓ Procedures to reduce the size of formulations.

PREPROCESSING WITHOUT CAPACITIES

- ✓ Computational study with two types of instances (small and large number of ties within the cost matrix)

Generalizations of the p-median problem

The p-median problem with upgrading of transportation costs and minimum travel time allocation

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March 12, 2021

Abstract

In this paper, we analyze the upgrading of arcs in the well known p-median problem on a bi-network. Associated to each arc, both travel times and transportation costs exist. Our goal consists of simultaneously finding p medians, allocating each node to the median of minimum travel time, and distributing a known budget among arcs of the network, to reduce their transportation cost, in order to minimize the total transportation cost of the system. The problem is motivated by the warehouse-to-locker structure of the distribution network of many ecommerces. We formulate it in two different ways as an Integer Programming Problem, derive some properties of any optimal solution, develop valid inequalities and present computational results.

Keywords: p-median; upgrading; bi-network.

The p-median problem with upgrading in bi-networks

The p-median problem with upgrading in bi-networks

- The upgrading of the vertices in a p-median context.

  Sepasian AR, Rahbarnia F (2015) Upgrading p-median problem on a path. Journal of Mathematical Modelling and Algorithms 14: 145-157.

- Works devoted to the optimization of bi-networks
  - *minimum cost flow problem*

    Holzhauser M, Krumke SO, Thielen C (2016) Budget-constrained minimum cost flows. Journal of Combinatorial Optimization 31: 1720-1745.
  - *Median Path Problem*

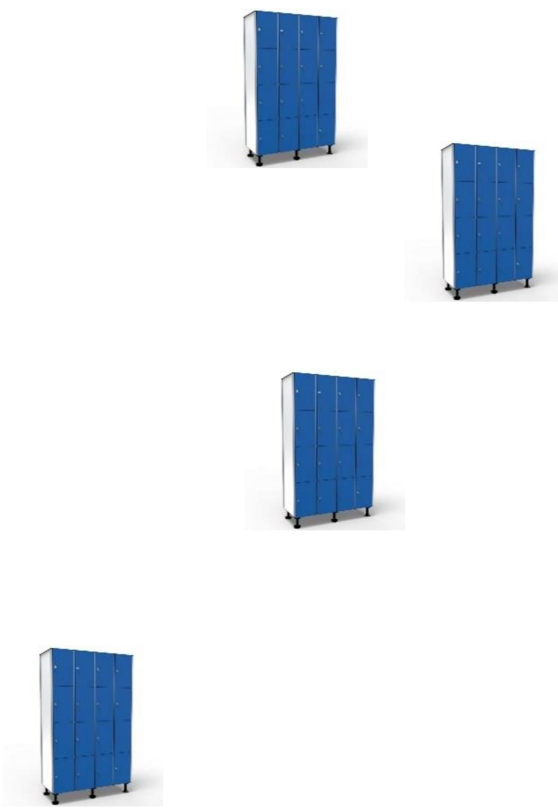
    Avella P, Boccia M, Sforza A (2005) A Branch-and-Cut Algorithm for the Median Path Problem. Computational Optimization and Applications 32: 215-230.

The p-median problem with upgrading of transportation costs and minimum travel time allocation

Distribution network of many ecommerces

Delivery points
(lockers)

Customers



The p-median problem with upgrading of transportation costs and minimum travel time allocation

Distribution network of many ecommerces

Delivery points
(lockers)

Customers

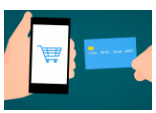
The p-median problem with upgrading of transportation costs and minimum travel time allocation

Distribution network of many ecommerces

Warehouse
(Fulfillment center)

Delivery points
(lockers)

Customers

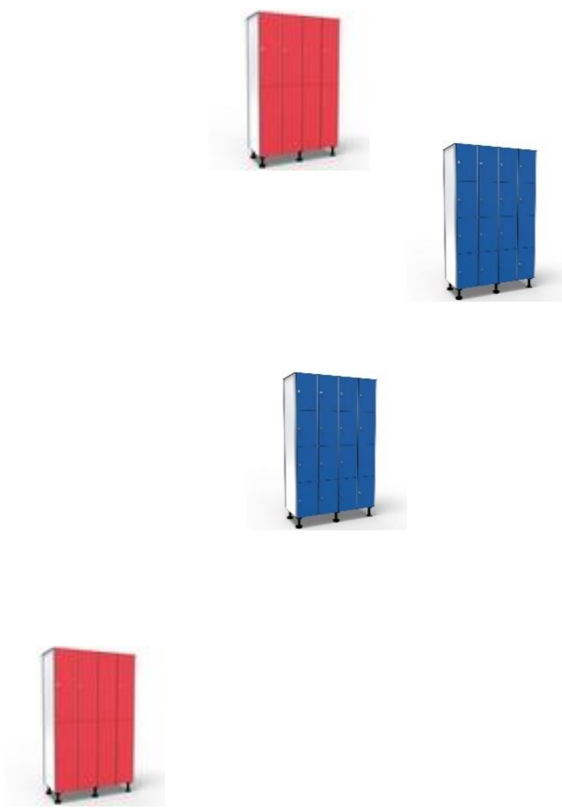


The p-median problem with upgrading of transportation costs and minimum travel time allocation

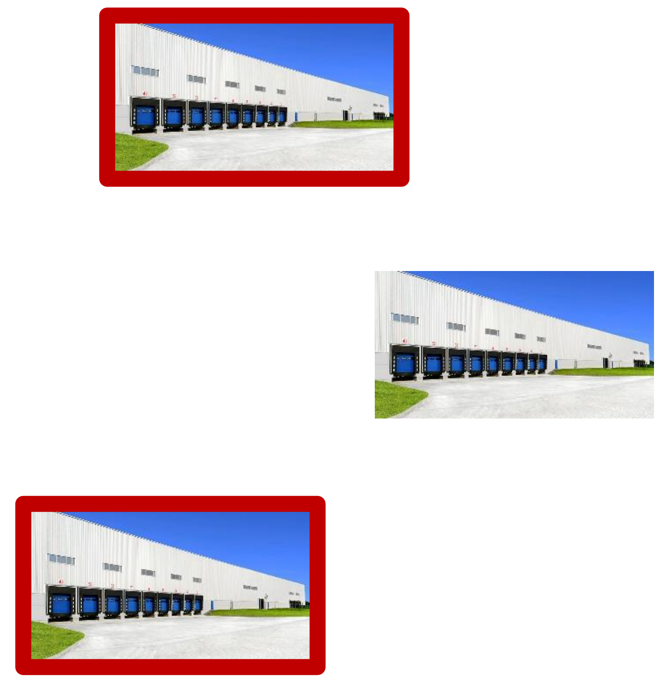
Distribution network of many ecommerces

Customers

Delivery points (lockers)

