Record-like observations and counters

IV Encuentros Desafíos de la Matemática Combinatoria
Priego de Córdoba, 27-29 Septiembre 2013

Índice

Particle counters

Records and record-like observations

The connection....

Particle Counters

Useful devices with many applications

- Geiger counters of radioactivity
- Quality of drinking water
- Air Pollutants
- Hydraulic liquids in industry
- etc..

Particle Counters

- A particle counter observes a sequence τ of events occurring at (random) times

$$\tau_1 < \tau_2 < \tau_3 \cdots$$

- Usually particle counters are blocked after the arrival of a new particle so that the counting is interrupted during this "busy" period.
- The particle counter records only a subsequence σ,

$$\sigma_1 < \sigma_2 < \sigma_3 \cdots$$

Types of counters

- *Type I or non-paralizable counters.*
  - The length of the busy period is a fixed amount $\delta > 0$.
  - The busy period is not extended by the arrival of particles during the busy period.
- *Type II or paralyzable counters.*
  - The busy period is always extended $\delta > 0$ units of time after the arrival of a particle

In both cases the detector counts only those particles arriving during idle periods.

- Arrival process homogeneous Poisson process or renewal process, Smith (1958) or Cox & Isham (1980)

- The Poisson and/or the equally distributed inter-arrival assumptions have been questioned in certain applications, Larsen & Kotinski (2009).

- The exact behavior of counters under time-varying arrival rates does not seem to be well-understood Teich (1978) or Sippl & Wiederstein (2012)

- Arrival process homogeneous Poisson process or renewal process, Smith (1958) or Cox & Isham (1980)

- The Poisson and/or the equally distributed inter-arrival assumptions have been questioned in certain applications, Larsen & Kotinski (2009).

- The exact behavior of counters under time-varying arrival rates does not seem to be well-understood Teich (1978) or Sippl & Wiederstein (2012)

- Arrival process homogeneous Poisson process or renewal process, Smith (1958) or Cox & Isham (1980)

- The Poisson and/or the equally distributed inter-arrival assumptions have been questioned in certain applications, Larsen & Kotinski (2009).

- The exact behavior of counters under time-varying arrival rates does not seem to be well-understood Teich (1978) or Sippl & Wiederstein (2012)

Poisson processes and Poisson random measures

A point process N is a Poisson process with rate $\lambda(t) \geq 0$ if (basically):

- Independent increments
- $N(B)$=number of points in $B \sim \text{Poisson}(\Lambda(B))$ where

$$\Lambda(B) = \int_B \lambda(t)\, dt$$

We denote

$$N \sim \text{PP}(\lambda(t))$$

One of our objectives is to study the probabilistic behavior of counters under time varying rates , i.e, when the arrival process of particles is a $PP(\lambda(t))$

Records

Let $\mathbf{X} = \{X_n\}_{n \geq 1}$ be sequence of random variables.

- $$\begin{aligned} T_1 &= 1, \\ T_k &= \min\{j > T_{k-1} : X_j > X_{T_{k-1}}\}, \ k \geq 2, \end{aligned} \quad (1)$$

  The *n*-th record is

  $$R_n = X_{T_n}$$

- if $X_n \overset{iid}{\sim} F$ then R_n is the n-th record from the parent F

Records

Let $\mathbf{X} = \{X_n\}_{n\geq 1}$ be sequence of random variables.

- $$\begin{aligned} T_1 &= 1, \\ T_k &= \min\{j > T_{k-1} : X_j > X_{T_{k-1}}\},\ k \geq 2, \end{aligned} \tag{1}$$

  The *n*-th record is

  $$R_n = X_{T_n}$$

- if $X_n \overset{iid}{\sim} F$ then R_n is the n-th record from the parent F

Record-like observations

Record-like observations are generalizations of the concept of records.

