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RESCHEDULING RAILWAY TIMETABLES IN PRESENCE OF PASSENGER TRANSFERS BETWEEN LINES WITHIN A TRANSPORTATION NETWORK

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Contents

1. INTRODUCTION

Line Planning is a key phase when designing a public transportation system.

It consists of determining a set of transit lines with their corresponding operating frequencies, such that most of trip demand can be satisfied.

PLANNING PUBLIC TRANSPORT	
RAILWAY NETWORK'S INFRASTRUCTURE	STRATEGICAL level (1)
LINE DESIGN	STRATEGICAL level(2)
SETTING FREQUENCIES	TACTICAL level
ROLLING STOCK	OPERATIONAL level (1)
CREW SCHEDULES	OPERATIONAL level (2)

There are two main perspectives:

Minimize operating costs (point of view of the operator)

Minimize riding and transfer times (perspective of passengers).

1. INTRODUCTION (i)

Given a railway infrastructure provided with different sections along a single transit line, the **Train Timetabling Problem** (TTP) consists of computing timetables that satisfy existing constraints and that optimize a single/multicriteria objective function for trains of both, passengers and /or cargo.

Timetable design is a central problem in railway planning with many interfaces with other classical problems: line planning, vehicle scheduling, and delay management.

The single-line Train Timetabling Problem is devoted to obtaining and optimizing timetables for trains with different levels of priority that share a railway line with single and multiple track sections.

1. INTRODUCTION (ii)

The **requirement for periodicity** of the timetables leads to the classification of TTP into Periodic (or cyclic) Train Timetabling and, on the other hand, Non-Periodic Train Timetabling.

In Periodic Timetabling, the timetable is easy to remember for the passengers although its solutions can become inefficient when planning resources such as crews and rolling stock.

Serafini, P., & Ukovich, W. (1989). A mathematical for periodic scheduling problems. *SIAM J. Discret. Math*., 2, 550–581.

Nachtigall, K., & Voget, S. (1996). A genetic algorithm approach to periodic railway synchronization. *Computers & Operations Research*, 23, 453–463.

Odijk, M. (1996). A constraint generation algorithm for the construction of periodic railway timetables. *Trans. Research Part B*, 30, 455–464.

Kroon, L., & Peeters, L. (2003). A variable time model for cycling railway timetabling. *Trans. Science*, 37, 198–212.

ARRIVAL project (http://arrival.cti.gr/, 2009).

1. INTRODUCTION (iii)

Non-Periodic Train Timetabling is relevant in presence of disturbances that can affect to the operativeness of train transit. The non-periodic train timetabling problem has been considered by most authors:

Szpigel, B. (1973). Optimal train scheduling on a single track railway. In: Roos, M. (ed.) *Proceedings of IFORS Conference on Operational Research 1972*, (pp. 343–352).

Jovanovic, D., & Harker, P.T. (1991). Tactical scheduling of rail operations: The SCAN-I system. *Trans. Science*, 25, 46–64.

Cai, X., & Goh, C.J. (1994). A fast heuristic for the train scheduling problem. *Computers & Operation Research*, 21, 499–510.

Carey, M., & Lockwood, D. (1995). A model, algorithms and strategy for train pathing. *Journal of the Operational Research Society*, 46, 988–1005.

Higgins, A., Kozan, E., & Ferreira, L. (1997). Heuristic techniques for single line train scheduling. *Journal of Heuristics* 3, 43–62.

Silva de Oliveira, E. (2001). *Solving Single-Track Railway Scheduling Problem Using Constraint Programming*. PhD thesis, The University of Leeds, School of Computing.

Caprara, A., Monaci, M., Toth, P., & Guida, P. (2006). A lagrangian heuristic algorithm for a real -world train timetabling problem. *Discrete Applied Mathematics*, 154, 738–753.

Ingolotti, L., Lova, A., Barber, F., Tormos, P., Salido, M.A., & Abril, M. (2006). New heuristics to solve the csop railway timetabling problem. *Lecture Notes in Computer Science*, 4031, 400–409.

Barber, F., Ingolotti, L., Lova, A., Tormos, P., & Salido, M.A. (2009). Meta-heuristic and Constraint-Based Approaches for Single-Line Railway Timetabling. *Lecture Notes in Computer Science*, 5868, 145–181.

Mesa, J.A., Ortega, F.A., & Pozo, M.A. (2009). Effective allocation of fleet frequencies by reducing intermediate stops and short turning in transit systems. *Lecture Notes in Computer Science*, 5868, 293–309.

Michaelis M., & Schöbel, A. (2009). Integrating line planning, timetabling, and vehicle scheduling: a customer-oriented heuristic. *Journal of Public Transport*, 1, 211–32.

2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (i)

Managers usually use running maps as graphic tools to plan train timetables. A **running map** is a time-space diagram where possible crossings of trains can be observed. Figure 1 shows the C4 line that belongs to the Madrid commuter railway network .
Figure 2 shows twenty-five instances of train schedules along the previous transit corridor.

