Models for rescheduling train timetables when passenger's arrivals follow dynamic patterns

Juan A. Mesa [1], Francisco A. Ortega [2], Miguel A. Pozo [3], Justo Puerto [4]

[1] Department of Applied Math II. University of Seville (Spain). E-mail: jmesa@us.es
[2] Department of Applied Math I. University of Seville (Spain). E-mail: riejos@us.es
[3] Department of Statistics and Operational Research - IMUS. University of Seville (Spain). E-mail:
miguelpozo@us.es
[4] Department of Statistics and Operational Research - IMUS. University of Seville (Spain). E-mail:
puerto@us.es

V Encuentro de trabajo del Grupo de Investigación:
"Nuevos Desafíos de la Matemática Combinatoria"
28-30 de Noviembre del 2014, Sanlúcar de Barrameda

Outline

Outline

Example: Line C4 of Renfe Cercanías Madrid

Line C4 (Parla-Atocha)

	Stations													
	$s=1$		$s=2$		$s=3$		$s=4$		$s=5$		$s=6$		$s=7$	
#k	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}
1	6:04	335	6:10	1	6:13	44	6:15	7	6:18	44	6:21	46	6:31	147
2	6:12	177	6:18	5	6:21	113	6:24	48	6:26	124	6:29	81	6:38	302
3	6:16	307	6:22	1	6:25	35	6:28	29	6:30	64	6:34	58	6:42	123
4	6:22	55	6:28	8	6:31	138	6:33	54	6:36	163	6:39	86	6:48	234
5	6:28	429	6:34	10	6:36	145	6:39	62	6:42	173	6:44	119	6:54	349
6	6:34	511	6:40	4	6:42	102	6:44	26	6:46	153	6:50	115	7:00	571
7	6:40	484	6:46	12	6:48	151	6:50	54	6:52	119	6:57	107	7:07	129
8	6:46	491	6:52	10	6:54	166	6:56	70	7:00	157	7:04	160	7:12	184
9	6:54	414	7:00	24	7:03	254	7:05	119	7:08	158	7:12	185	7:20	490
10	7:01	476	7:07	17	7:10	195	7:12	88	7:15	106	7:18	145	7:26	514
11	7:09	421	7:15	33	7:18	260	7:20	111	7:24	146	7:27	209	7:35	491
12	7:16	550	7:22	38	7:25	218	7:27	158	7:30	123	7:34	298	7:43	574
13	7:22	414	7:28	36	7:31	247	7:34	119	7:38	136	7:41	243	7:48	451
14	7:28	421	7:34	26	7:37	127	7:41	104	7:44	97	7:47	154	7:54	276
15	7:34	386	7:40	31	7:42	145	7:45	113	7:48	103	7:53	144	8:00	424
16	7:40	384	7:46	47	7:48	171	7:50	108	7:54	99	7:58	180	8:06	284
17	7:46	323	7:52	31	7:54	202	7:57	134	8:00	128	8:04	231	8:12	647
18	7:52	408	7:58	19	8:01	190	8:02	77	8:05	84	8:09	192	8:18	446
19	7:58	441	8:03	49	8:06	210	8:09	91	8:13	119	8:15	223	8:24	335
20	8:04	165	8:10	47	8:13	229	8:15	110	8:19	126	8:23	259	8:30	338
21	8:11	347	8:15	44	8:18	225	8:21	134	8:25	98	8:28	165	8:36	302
22	8:16	336	8:22	38	8:25	294	8:28	112	8:30	79	8:33	158	8:42	271
23	8:22	317	8:28	33	8:31	230	8:34	119	8:36	119	8:40	156	8:48	410
24	8:28	335	8:34	36	8:37	119	8:39	91	8:41	61	8:45	144	8:54	364
25	8:34	265	8:40	13	8:43	117	8:45	66	8:47	43	8:52	153	9:00	381

Timetables and boarding in Line C4

Train timetabling problem

Train timetabling problem (TTP)

Obtaining and optimizing departure and arrival times for each service (trip from an origin to a final destination) to and from each station over a planning horizon imposing/optimizing different constraints and objectives.

Main constraints and objectives: User

1. Timetables should match the demand at best so as to avoid overcrowding and large headways, and thereby reduce waiting and transfer times.