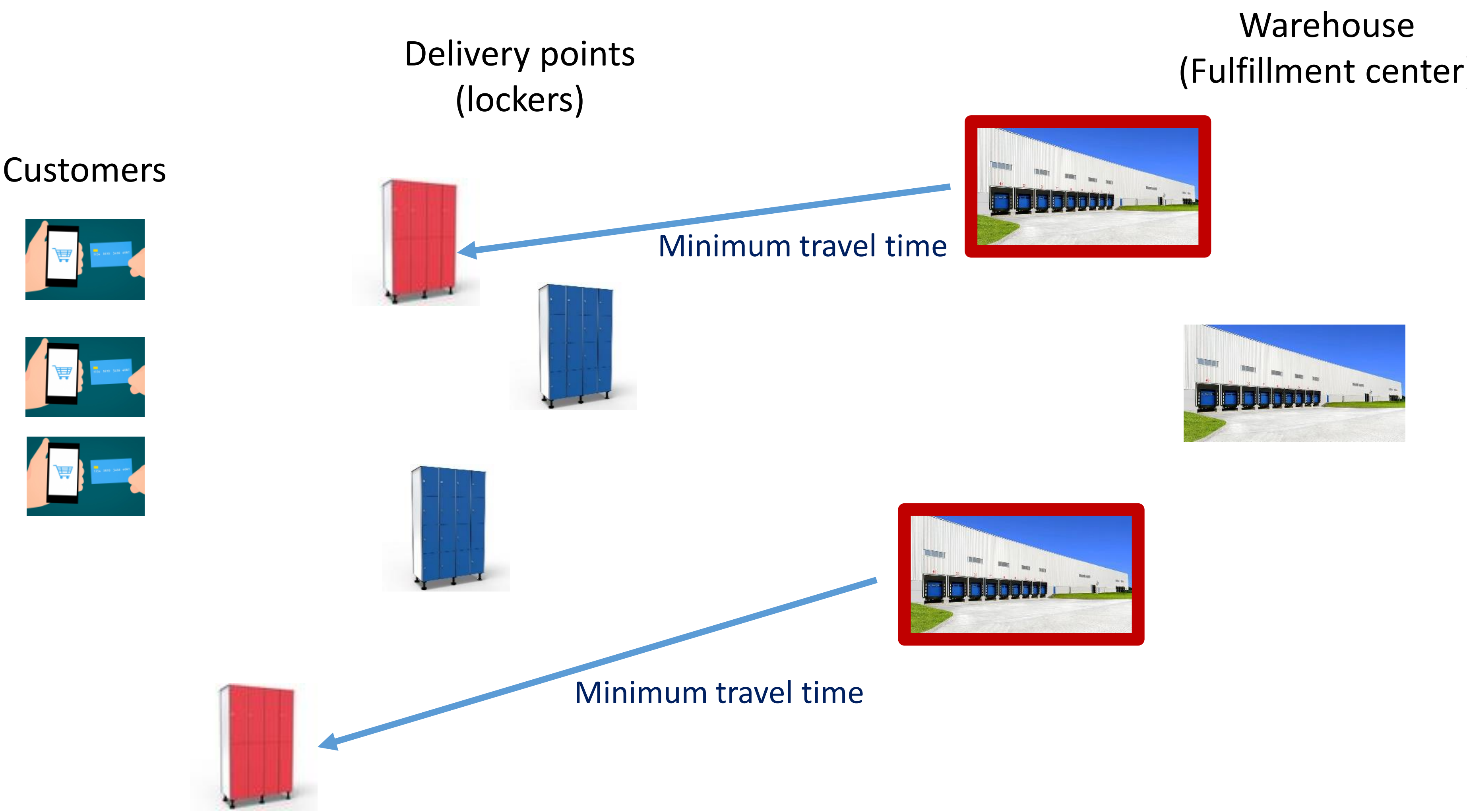


Warehouse (Fulfillment center)



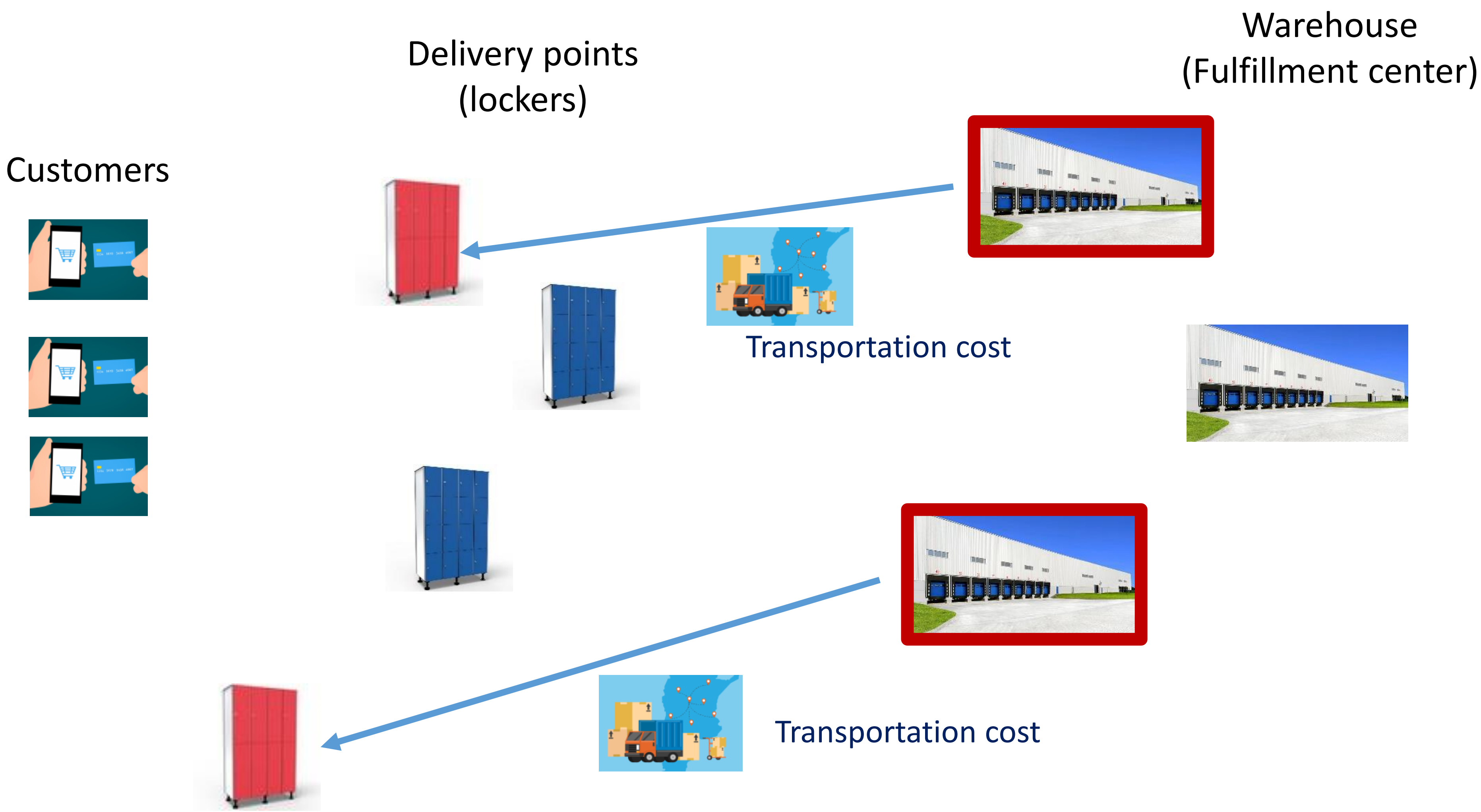
The p-median problem with upgrading of transportation costs and minimum travel time allocation