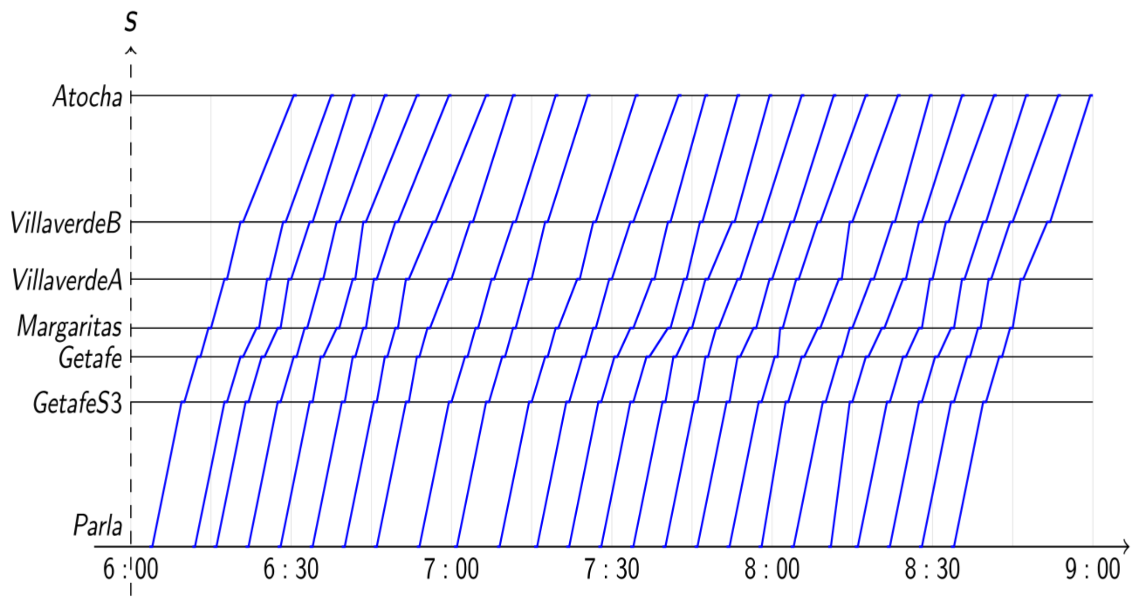
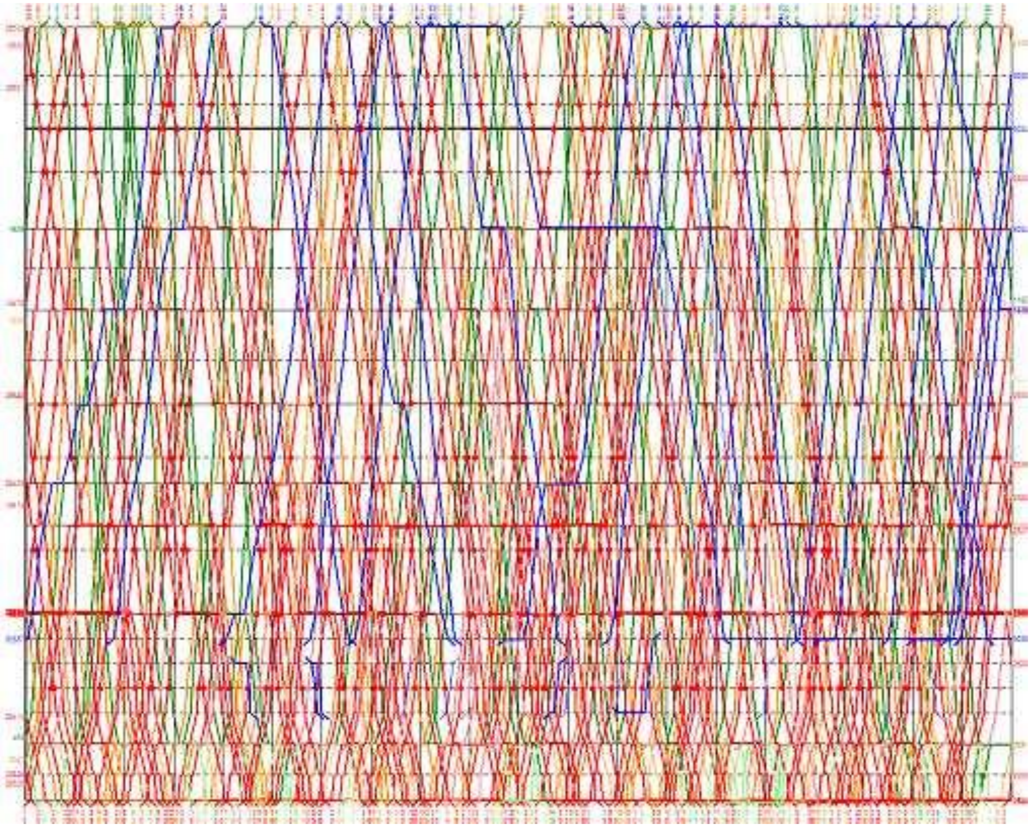


Fig 1.: Line C4 (Parla-Atocha).

Fig 2.: Train schedule

2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (ii)

A transit line of **high traffic density** will generate in a **labyrinthine tangle** of polygonal lines, each of which will correspond to the hours of operation of a train, **making infeasible a non-automated assessment of the possible alternatives**.



2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (iii)

MESA, J.A., ORTEGA, F.A., & POZO, M.A. (2013). A geometric model for an effective rescheduling after reducing service in public transportation systems. *Computers & Operations Research*, 40, 737-746.

Context: SINGLE RAILWAYS CORRIDOR

H1. All trains run by the same railways corridor in one direction and at **constant commercial speed** along the way.

H2. There is a **common time period (h)** that is used as unit for quantifying the time required **for all service tasks** sequenced.

H3. **Time** taken to travel without stopping **between two consecutive** stations is h.

H4. Minimum **time required for boarding and alighting** passengers on/from train is also h.