2. Minimizing total travel time

3. The global network TTP should permit smooth transfers between lines.

Main constraints and objectives: Operator

1. Most of the papers optimize an objective function relevant to the operator (see Cacchiani et al., 2012; Barrena2014 et al., 2014).

2. Track capacities should be respected

3. The operator can be imposed minimum and/or maximum headways on some lines or areas by regulating authorities.

4. (Non)Periodicity of the timetables.

Railway rescheduling problem (1/2)

Rescheduling

Rescheduling is the process of updating an existing production plan in response to disturbances or disruptions (Vieira et al. 2003).

📄 Cacchiani, V., Huisman, D., Kidd, M., Kroon, L., Toth, P., Veelenturf, L. and Wagenaar, J. (2014) "An overview of recovery models and algorithms for real-time railway rescheduling". *Transportation Research Part B: Methodological*, vol. 63(0): 15 – 37.

Disturbances and disruptions

Disturbances are relatively small perturbations of the railway system that can be handled by modifying the timetable, but without modifying the duties for rolling stock and crew. Disruptions are relatively large incidents, requiring both the timetable and the duties for rolling stock and crew to be modified.

Macroscopic and microscopic levels

The macroscopic level considers the railway network at a higher level, in which stations can be represented by nodes of a graph and tracks by arcs, and the details of block sections and signals are not taken into account. In a microscopic level these aspects are considered in detail.

Railway rescheduling problem (2/2)

Main objectives and constraints

- ▶ minimize the deviations from the initial timetable proposed by the operator
- ▶ minimize the dissatisfaction experienced by the passengers due to disturbances in terms of waiting times and overflows (Kumazawa et al., 2010)
- ▶ preserve connections with other lines in normal operation

Differences with the scheduling phase

- ▶ Some services are canceled and/or new ones are inserted
- ▶ Demand dissatisfaction has to be modelled by using specific measures (Kanai et al. 2011)
- ▶ There is less flexibility in the rescheduling stage since connections have to be preserved

Outline

Infrastructure

$s \in S$ index identifying stations of set $S = \{1, ..., |S|\}$

l directed transit line running along set S

$\langle s, l \rangle \in S_l$ station in position $s \in \{1, ..., |S_l|\}$

$\langle s, l \rangle^{-1} \in S_l$ position of station $s \in S$

$t \in T$ index identifying the time horizon discretization T

$k \in K$ index identifying vehicles of set K ($|K| = \kappa$)

$k_{st} \in K$ vehicle that departs (strictly) after time $t - 1$ from s

$\overline{K} \subset K$ set of vehicles to be rescheduled ($|\overline{K}| = \overline{\kappa}$)

Demand (1/4): arrival pattern with unknown timetables

Passengers enter in a station s and wait until a train arrives. A passenger that arrived to station s at time t is served by the next vehicle that departs (strictly) after time $t - 1$ (denoted by $k_{st} \in K$).
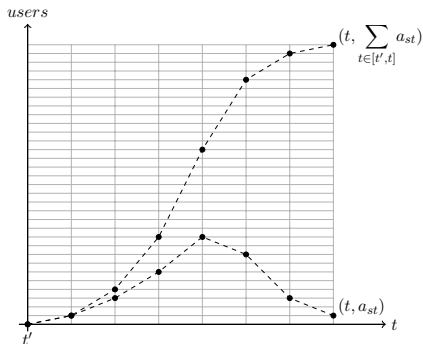
a_{st} number of passengers that access to station s at time t
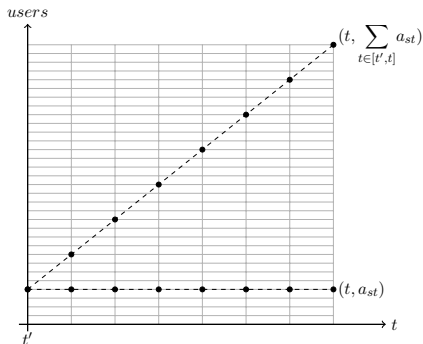
Demand (2/4): arrival pattern with known timetables

Passengers enter in a station s and wait until a train arrives. A passenger that arrived to station s at time t is served by the next vehicle that departs (strictly) after time $t - 1$ (denoted by $k_{st} \in K$).

a_{st} number of passengers that access to station s at time t

Timetables (1/2)

Arrival-departure timetable

Given the set of vehicles $k \in K$ defined in line l, a timetable Θ along partition T is defined as the set of arrival/departure times at each station for each vehicle: $\Theta = \{(\theta_{sk}^+, \theta_{sk}^-), s \in S, k \in K\}$.