Distribution network of many ecommerces



The p-median problem with upgrading of transportation costs and minimum travel time allocation

Distribution network of many ecommerces



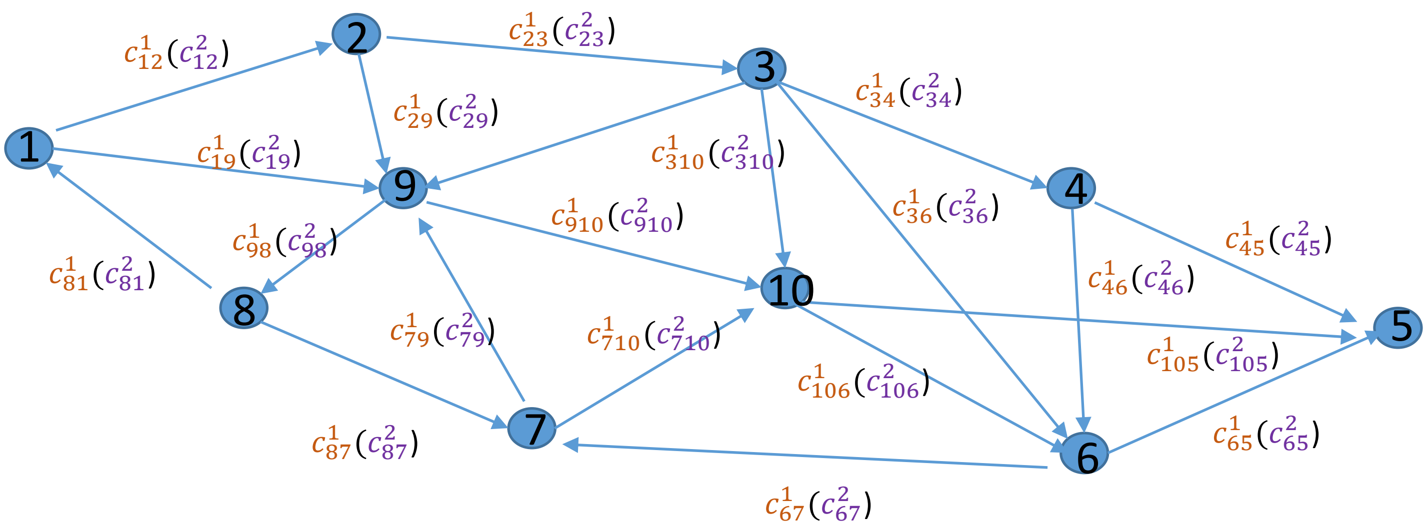
The p-median problem with upgrading of transportation costs and minimum travel time allocation

Given a directed bi-network $(V; A;\ c^1; c^2)$ strongly connected

- $V = \{1, \dots, n\}$: set of nodes representing users (lockers) and candidates of medians (fulfillment centers).
- Demand $\omega_i \geq 0\ i \in V$.
- A: set of arcs.

  c^1**: travel times.**

  c^2**: transportation costs** (per unit transported).



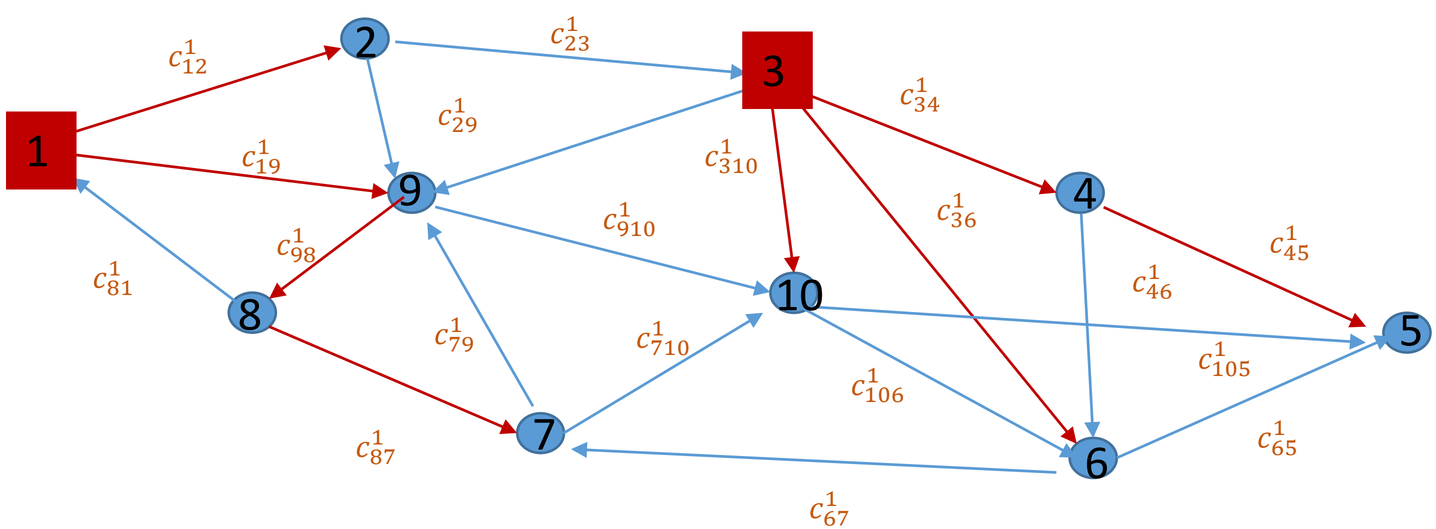
The p-median problem with upgrading of transportation costs and minimum travel time allocation