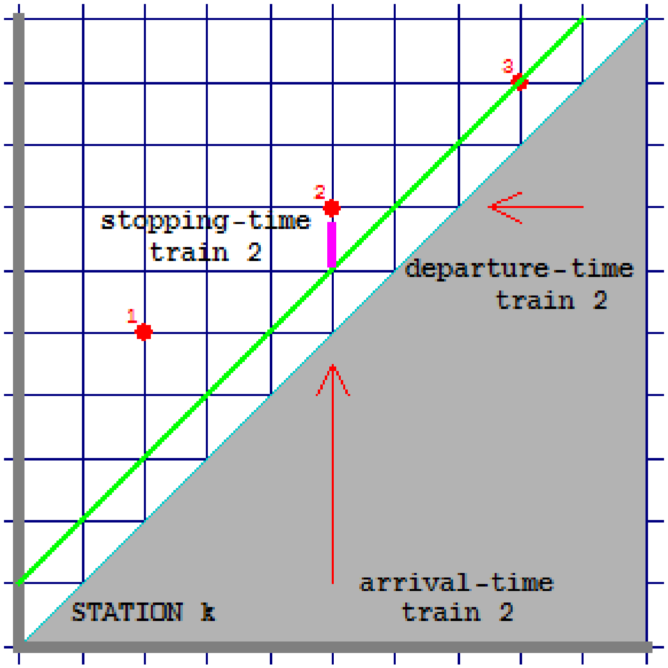
H5. Temporary **security margin** between each pair of consecutive trains is **a multiple of h**.

The **above assumptions can be relaxed** without altering the validity of the model.

2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (iv)

Event-activity maps at stations along the corridor.

A uniform grid of squares of length h (in terms of time) establishes feasible times for locating train maneuvers at each station of the line. Each active point in the event-activity map will indicate, simultaneously, arrival time (X-coordinate) and departure time (Y-coordinate) of an specific train.

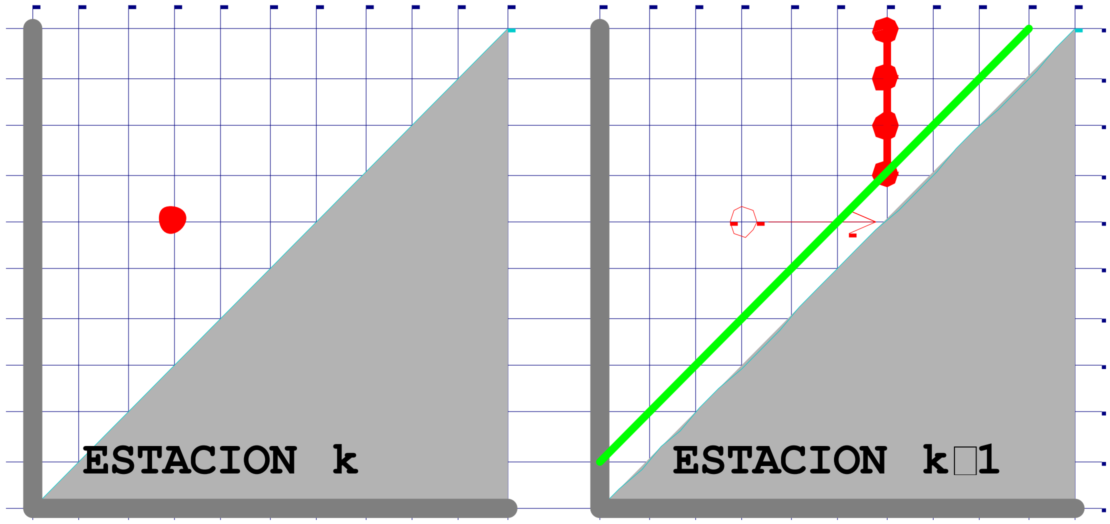


Three trains passing through the station k

2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (v)

The **sequence of stations** (with stopping or not) along the railway line **will correspond to a succession of temporary diagrams** with active points indicating arrival-departure timetables.

Each timetable-point in the k-th diagram of activity will **match** to some other feasible point of the vertical segment that starts from its **projection** on the diagonal in the (k +1)-th activity-map.



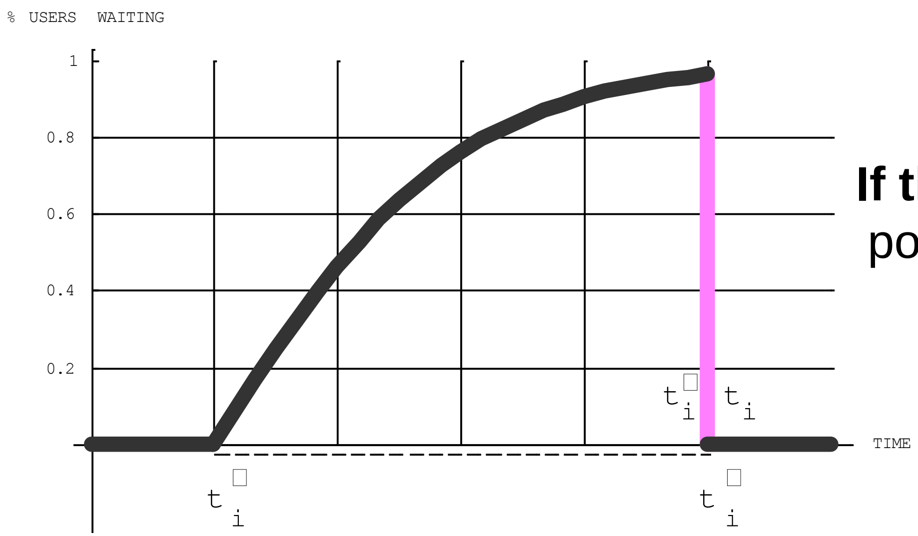
Feasibility zone of timetable-points between consecutive stations

3. PATTERN OF DEMAND BEHAVIOR (i)

Assume that **arrival / departure times of trains at stations** were **previously set** and are **known by users**.