Denoting by λ_{sk} the waiting time of vehicle k at station $s \in S$ and by μ_{sk} the travel time from station s to the next station (that is, the travel time between stations s and $\langle \langle s \rangle^{-1} + 1 \rangle$ we can assume that:

1 $\lambda_{sk} = \theta_{sk}^- - \theta_{sk}^+$

2 $\mu_{sk} = \theta_{\langle \langle s \rangle^{-1} + 1 \rangle, k}^+ - \theta_{sk}^-, \qquad s \in S : \langle s \rangle < |S|, k \in K$

Departure timetable

$\Theta \equiv x = \{x_{st}, t \in T, s \in S : \langle s \rangle < |S|\}$ where $x_{st} \in \{0, 1\}$ is equal to 1 if and only if a vehicle departs from station s at time t and:

▶ The departure time of vehicle k from station s:
$\theta_{sk}^- = \langle \{t : x_{st} = 1, t \in T\} \rangle_k$ (where $\langle \cdot \rangle_k$ denotes the k-th element of a set that is sorted in non decreasing order).

▶ The arrival time of vehicle k to station s: $\theta_{sk}^+ = \theta_{\langle \langle s \rangle^{-1} - 1 \rangle k}^- + \mu_{\langle \langle s \rangle^{-1} - 1 \rangle k}$.

▶ The waiting time of vehicle k at station $\lambda_{sk} = \theta_{sk}^- - \theta_{sk}^+$.

Timetables (2/2)

Example. Arrival-departure timetable for two vehicles in a directed transit line running along stations 12, 7, 9, 13 that occupy positions 1, 2, 3 and 4 in the line, respectively. We also show the departure timetable (in terms of variables x). By means of set T_s we can reduce the feasible time slots for each station as it is indicated.
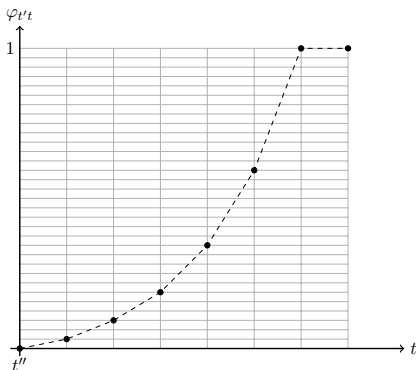
		$k=1$			$k=2$		
s	$\langle s \rangle^{-1}$	$(\theta_{sk}^+, \theta_{sk}^-)$	λ_{sk}	μ_{sk}	$(\theta_{sk}^+, \theta_{sk}^-)$	λ_{sk}	μ_{sk}
12	1	(1,2)	1	2	(1,8)	7	2
7	2	(4,7)	3	4	(10,12)	2	4
9	3	(11,13)	2	3	(16,19)	3	3
13	4	(16,23)	7	0	(22,23)	1	0

Table: Arrival-departure timetable

| s	$\langle s \rangle^{-1}$	$x = (x_{s1}, ..., x_{s|T|})$	T_s
12	1	$(x_{st}) : x_{st} = 1, t \in \{2, 8\}; x_{st} = 0, o.c.$	$[2, 10]$
7	2	$(x_{st}) : x_{st} = 1, t \in \{7, 12\}; x_{st} = 0, o.c.$	$[5, 13]$
9	3	$(x_{st}) : x_{st} = 1, t \in \{12, 19\}; x_{st} = 0, o.c.$	$[10, 18]$
13	4		

Table: Departure timetable

Demand (3/4): (in)convenience function in scheduling

$\varphi_{t't}$ inconvenience suffered by passengers that have entered to a station at time t' and have to wait until time t for departing

$$\varphi_{t't} = \begin{cases} 0, & t' < t \leq \theta^-_{s,k_{st'}}; \\ \alpha_t \in [0,1], & \theta^-_{s,k_{st'}} < t \end{cases}$$