Given a directed bi-network $(V; A;\ c^1; c^2)$ strongly connected

- Choose p medians (fulfillment centers).
- Allocation of users to median: minimum travel time (c^1) *from the median to the user.*

$FP(i,j)$: Fastest Path (minimum travel time path from median j to user i)

C^1_{ij}: total travel time of $FP(i,j)$

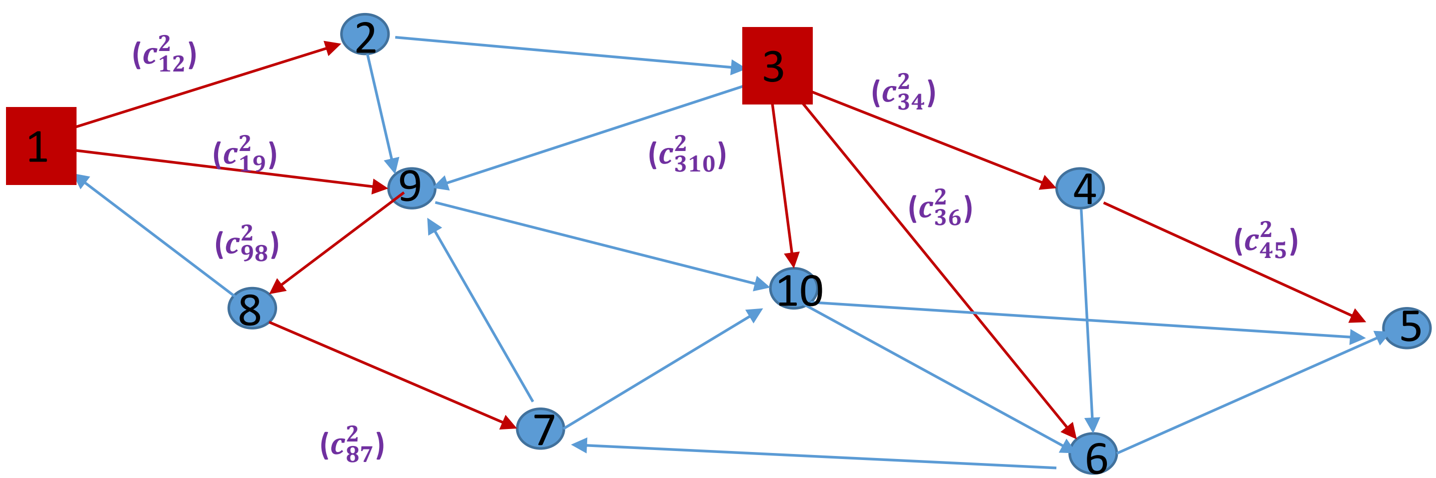


The p-median problem with upgrading of transportation costs and minimum travel time allocation

C_{ij}^{2}**: transportation cost from median** j **to user** i **through** $FP(i, j)$

A budget B > 0 is given to reduce the transportation costs,

The reduction in each arc a is limited to u_a.



The p-median problem with upgrading of transportation costs and minimum travel time allocation

The Induced p-median Problem with Upgrading

Simultaneously

finding p medians and

distributing the budget B among the arcs of A (so reducing their transportation costs $\boldsymbol{c}^{\mathbf{2}}$)

to ***minimize the sum of the upgraded transportation costs*** to users from their corresponding medians (with minimum travel times $\boldsymbol{c}^{\mathbf{1}}$).

The p-median problem with upgrading of transportation costs and minimum travel time allocation

INITIAL FORMULATION (NON LINEAR)

$$\min \sum_{i \in V} \boldsymbol{\omega_i} \sum_{j \in V} \left(\boldsymbol{C^2_{ij}}_{ij} - \sum_{a \in FP(i,j)} \boldsymbol{b_a} \right) \boldsymbol{x_{ij}}$$

(1) $$\sum_{j \in V} x_{ij} = 1 , \forall i \in V$$

(2) $$x_{ij} \leq x_{jj}, \forall i \neq j \in V$$

(3) $$\sum_{j \in V} x_{jj} = p$$

(4) $$x_{jj} + \sum_{\substack{s \in V \\ C^1_{is} > C^1_{ij}}} x_{is} \leq 1, \forall i, j \in V$$

(5) $$\sum_{a \in V} b_a \leq B$$

(6) $$b_a \leq u_a, \forall a \in A$$

(7) $$b_a \geq 0, \forall a \in A$$

(8) $$x_{ij} \in \{0,1\}, \forall i \neq j \in V$$

Variables

b_a: *the reduction of the transportation cost of arc* $a \in A$

$$x_{jj} = \begin{cases} 1 & j \text{ is chosen as a median,} \\ 0 & \text{otherwise,} \end{cases} \qquad \forall j \in V$$

$$x_{ij} = \begin{cases} 1 & j \text{ is the closest (minimum travel time) median for } i \\ 0 & \text{otherwise} \end{cases}$$

$\forall i, j (i \neq j) \in V$

The p-median problem with upgrading of transportation costs and minimum travel time allocation

First linear formulation: FL1

$$\min \sum_{i \in V} \boldsymbol{\omega_i} \left(\sum_{j \in V} \boldsymbol{C_{ij}^2 x_{ij}} - \sum_{(k,l) \in FP(i)} \boldsymbol{z_{ikj}} \right)$$

(1) - (8)

$$z_{ikl} \leq b_{kl}, \qquad \forall i \in V, \forall (k,l) \in FP(i)$$

$$z_{ikl} \leq u_{kl} \sum_{\substack{j \in V \\ (k,l) \in FP(i,j)}} x_{ij} \quad \forall i \in V, \forall (k,l) \in FP(i)$$

$$z_{ikl} \geq 0, \qquad \forall i \in V, \forall (k,l) \in FP(i)$$

$\forall \boldsymbol{i} \in \boldsymbol{V}, \boldsymbol{FP}(\boldsymbol{i}) \subseteq \boldsymbol{A}$:
arcs that belong to any fastest path to i,