Figure explains in percentage terms the **travelers' accumulation on the platform** of station *k*, due to the imminent arrival of the scheduled train *i* at time t_i.

Time interval associated with the arrival of travelers to the platform is $\left[t_i^-, t_i^+\right]$

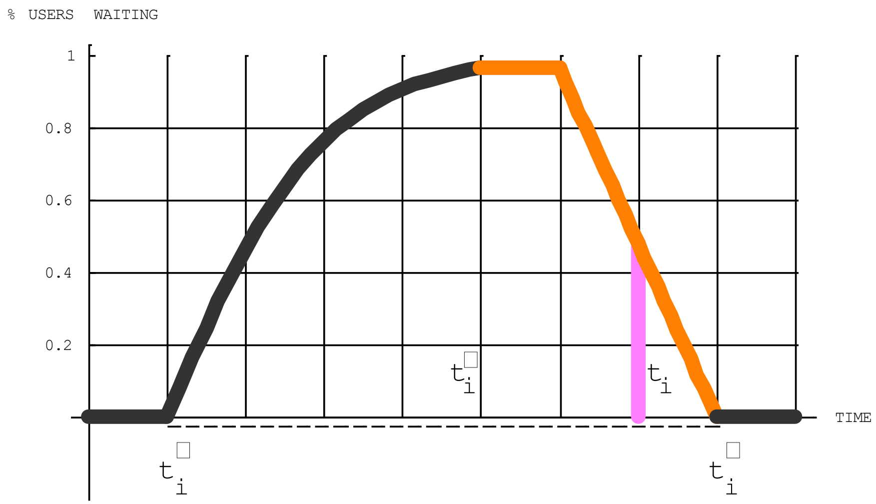


Usual demand behavior in terms of percentage of user´s presence at platform

If the train *i* arrived on time, the whole population placed on platform could be transported, as shows the figure.

3. PATTERN OF DEMAND BEHAVIOR (ii)

Nevertheless, **if train *i* were delayed, the reaction of users** when they know the existence of such delay **would consist of initially waiting along a short certain period of time**. Subsequently, the curve that models the percentage of population waiting would appear stabilized. **After this period, the traveler population gradually decreases** until disappearing.

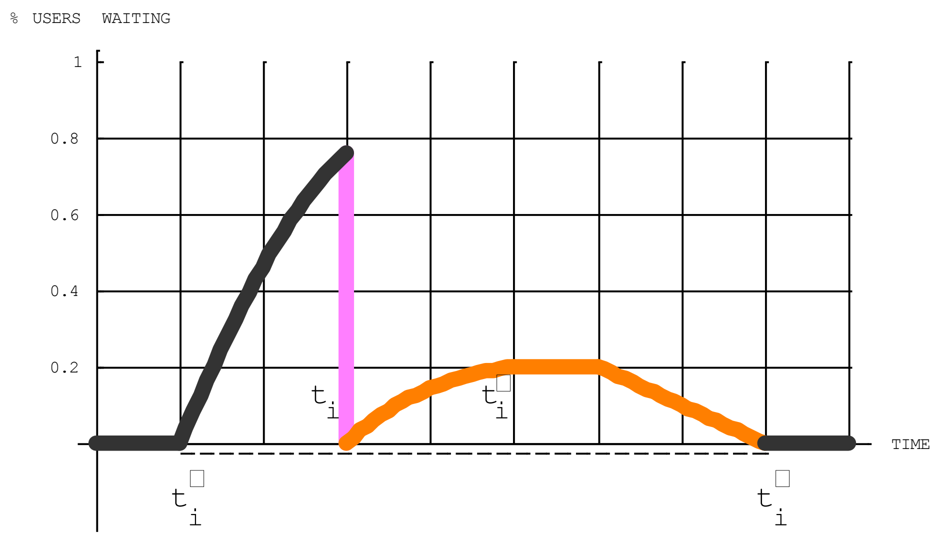


Demand behavior when train is delayed.

If the train arrived late, **only a portion of the population** that normally waits could be transported.

3. PATTERN OF DEMAND BEHAVIOR (iii)

Finally, **if the train arrived and departed in advance, only users who were already placed on the platform could take the train**. The other passengers will be coming in the usual way, because they were unaware of this schedule change (see figure).

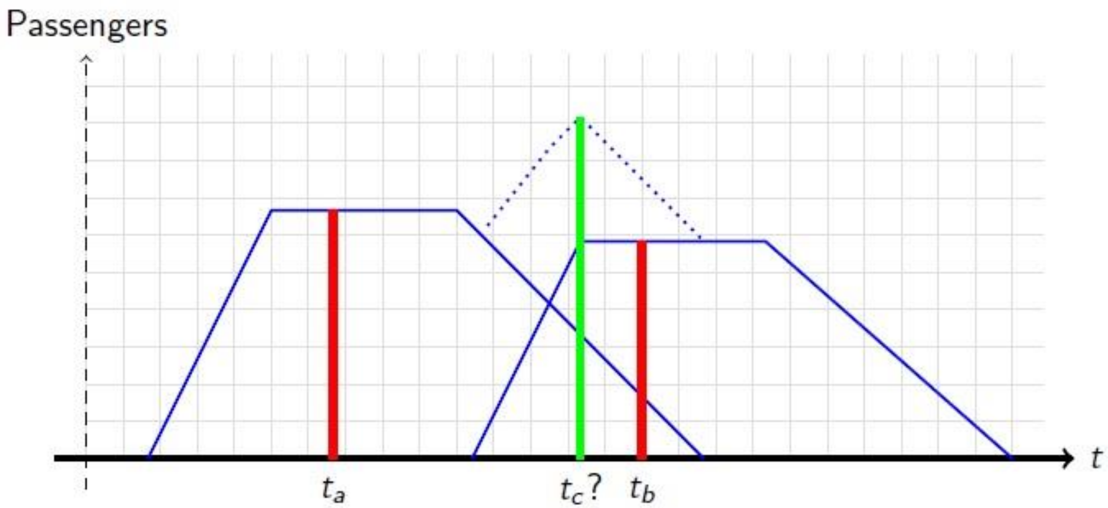


Demand behavior if train departed in advance.