Demand (4/4): (in)convenience function in rescheduling
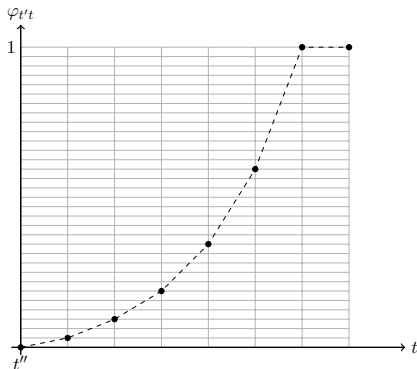
$\varphi_{t't}$ inconvenience suffered by passengers that have entered to a
station at time t' and have to wait until time t for departing

$$\varphi_{t't} = \begin{cases} 0, & t' < t \leq \theta^-_{s,k_{st'}}; \\ \alpha_t \in [0,1], & \theta^-_{s,k_{st'}} < t \leq \theta^-_{s,k_{st'}+1}; \\ 1, & \theta_{s,k_{st'}+1} < t; \end{cases}$$

Outline

Scheduling model: non-linear model

$$F^x: \quad z \equiv min \sum_{s \in S} \sum_{t' \in T_s} \sum_{t \in T_s : t' < t} a_{st'} \varphi_{t't} x_{st} \prod_{t' \le t'' < t} (1 - x_{st''}) \tag{1a}$$

$$s.t. \quad \sum_{t \in T_s} x_{st} = \bar{\kappa} \qquad\qquad s \in S \tag{1b}$$

$$\sum_{t' \le t} x_{st'} \le \sum_{t' \le t + \mu_{\langle s \rangle}} x_{\langle \langle s \rangle^{-1} + 1 \rangle t'} \qquad s \in S, t \in T_s \tag{1c}$$

$$x_{st} \in \{0, 1\} \qquad\qquad s \in S, t \in T_s \tag{1d}$$

Scheduling model: linear model

$$F^{xy}: \quad z \equiv min \sum_{s \in S} \sum_{t' \in T_s} \sum_{t \in T_u : t' < t} a_{st'} \varphi_{t't} y_{st't} \tag{2a}$$

$$s.t. \quad (1b) \tag{2b}$$

$$(1c) \tag{2c}$$

$$(1d) \tag{2d}$$

$$y_{st't} \leq x_{st} \qquad\qquad t', t \in T_s : t' \leq t, s \in S \tag{2e}$$

$$\sum_{t > t'} y_{st't} = 1 \qquad\qquad t' \in T_S, s \in S \tag{2f}$$

$$y_{st't} \in \{0, 1\} \qquad\qquad t', t \in T_s : t' \leq t, s \in S \tag{2g}$$

📄 Mesa, J. A., Ortega, F. A. and Pozo, M. A. (2014) "Locating optimal timetables and vehicle schedules in a transit line".
Annals of Operations Research, vol. 222: 439–455.

Rescheduling model: Considering all vehicles equal

$$F^{st}: \quad z \equiv \min \sum_{s \in S} \sum_{t' \in T_s} \left(\sum_{t \in T_s: t' < t < \theta_{s,k_{s,t'}+1}} a_{st'} \varphi_{t't} x_{st} + a_{st'} (1 - \sum_{t \in T_s: t' < t < \theta_{s,k_{s,t'}+1}} x_{st}) \right)$$

$$(3a)$$

$$s.t. \quad (1b) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3b)$$

$$(1c) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3c)$$

$$(1d) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3d)$$

$$\sum_{t \in T_s: \theta_{s,k-1} < t < \theta_{s,k+1}} x_{st} \leq 1 \qquad\qquad k \in K, s \in S \qquad (3e)$$

Rescheduling model: Considering all vehicles equal

Definition 1. The convenience cost function φ is defined as $\psi = 1 - \varphi$.