$\forall (\boldsymbol{k}, \boldsymbol{l}) \in \boldsymbol{FP}(\boldsymbol{i})$, we define variables

z_{ikl}: *reduction obtained in the path to node i from the closest median when the arc* $a = (k,l) \in A$ *is upgraded*

$$z_{ikl} := \sum_{\substack{j \in V \\ (k,l) \in FP(i,j)}} b_{kl} x_{ij}$$

Proposition

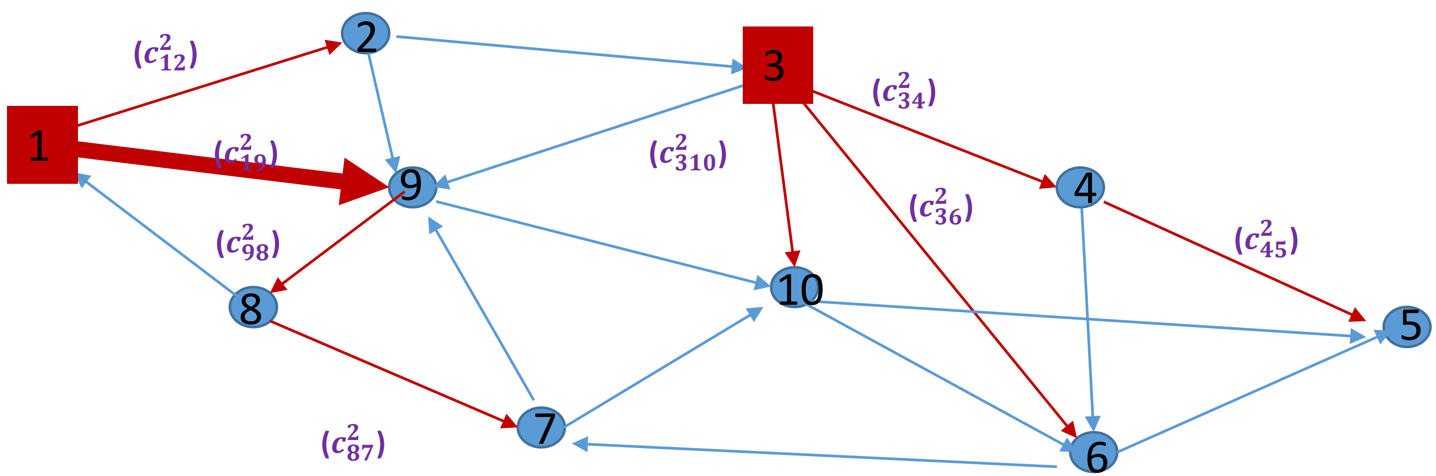
The integrality of the x_{ij}-variables, $\forall i \neq j \in V$, can be relaxed.

The p-median problem with upgrading of transportation costs and minimum travel time allocation

Properties and Valid Inequalities

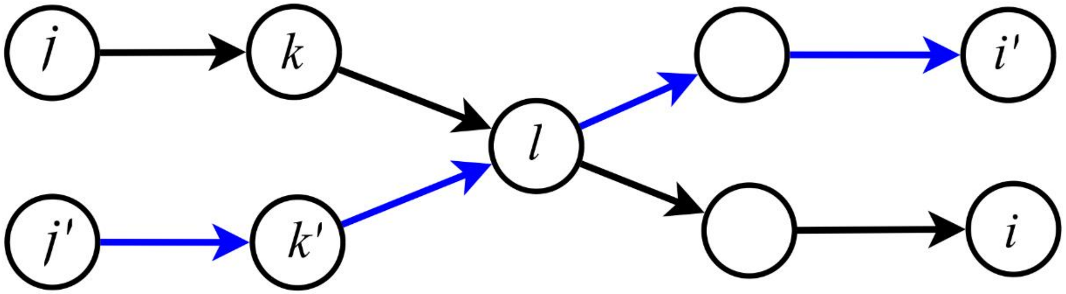
When is optimal a positive reduction of the cost of arc (k; l) in the fastest path to a node i from its median j?

When the maximal reduction has been applied to the previous arc (s; k) in the aforementioned path.



The p-median problem with upgrading of transportation costs and minimum travel time allocation

Properties and Valid Inequalities



Proposition

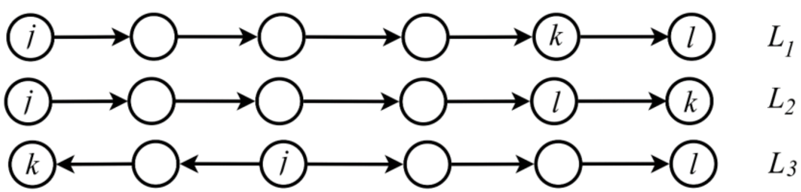
Let $(x^*; z^*)$ be an optimal solution to $(FL1)$ and $i, j \in V$ such that $(k, l) \in FP(i, j)$ with $x^*_{ij} = 1$.
Then $\forall \mathrm{i}', \mathrm{j}', \mathrm{k}' \in V (k \neq k')$ such that $(k', l) \in FP(i', j')$, it holds $x^*_{i'j'} = 0$, whenever $C^1_{lj} \neq C^1_{lj'}$.

The p-median problem with upgrading of transportation costs and minimum travel time allocation

Properties and Valid Inequalities

$$\sum_{\substack{k \in V \\ (k,l) \in A}} b_{kl} \leq max\{u_{kl} : (k,l) \in A\} \qquad (16)$$

$Let\ k, l \in V\ such\ that\ (k,l) \in A\ or\ (l,k) \in A$



U_{kj}: $maximum\ reduction\ in\ FP(k,j)$

$$b_{kl} \leq \sum_{j \in L_1} min\left\{\left(B - U_{kj}\right)^+, u_{kl}\right\} x_{lj} + \sum_{j \in L_1} min\left\{\left(B - U_{lj}\right)^+, u_{kl}\right\} x_{lj} + u_{kl} \sum_{j \in L_3} x_{lj} \qquad (17)$$

$$b_{kl} + b_{lk} \leq max\{u_{kl}, u_{lk}\}, \forall k, l \in V: (k,l) \in A\ \ or\ \ (l,k) \in A \qquad (21)$$

The p-median problem with upgrading of transportation costs and minimum travel time allocation

Properties and Valid Inequalities

$$\sum_{\substack{k \in V \\ (k,l) \in FP(i)}} z_{ikl} \leq max\{u_{kl} : (k,l) \in FP(i)\} \sum_{\substack{j \in V \\ (k,l) \in FP(i,j)}} x_{ij}, \forall i \in V, (k,l) \in FP(i) \tag{18}$$

$$z_{ikl} \leq \sum_{\substack{j \in V \\ (k,l) \in FP(i,j)}} min\left\{\left(B - U_{kj}\right)^{+}, u_{kl}\right\} x_{ij}, \forall i \in V, (k,l) \in FP(i) \tag{19}$$

$$x_{ij} \leq x_{kj} \qquad \forall i,j \in V, \forall k \neq j: \quad k \in FP(i,j) \tag{20}$$

The p-median problem with upgrading of transportation costs and minimum travel time allocation

Second linear formulation: FL2

$$\min \sum_{i \in V} \boldsymbol{\omega_i} \left(\sum_{j \in V} \boldsymbol{C^2_{ij} x_{ij}} - \boldsymbol{s_i} \right)$$