The option to wait a certain interval of time leads to the possibility of taking the next train.

3. PATTERN OF DEMAND BEHAVIOR (iv)

Assuming this behavior pattern, **a new time for the train arrival/departure** at the station can be determined, taking advantage of these overlapping demand curves.



Overlapping curves of population behaviour

The subsequent rescheduling of train timetables will have **the objective of minimizing the loss of passengers**.

Two scenarios can be considered depending on that passengers require transfers toward / from other network lines are (or not) at particular times.

4. FORMULATION OF THE MODEL WITHOUT TRANSFERS (i)

Indices and Sets

$i \in I$	index identifying trains of set I
$k \in K$	index identifying cantons (or stations) of set K
$u, v \in T$	indices identifying the time horizon discretization T
$(u, v) \in M_k$	coordinates corresponding to temporary map *M* at station *k*

Parameters

a_v^{ik}	population available to boarding to train *i* at station *k* and at time *v*

Variables

x_{uv}^{ik}	binary variable equals to 1 if train *i* is located at point (u, v) at station k; 0, otherwise

FORMULATION

(1)
$$\text{Max} \sum_{i \in I} \sum_{k \in K} \sum_{(u,v) \in M_k} a_v^{ik} \; x_{uv}^{ik}$$

The objective function maximizes customers' mobility by using the train system

4. FORMULATION OF THE MODEL WITHOUT TRANSFERS (ii)

$$\sum_{i\in I}\sum_{(u,v)\in M_k} x_{uv}^{ik} = |I|; \quad k\in K$$

(1) The number of train schedules to be located must be exactly |*I*|.

$$\sum_{k\in K}\sum_{(u,v)\in M_k} x_{uv}^{ik} = |K|; \quad i\in I$$

(2) Forced passage through each station (with or without stopping) for all trains to be determined

$$\sum_{i\in I}\sum_{u<v} x_{uv}^{ik} \le 1, \sum_{i\in I}\sum_{v>u} x_{uv}^{ik} \le 1; \quad (u,v)\in M_k,\ k\in K$$

(3, 4) There can be no train arriving/departing from the k-th station if there was just another train operating

$$x_{uv}^{ik} \le \sum_{v'>v} x_{v+1\,v'}^{i\,k+1}; (u,v)\in M_k, i\in I, k\in K\,(k\ne |K|)$$

(5) If there is a timetable-point located at position (u, v) of the temporary map for the k-th station, then there must be another timetable point, at the (k + 1)-th station, on the v-th column

$$\sum_{i\in I}\sum_{\substack{(u',v')\in M_k\\ u'<v;v'>u}} x_{u'v'}^{ik} \le n_k - x_{uv}^{ik}; \quad ; \quad (u,v)\in M_k,\ k\in K$$

(6) Limitation of the number of trains that can operate, according to the existing number of tracks

$$x_{uv}^{ik} \in \{0,1\}; \quad (u,v)\in T, i\in I, k\in K$$

(7) Binary nature of the decision variables

4. GREEDY ALGORITHM for selecting the |I| better solutions in sequence

[Step 1] Set the mesh density of parameter **h**. **Generate** the sequence of **temporary maps** corresponding to the sections of railway line. **Locate** the **existing timetable-points** *(u,v)*.

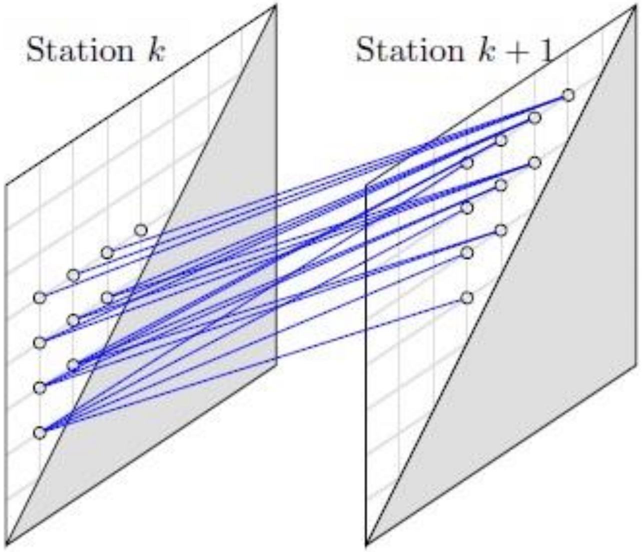
[Step 2] Estimate populations a_{uv}^{k} for the remaining unmeasured timetable-points, according to the previous procedure, and obtain the maximum value $a^{\max}$

[Step 3] Build an initial feasible graph G_1, composed of a sequence of maps ranging from the map k = 1 to $k = |K| - 1$ and whose edges connect points (u, v) of the map k-th with points of the (k +1)-th map, according to feasibility criteria.

[Step 4] i=1.