Property 1. Let ψ denote the convenience cost function of φ. Then, objective function (3a) is equivalent to:

$$z \equiv \max \sum_{s \in S} \sum_{t' \in T_s} \sum_{t \in T_s : t' < t} (a_{st'} \psi_{t't}) x_{st} \tag{4}$$

and, in particular, equivalent to:

$$z \equiv \max \sum_{s \in S} \sum_{t' \in T_s} \sum_{t \in T_s : t' < t < \theta_{s,k_{s,t'+1}}} (a_{st'} \psi_{t't}) x_{st} \tag{5}$$

Rescheduling model: Considering all vehicles equal

$$F^{st}: \quad z \equiv \max \sum_{s \in S} \sum_{t' \in T_s} \sum_{t \in T_s : t' < t < \theta_{s,k_{s,t'+1}}} (a_{st'} \psi_{t't}) x_{st} \tag{6a}$$

$$s.t. \quad (1b) \tag{6b}$$

$$(1c) \tag{6c}$$

$$(1d) \tag{6d}$$

$$\sum_{t \in T_s : \theta_{s,k-1} < t < \theta_{s,k+1}} x_{st} \leq 1 \qquad k \in K, s \in S \tag{6e}$$

Rescheduling model: Considering all vehicles different

$$F^{stk}: \quad z \equiv \max \sum_{k \in K} \sum_{s \in S} \sum_{t' \in T_s} \sum_{t \in T_s : t' < t < \theta_{s,k_{s,t'+1}}} (a_{st'} \psi_{t't}) x_{stk} \tag{7a}$$

$$s.t. \quad \sum_{t \in T_s} x_{stk} = 1 \qquad\qquad\qquad s \in S, k \in \overline{K} \tag{7b}$$

$$\sum_{t' \le t} x_{st'k} \le \sum_{t' \le t + \mu_s} x_{\langle\langle s \rangle^{-1}+1\rangle t' k} \qquad s \in S, t \in T_s, k \in K \tag{7c}$$

$$\sum_{k' \in K} \sum_{t \in T_s : \theta_{s,k-1} < t < \theta_{s,k+1}} x_{stk'} \le 1 \qquad s \in S, k \in K \tag{7d}$$

$$x_{stk} \in \{0,1\} \qquad\qquad\qquad s \in S, t \in T_s, k \in K \tag{7e}$$

A/D rescheduling model: considering all vehicles equal

$$F^{sut}: \quad z \equiv \max \sum_{s \in S} \sum_{t' \in T_s} \sum_{(u,t) \in T_s^2 : t' < t < \theta_{s,k_{s,t'+1}}} (a_{st'} \psi_{t't}) x_{sut} \tag{8a}$$

$$s.t. \quad \sum_{(u,t) \in T_s^2} x_{sut} = \kappa \qquad\qquad\qquad\qquad s \in S \tag{8b}$$

$$\sum_{(u,t') \in T_s^2 : t' \leq t} x_{sut'} \leq \sum_{(u,t') \in T_s^2 : t' \leq t + \mu_{\langle s \rangle}} x_{\langle \langle s \rangle^{-1}+1 \rangle ut'} \quad s \in S, t \in T_s \tag{8c}$$

$$\sum_{(u,t) \in T_s^2 : \theta_{s,k-1} < t < \theta_{s,k+1}} x_{sut} \leq 1 \qquad\qquad s \in S, k \in K \tag{8d}$$

$$x_{sut} \in \{0, 1\} \qquad\qquad\qquad\qquad s \in S, t \in T_s \tag{8e}$$

A/D rescheduling model: Considering all vehicles different

$$F^{sutk}: \quad z \equiv \max \sum_{k \in \bar{K}} \sum_{s \in S} \sum_{t' \in T_s} \sum_{(u,t) \in T_s^2 : t' < t < \theta_{s,k_{s,t'+1}}} (a_{st'} \psi_{t't}) x_{sutk} \quad (9a)$$

$$s.t. \sum_{(u,t) \in T_s^2} x_{sutk} = 1 \qquad\qquad s \in S, k \in \bar{K}$$
$$(9b)$$

$$\sum_{(u,t') \in T_s^2 : t' \le t} x_{sut'k} \le \sum_{(u,t') \in T_s^2 : t' \le t + \mu_{\langle s \rangle}} x_{\langle \langle s \rangle^{-1}+1 \rangle ut'k} \quad s \in S, t \in T_s, k \in \bar{K}$$
$$(9c)$$