(1) - (8)

$$\boldsymbol{s_i \leq \sum_{a \in FP(i,j)} b_a + \sum_{j' \neq j} M_{ijj'} \, x_{ij'} \quad \forall i, j \in V}$$

$$\boldsymbol{s_i \geq 0, \qquad \forall i \in V}$$

Particular case of valid inequalities

$$\boldsymbol{s_i \leq \sum_{a \in A'} b_a + \sum_{a \in FP(i) \setminus A'} \sum_{\substack{j' \in V \\ a \in FP(i,j')}} u_a \, x_{ij'} \quad \forall i \in V, A' \subset FP(i)}$$

Variables

s_i: *reduction associated to user i*

$$s_i := \sum_{(k,l) \in FP(i)} z_{ikl} = \sum_{(k,l) \in FP(i)} b_{kl} \sum_{\substack{j \in V \\ (k,l) \in FP(i,j)}} x_{ij}$$

$$\boldsymbol{M_{ijj'} := min \left\{ B, \sum_{a \in FP(ij') \setminus \mathrm{FP}(i,j)} u_a \right\}}$$

Separation procedure

The p-median problem with upgrading of transportation costs and minimum travel time allocation

Second linear formulation: FL2

$$\min \sum_{i \in V} \omega_i \left(\sum_{j \in V} C_{ij}^2 x_{ij} - s_i \right)$$

(1) - (8)

$$s_i \leq \sum_{a \in FP(i,j)} b_a + \sum_{j' \neq j} M_{ijj'} x_{ij'} \quad \forall i, j \in V$$

$$s_i \geq 0, \quad \forall i \in V$$

$$M_{ijj'} := min \left\{ B, \sum_{a \in FP(ij') \setminus \mathrm{FP}(i,j)} u_a \right\}$$

FL3

$$\min \sum_{i \in V} \omega_i \left(\sum_{j \in V} C_{ij}^2 x_{ij} - s_i \right)$$

(1) - (8)

$$s_i \leq \sum_{a \in A'} b_a + \sum_{a \in FP(i) \setminus \mathrm{A}'} \sum_{\substack{j' \in V \\ a \in FP(i,j')}} u_a \, x_{ij'} \quad \forall i \in V, A' \subset FP(i)$$

$$s_i \geq 0, \quad \forall i \in V$$

The p-median problem with upgrading of transportation costs and minimum travel time allocation

FL1

$$\min \sum_{i \in V} \boldsymbol{\omega_i} \left(\sum_{j \in V} \boldsymbol{C_{ij}^2 x_{ij}} - \sum_{(k,l) \in FP(i)} \boldsymbol{z_{ikj}} \right)$$

(1) - (8)

$$z_{ikl} \leq b_{kl}, \qquad \forall i \in V, \forall (k,l) \in FP(i)$$

$$z_{ikl} \leq u_{kl} \sum_{\substack{j \in V \\ (k,l) \in FP(i,j)}} x_{ij} \qquad \forall i \in V, \forall (k,l) \in FP(i)$$

$$z_{ikl} \geq 0, \qquad \forall i \in V, \forall (k,l) \in FP(i)$$

FL3

$$\min \sum_{i \in V} \boldsymbol{\omega_i} \left(\sum_{j \in V} \boldsymbol{C_{ij}^2 x_{ij}} - \boldsymbol{s_i} \right)$$

(1) - (8)

$$s_i \leq \sum_{a \in A'} b_a + \sum_{a \in FP(i) \setminus A'} \sum_{\substack{j' \in V \\ a \in FP(i,j')}} u_a \, x_{ij'} \qquad \forall i \in V, A' \subset FP(i)$$

$$s_i \geq 0, \qquad \forall i \in V$$

Proposition

The linear relaxation of FL1 is the same as FL3

The p-median problem with upgrading of transportation costs and minimum travel time allocation

- Intel Xeon(R) CPU E5-2623 v3, 3.00GHz x 8 processor with 16 GB of RAM
  Xpress Mosel v. 5.0.2 (under Linux)
- All cuts and preprocessing from Xpress disabled.
- Time limit: 1 hour

DATA SET R

- 480 instances
- Travel times C1 and transportation costs C2 independent in [0,100]

DATA SET P

- 480 instances
- Travel times C1 and transportation costs C2 correlated

$n \in \{20,40,60,80\}$ $\quad$ $m \in \{100,500\}$

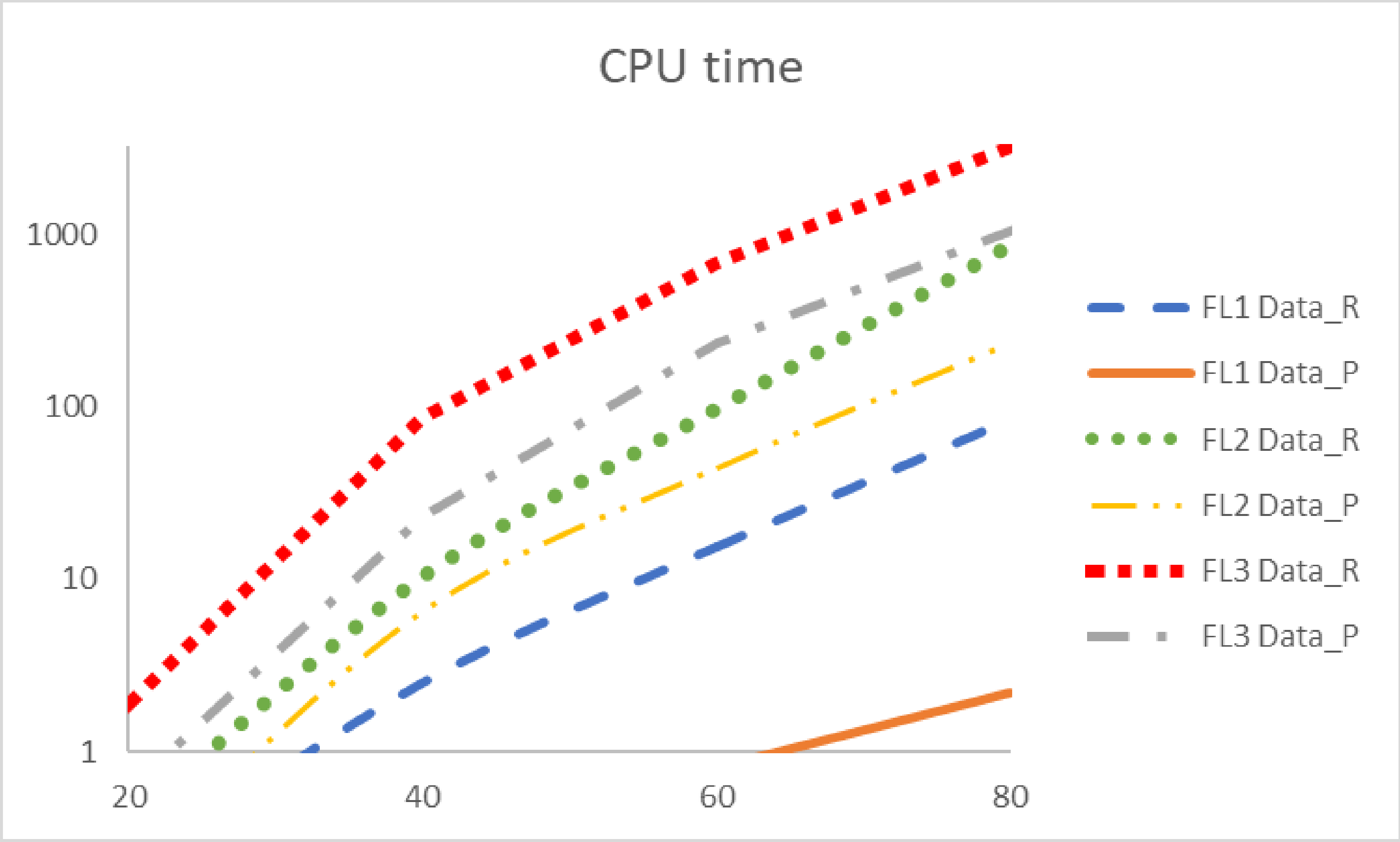
$2 \leq p \leq 5$ $\quad$ $B \in [50; 100]$