While i is less than $|I|$

- Using a shortest path algorithm, **determine the i-th optimal path connecting the two terminal stations of the line through the sequence of maps that represents graph G_i**
- **Remove the feasible arcs** used in the i-th path and those infeasible (isolated) arcs arising from the previous reduction. The new graph is denoted by G_{i+1} for the next iteration.
- $i:=i+1$



5. AN EXAMPLE (i)

- Assume a railway line that consists of 7 equi-spaced stations, separated from each other by a distance (travel time) equal to h. There are 3 vehicles crossing the line.
- Operating time [8:20 to 9:30] is discrete with periodicity of size h = 2 minutes.
- The arrival / departure timetables at stations are known by users (Table 1) in addition to the matrix of users' arrivals at each station (Table 2)

Station Number	1	2	3	4	5	6	7
Train 1: Arrival/Departure	8:26/8:28	8:30/8:32	8:34/8:36	8:38/8:42	8:44/8:48	8:50/8:52	8:54/8:56
Train 2: Arrival/Departure	8:38/8:40	8:42/8:44	8:46/8:48	8:50/8:54	8:56/9:00	9:02/9:04	9:06/9:08
Train 3: Arrival/Departure	8:50/8:52	8:54/8:56	8:58/9:00	9:02/9:06	9:08/9:12	9:14/9:16	9:18/9:20

Station Number	1	2	3	4	5	6	7
Train 1: Passengers	1417	1153	664	281	77	39	0
Train 2: Passengers	1143	756	359	113	23	10	0
Train 3: Passengers	2131	1204	488	117	18	7	0

5. AN EXAMPLE (ii)

- Assume that, as consequence of an incident, the system operator must reduce the fleet size by one unit. Rescheduling train timetables must minimize the loss of users, by introducing advances or delays in the original schedules of the two vehicles which will remain operative. According to the previous model, the following **distribution of passengers that access to stations** is shown in table 1.

TIMING	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
ST 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ST 6	0	0	0	0	0	0	0	0	0	0	0	0	15	25	32	36	39	0	9	6	8	9	10	0	6	6	6	6	7	0	0	0	0	0	0
ST 5	0	0	0	0	0	0	0	0	0	0	30	50	63	72	77	0	21	15	19	21	23	0	7	12	15	17	18	0	0	0	0	0	0	0	0
ST 4	0	0	0	0	0	0	0	110	182	229	261	281	0	44	73	92	105	113	0	46	76	96	108	117	0	0	0	0	0	0	0	0	0	0	0
ST 3	0	0	0	0	250	430	542	616	664	0	140	233	293	333	359	0	191	316	398	453	488	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ST 2	0	0	451	747	942	1069	1153	0	296	490	617	701	756	0	471	780	983	1116	1204	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ST 1	554	918	1157	1314	1417	0	447	740	933	1060	1143	0	833	833	1381	1740	2131	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

- **If train were not punctual**, population waiting for boarding could be deterministically estimated (see table 2). Since it is assumed that the user loss for railway system is only caused by decisions of putting advanced or delayed schedules, the sequence of blue cells indicates optimal reprogramming of the two feasible schedules.

TIMING	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
ST 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ST 6	0	0	0	0	0	0	0	0	0	0	0	0	15	25	32	36	39	36	36	32	23	9	10	9	10	10	9	5	7	5	5	4	2	0	0
ST 5	0	0	0	0	0	0	0	0	0	0	30	50	63	72	77	72	72	65	49	22	23	22	26	26	23	16	18	16	14	11	7	0	0	0	0
ST 4	0	0	0	0	0	0	0	110	182	229	261	281	251	274	255	202	105	113	105	138	149	140	108	117	108	95	76	46	0	0	0	0	0	0	0
ST 3	0	0	0	0	250	430	542	616	664	616	683	653	553	333	359	333	484	549	538	452	488	452	396	316	191	0	0	0	0	0	0	0	0	0	0
ST 2	0	0	451	747	942	1069	1153	1069	1237	1237	1068	701	756	701	1088	1269	1278	1116	1204	1116	983	780	471	0	0	0	0	0	0	0	0	0	0	0	0
ST 1	554	918	1157	1314	1417	1314	1603	1658	1486	1059	1143	1059	1766	2120	2186	1978	2131	1976	1740	1380	833	0	0	0	0	0	0	0	0	0	0	0	0	0	0