$$\sum_{k' \in \bar{K}} \sum_{(u,t) \in T_s^2 : \theta_{s,k-1} < t < \theta_{s,k+1}} x_{sutk'} \le 1 \qquad s \in S, k \in K$$
$$(9d)$$

$$x_{sutk} \in \{0,1\} \qquad\qquad s \in S, t \in T_s$$
$$(9e)$$

A/D rescheduling model: Considering all vehicles different and transfers

$$z \equiv \max \sum_{k \in \bar{K}} \sum_{s \in S} \sum_{t' \in T_s} \sum_{(u,t) \in T_s^2 : t' < t < \theta_{s,k_{s,t'+1}}} (a_{st'} - (\sum_{s' \in \bar{S}} a_{ss't'}) \psi_{t't}) x_{sutk} +$$

$$+ \sum_{k \in \bar{K}} \sum_{s \in S} \sum_{s' \in \bar{S}} \sum_{t' \in T_s} \sum_{(u,t) \in T_s^2 : t' < t < \theta_{s,k_{s,t'+1}}} \sum_{(u'',t'') \in T_{s'}^2 : u'' \leq \tau_{s' \theta_{s,k_{s,t'}}}} a_{ss't'} x_{sutk} x_{s'u''t''k}$$

$$\tag{10a}$$

$$s.t. (9b) \tag{10b}$$
$$(9c) \tag{10c}$$
$$(9d) \tag{10d}$$
$$(9e) \tag{10e}$$

Outline

Computational experience

We have successfully applied our models in random instances of sizes
$|T| = \{60, 120, 180, 240\}$ and $|\bar{K}| = \{1, ..., 19\}$

Further details at:

📄 Mesa, J. A., Ortega, F. A. and Pozo, M. A. (2013) "A geometric model for an effective
rescheduling after reducing service in public transportation systems".
Computers & Operations Research, vol. 40(3): 737–746.

📄 Mesa, J. A., Ortega, F., Ángel Pozo, M. and Puerto, J. (2014) "Rescheduling railway
timetables in presence of passenger transfers between lines within a transportation network".
In J. F. Sousa and R. Rossi, eds., *Computer-based Modelling and Optimization in
Transportation*, vol. 262 of *Advances in Intelligent Systems and Computing*, pp. 347–360.
Springer International Publishing Switzerland.

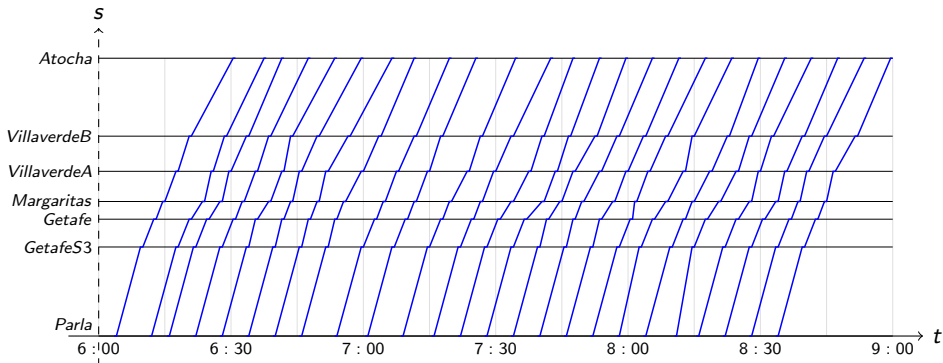## Application to Madrid local trains (RENFE cercanías)

Line C4 (Parla-Atocha)