$\omega \in [0; 40]$ $\quad$ $u_a = 0.5c_a^2$

The p-median problem with upgrading of transportation costs and minimum travel time allocation

FIRST STUDY: FL1, FL2, FL3 for DATASET R and DATASET P

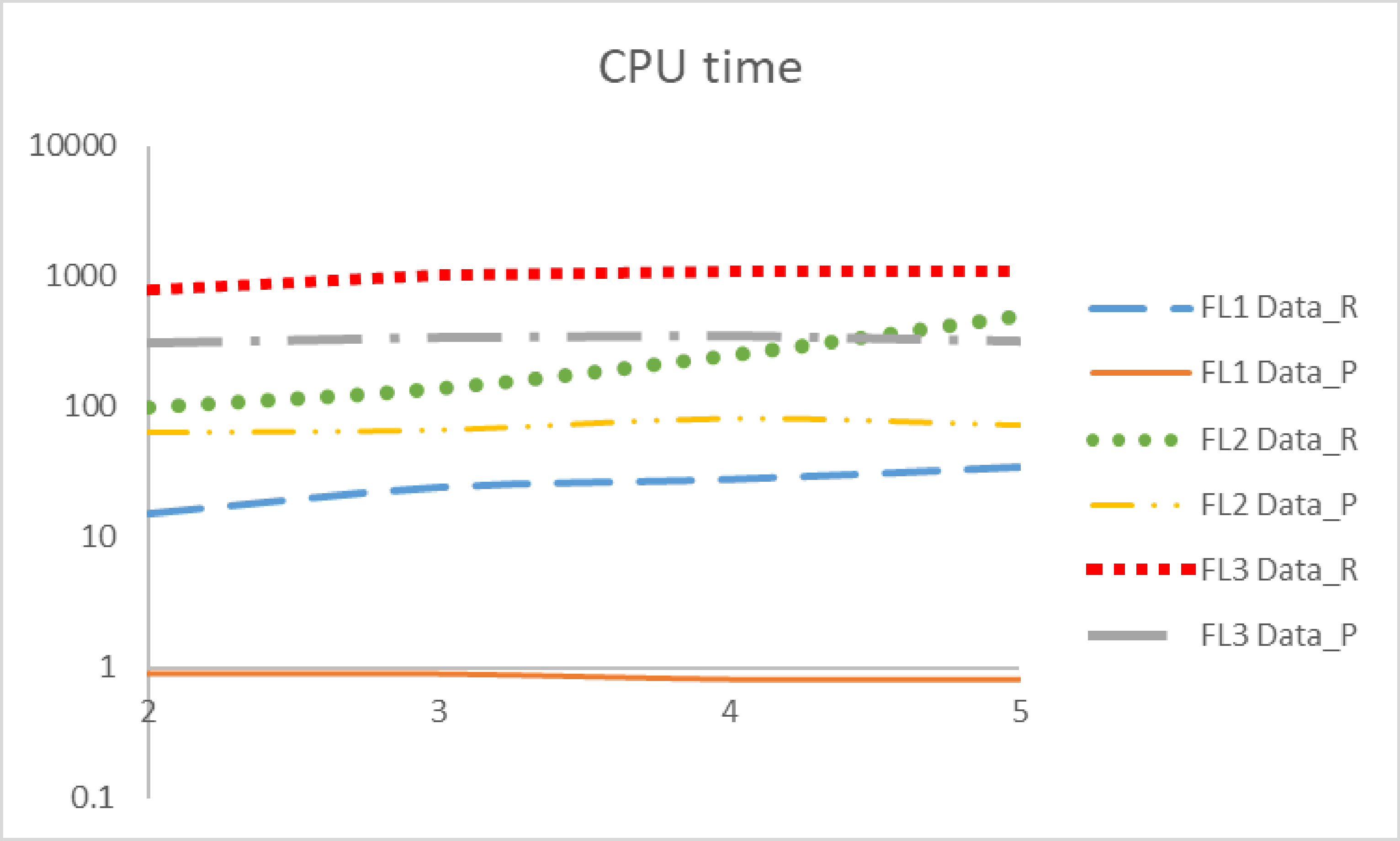
Summary of results by **size of n** (logarithmic scale)



The p-median problem with upgrading of transportation costs and minimum travel time allocation

FIRST STUDY: FL1, FL2, FL3 for DATASET R and DATASET P

Summary of results by **size of p** (logarithmic scale)