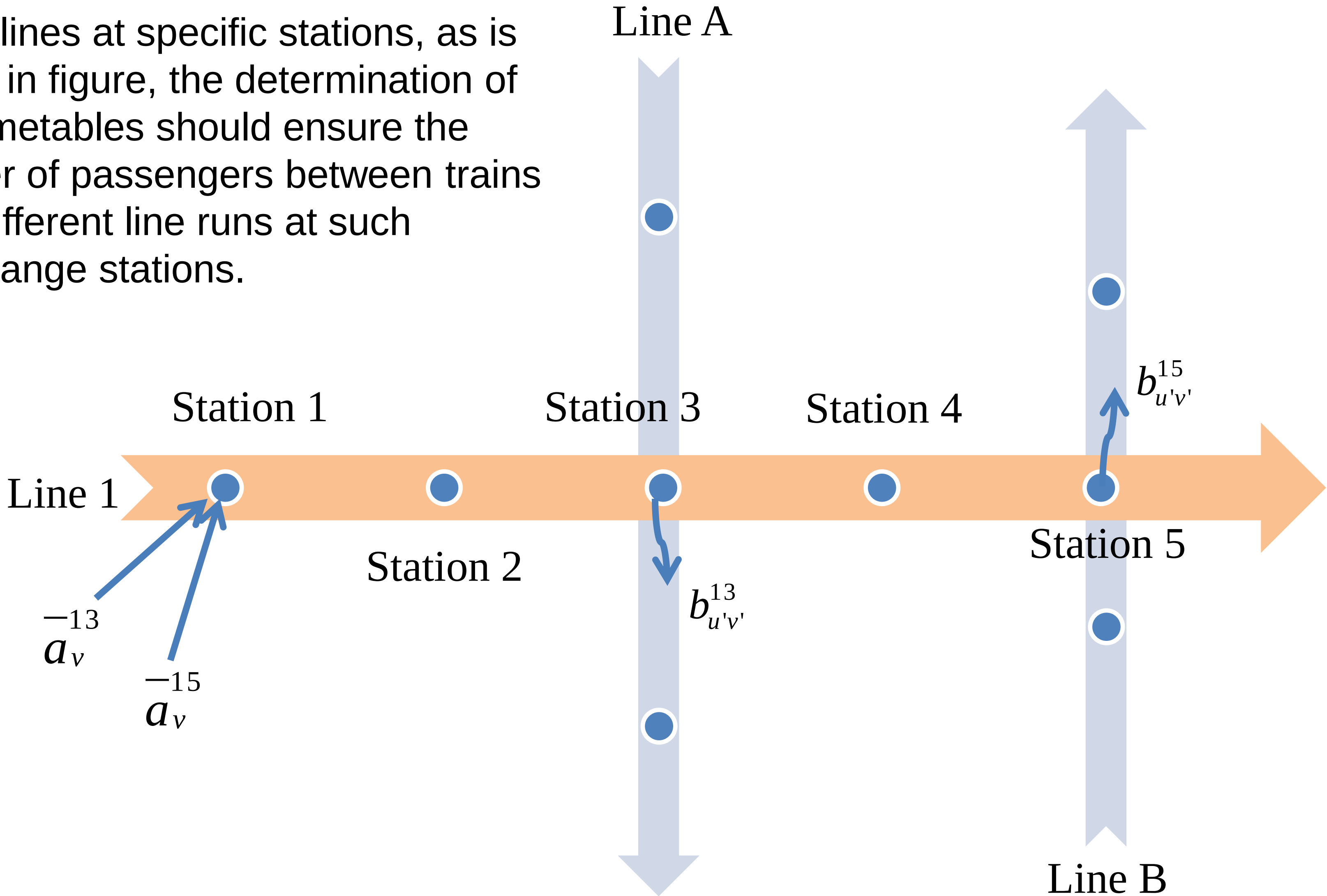
5. AN EXAMPLE (iii)

- The solution after applying a myopic methodology (cancel the train that serves the smallest number of users) can be compared with that obtained by applying the model (by introducing small advances or delays in the starting times to reduce the loss of users).
- The results obtained are summarized in Table 4

TABLE 4: PASSENGERS			
	INITIAL STATUS	MYOPIC SOLUTION	MODEL SOLUTION
TRAIN 1	3631	3631	3940 (1st.)
TRAIN 2	2404	..	4187 (2nd.)
TRAIN 3	3965	3965	..
TOTAL	10000	7596	8122
% LOSS	0%	-24,04%	-18,73%

6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (i)

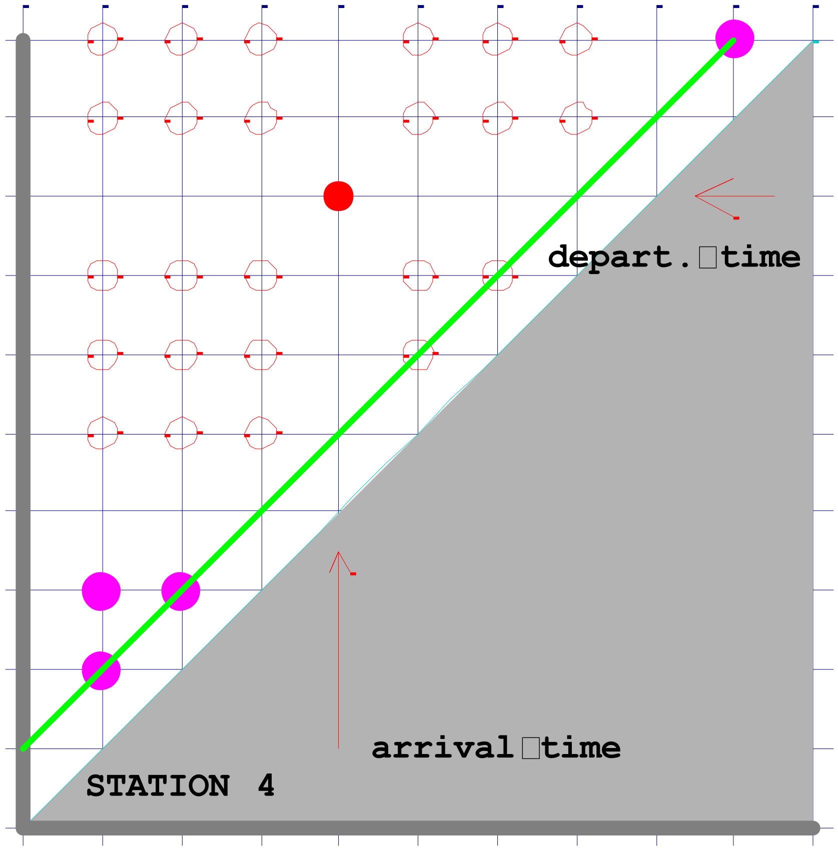
If transit corridor intersects with others transit lines at specific stations, as is shown in figure, the determination of new timetables should ensure the transfer of passengers between trains from different line runs at such interchange stations.