| #k | Stations | | | | | | | | | | | | | |
| | $s = 1$ | | $s = 2$ | | $s = 3$ | | $s = 4$ | | $s = 5$ | | $s = 6$ | | $s = 7$ | |
	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}	t	a_{stk}
1	6:04	335	6:10	1	6:13	44	6:15	7	6:18	44	6:21	46	6:31	147
2	6:12	177	6:18	5	6:21	113	6:24	48	6:26	124	6:29	81	6:38	302
3	6:16	307	6:22	1	6:25	35	6:28	29	6:30	64	6:34	58	6:42	123
4	6:22	55	6:28	8	6:31	138	6:33	54	6:36	163	6:39	86	6:48	234
5	6:28	429	6:34	10	6:36	145	6:39	62	6:42	173	6:44	119	6:54	349
6	6:34	511	6:40	4	6:42	102	6:44	26	6:46	153	6:50	115	7:00	571
7	6:40	484	6:46	12	6:48	151	6:50	54	6:52	119	6:57	107	7:07	129
8	6:46	491	6:52	10	6:54	166	6:56	70	7:00	157	7:04	160	7:12	184
9	6:54	414	7:00	24	7:03	254	7:05	119	7:08	158	7:12	185	7:20	490
10	7:01	476	7:07	17	7:10	195	7:12	88	7:15	106	7:18	145	7:26	514
11	7:09	421	7:15	33	7:18	260	7:20	111	7:24	146	7:27	209	7:35	491
12	7:16	550	7:22	38	7:25	218	7:27	158	7:30	123	7:34	298	7:43	574
13	7:22	414	7:28	36	7:31	247	7:34	119	7:38	136	7:41	243	7:48	451
14	7:28	421	7:34	26	7:37	127	7:41	104	7:44	97	7:47	154	7:54	276
15	7:34	386	7:40	31	7:42	145	7:45	113	7:48	103	7:53	144	8:00	424
16	7:40	384	7:46	47	7:48	171	7:50	108	7:54	99	7:58	180	8:06	284
17	7:46	323	7:52	31	7:54	202	7:57	134	8:00	128	8:04	231	8:12	647
18	7:52	408	7:58	19	8:01	190	8:02	77	8:05	84	8:09	192	8:18	446
19	7:58	441	8:03	49	8:06	210	8:09	91	8:13	119	8:15	223	8:24	335
20	8:04	165	8:10	47	8:13	229	8:15	110	8:19	126	8:23	259	8:30	338
21	8:11	347	8:15	44	8:18	225	8:21	134	8:25	98	8:28	165	8:36	302
22	8:16	336	8:22	38	8:25	294	8:28	112	8:30	79	8:33	158	8:42	271
23	8:22	317	8:28	33	8:31	230	8:34	119	8:36	119	8:40	156	8:48	410
24	8:28	335	8:34	36	8:37	119	8:39	91	8:41	61	8:45	144	8:54	364
25	8:34	265	8:40	13	8:43	117	8:45	66	8:47	43	8:52	153	9:00	381

Timetables and boarding in Line C4

Application to Madrid local trains (RENFE cercanías)

Timetables corresponding to 25 vehicles of line C4 running along period [6:30,8:35]
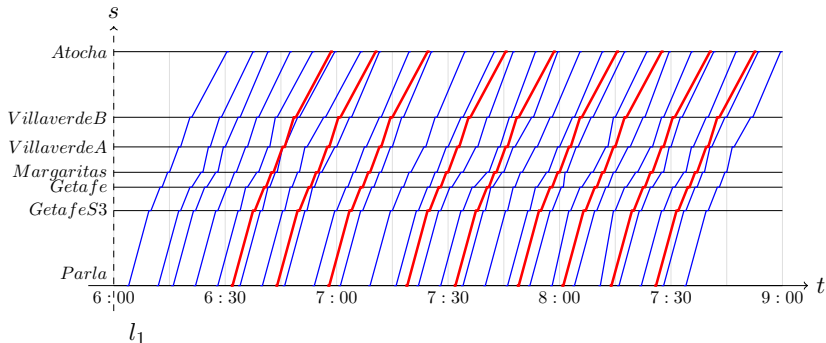
Application to Madrid local trains (RENFE cercanías)

Figure: Timetables (in blue) and 9 rescheduled timetables (bold red)

▶ The solution obtained from rescheduling the remaining 9 vehicles outperforms the best solution obtained just canceling services in a 20.9%.

Outline

Conclusions and future research

- ▶ We have presented a new approach for implementing a timetable rescheduling along a railway line after a fleet size reduction.
- ▶ A general modeling framework has been proposed for the rescheduling problem in order to add side constraints or further objectives as necessary.
- ▶ Transfers with other transportation systems have been considered.

Thanks for your attention
Questions, comments, suggestions… are welcome.