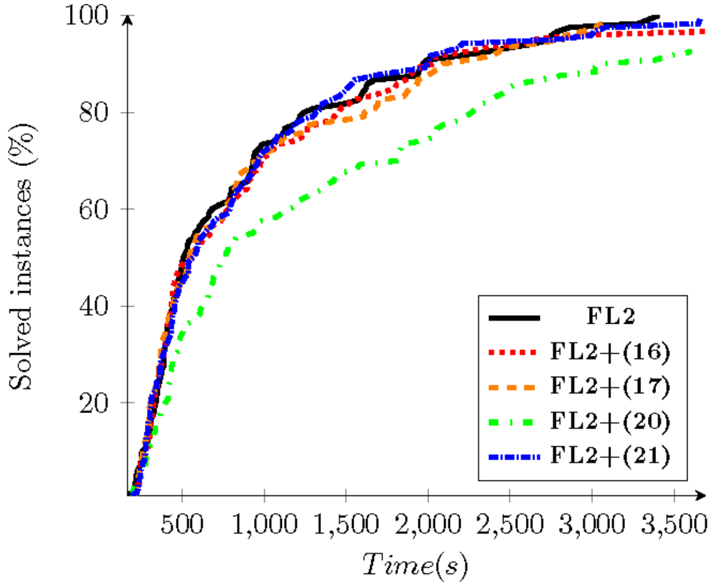


The p-median problem with upgrading of transportation costs and minimum travel time allocation

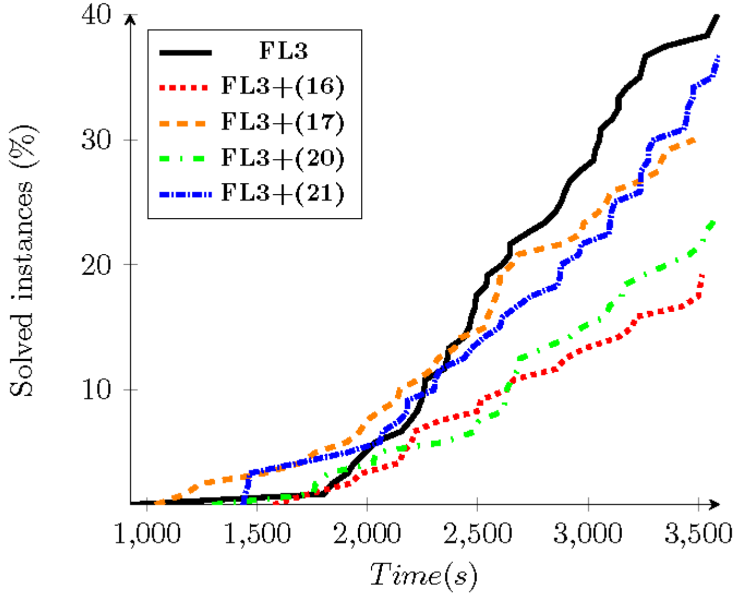
SECOND STUDY: FL1, FL2, FL3 with VALID INEQUALITIES for DATASET R

Percentage of instances solved with n = 80

FL2



FL3

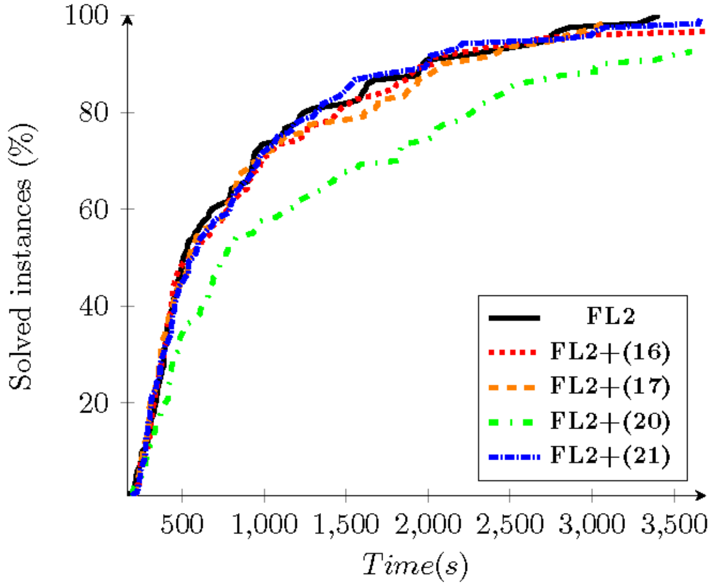


The p-median problem with upgrading of transportation costs and minimum travel time allocation

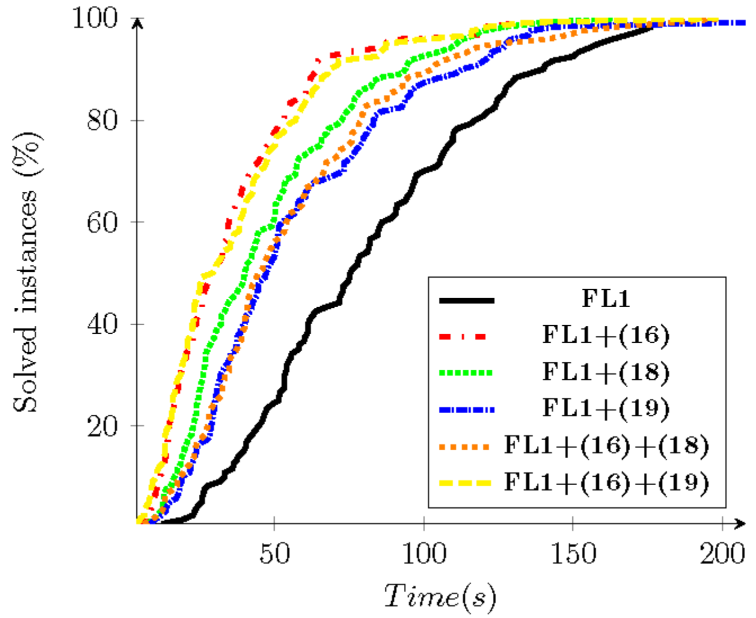
SECOND STUDY: FL1, FL2, FL3 with VALID INEQUALITIES for DATASET R

Percentage of instances solved with n = 80

FL2



FL1



The p-median problem with upgrading of transportation costs and minimum travel time allocation

THIRD STUDY: FL1 with MOST PROMISING VALID INEQUALITIES for DATASET R

p = 5 and B = 100

n	m	#	% GAP	TIME	NODES
100	500	3(3)	25.5	485	779
100	1000	3(3)	30.8	889	1608
125	500	3(3)	24.8	1417	1172
125	1000	3(3)	29.3	2002	1707
150	500	1(3)	13.8	1161	281
200	500	1(3)	15.1	3170	315

The p-median problem with upgrading of transportation costs and minimum travel time allocation

CONCLUDING REMARK

- Initial attempt to address p-median location problems considering two costs associated to the arcs of a network and the upgrading of arcs.
- Two different formulations considering variables with three and one indexes.
- Valid inequalities and a separation procedure.

The percentage of instances solved in less than 50 seconds (for a size of 80) increased by 50% after including some valid inequalities.

The three-indexed formulation with the most promising valid inequalities can optimally solve instances with up to 200 nodes in reasonable computational time.