6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (ii)

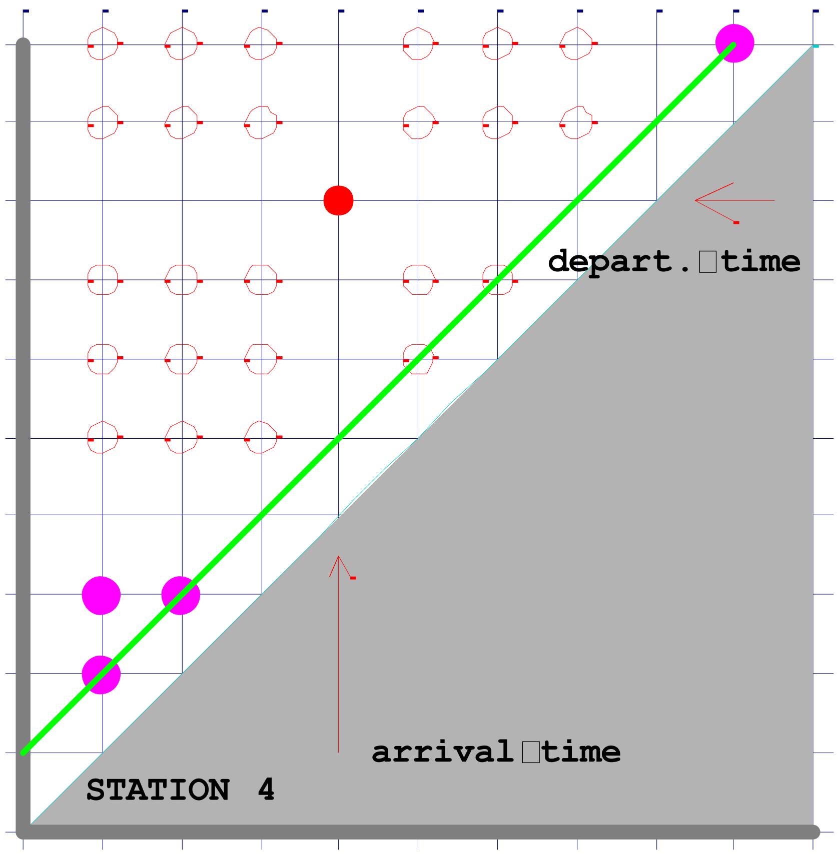
Two strategies can be considered:

- Imposing synchronization between the timetables of these lines; that is, a solution can be accepted only if the connection between them is feasible (Scenario 2.1).
- Rewarding the possibility of providing transfers for passengers of external lines towards concurrent expeditions of the internal line by means of a weighting factor (Scenario 2.2).



6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (iii)

For instance, Figure shows the timetable-point (filled in red) of another line (line A) when arrives/departs at/from station *4* at times *u=4* and *v=8,* respectively. If the synchronization between the timetables of these lines were imposed, the feasible subset of timetable-points (i.e.,), where transfer is preserved, would coincide with the set of unfilled points in magenta color. Consistently with the notation used for decision variables in the model, let be a binary input data which is equal to 1 if train *j* (of an external line whose arrival/depart timetables are given) is located at timetable-point (*u, v*) at station; otherwise, its value would be 0.



6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (iv)

NEW Indices and Sets

$j \in J$ index that identifies trains of other transit lines concurrent with lines runs of set *I*.

$s \in S \subset K$ index that enumerates the subset of stations that allow transfers to the travelers.

$F_s(u,v) \subset M_s$ subset of timetable-points in the temporary map *M* of station *s* where transfers between two transit lines can be carried out

FORMULATING MODEL FOR SCENARIO 2.1

Objective (1), constraints (2)-(7) , and additionally:

(8) $$y_{uv}^{js} \le \sum_{i \in I} \sum_{(u',v') \in F_s(u,v)} x_{u'v'}^{is}; \qquad j \in J, (u,v) \in M_s, s \in S$$

Constraints (8) establish that if there is an active (i.e., $y_{uv}^{js} = 1$) timetable-point located at position (*u, v*) of the temporary map for the *s*-th station of an outside line *j*, then there must be at least another active timetable-point at the same station for synchronizing transfers from/toward line runs *i* of the inner transit line *I*.

6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (v)

FORMULATING MODEL FOR SCENARIO 2.2

For this context, it is necessary to distinguish between users who enter in the system from outside and passengers who previously entered into the system with the certainty of being able to make a transfer to another line already. Objective to maximize must take into account this division of populations and asymmetrically favor one over the other population by using a weighting factor $\mu \geq 1$.

Let b_{uv}^{jk} be a real input data which represents the population available to transferring from train *j* at station *k* and at timetable-point (*u, v*).

Redefining the objective (1'):

$$(1') \quad Max \sum_{i \in I} \sum_{k \in K} \sum_{(u,v) \in M_k} \left(a_v^{ik} + \mu \sum_{j \in J} \sum_{(u',v') \in F_k(u,v)} b_{u'v'}^{jk} \right) x_{uv}^{ik}$$

If *k* is not an interchange station, then $F_k(u,v) \equiv \varnothing$ and the second additive term is cancelled.

Therefore, objective (1') and constraints (2)-(8) constitute a procedure for maximizing mobility of travelers who enter in the system after rescheduling, by ensuring the option of transferring from/towards other external lines at interchange stations.

7. CONCLUSIONS

A **geometric approach** to determine the redistribution of service along a rail corridor has been introduced.

Motivation for rescheduling railway timetables is caused by the forced reduction of fleet size due to accidents, strikes and other sources of train delays and cancellations.

Two scenarios have been modelled: a context without considering transfers from/towards other transit lines, and a setting where the existence of transfers between lines must be preserved although the service must be rescheduled.

A **common approach for these scenarios** has been developed by using **a geometrical representation** of train timetables at stations. The associated formulations are **Integer Linear Programming models**, where the number of decision variables can be reduced according to different constraints imposed by the structural and fleet capacities.

The theoretical development has been illustrated with a non-sophisticated example in order to clarify the concepts used through the paper.