Among them:

- δ-exceedances of records, Balakrishnan et al. (1996)
- δ-records, Gouet et al. (2007, 2012, 2013)
- Geometric records, Eliazar (2005)

Record-like observations

Record-like observations are generalizations of the concept of records.
Among them:

- δ-exceedances of records, Balakrishnan et al. (1996)
- δ-records, Gouet et al. (2007, 2012, 2013)
- Geometric records, Eliazar (2005)

δ-exceedances, Balakrishnan et al. (1996)

Let $\mathbf{X} = \{X_n\}_{n\geq 1}$ be sequence of random variables.
$\delta \in \mathbb{R}$

$$T_{1,\delta} = 1,$$
$$T_{k,\delta} = \min\{j > T_{k-1,\delta} : X_j > X_{T_{k-1,\delta}} + \delta\}, \; k \geq 2, \qquad (2)$$

The *n-th δ-exceedance* is

$$A_{n,\delta} = X_{T_{n,\delta}}$$

δ-exceedances, Balakrishnan et al. (1996)

Let $\mathbf{X} = \{X_n\}_{n\geq 1}$ be sequence of random variables.
$\delta \in \mathbb{R}$

$$T_{1,\delta} = 1,$$
$$T_{k,\delta} = \min\{j > T_{k-1,\delta} : X_j > X_{T_{k-1,\delta}} + \delta\}, \; k \geq 2, \qquad (2)$$

The n-th δ-exceedance is

$$A_{n,\delta} = X_{T_{n,\delta}}$$

δ-records, Gouet et al.

Let $\mathbf{X} = \{X_n\}_{n \geq 1}$ be sequence of random variables.
$\delta \in \mathbb{R}$

$$\begin{aligned} T_{1,\delta} &= 1, \\ T_{k,\delta} &= \min\{j > T_{k-1,\delta} : X_j > M_{j-1} + \delta\}, \ k \geq 2, \end{aligned} \tag{3}$$

with $M_n = \max\{X_1, \ldots, X_n\}$ The *n-th δ-record* is

$$R_{n,\delta} = X_{T_{n,\delta}}$$

δ-records, Gouet et al.

Let $\mathbf{X} = \{X_n\}_{n\geq 1}$ be sequence of random variables.
$\delta \in \mathbb{R}$

$$\begin{aligned} T_{1,\delta} &= 1, \\ T_{k,\delta} &= \min\{j > T_{k-1,\delta} : X_j > M_{j-1} + \delta\}, \ k \geq 2, \end{aligned} \tag{3}$$

with $M_n = \max\{X_1, \ldots, X_n\}$ The n-th δ-record is

$$R_{n,\delta} = X_{T_{n,\delta}}$$

Geometric records, Eliazar(2005)

Let $\mathbf{X} = \{X_n\}_{n\geq 1}$ be sequence of random variables.
$\lambda > 0$

$$T_{1,\lambda} = 1,$$
$$T_{k,\lambda} = \min\{j > T_{k-1,\lambda} : X_j > \lambda M_{j-1}\}, \ k \geq 2, \quad (4)$$

with $M_n = \max\{X_1, \ldots, X_n\}$ The *n-th λ-geometric record* is

$$G_{n,\delta} = X_{T_{n,\lambda}}$$

Geometric records, Eliazar(2005)

Let $\mathbf{X} = \{X_n\}_{n\geq 1}$ be sequence of random variables.
$\lambda > 0$

$$\begin{aligned} T_{1,\lambda} &= 1, \\ T_{k,\lambda} &= \min\{j > T_{k-1,\lambda} : X_j > \lambda M_{j-1}\}, \ k \geq 2, \end{aligned} \tag{4}$$

with $M_n = \max\{X_1, \ldots, X_n\}$ The n-th λ-geometric record is

$$G_{n,\delta} = X_{T_{n,\lambda}}$$

The conection...

- Assume that the arrival process of particles is PP($\lambda(t)$)
- A remarkable property of records :

  The points of a Poisson process with rate $\lambda(t)$, $t \geq 0$, are the records from the parent distribution $F(x) = 1 - \exp \Lambda(x)$, where $\Lambda(x) = \int_0^x \lambda(t)dt$

The conection...

- Assume that the arrival process of particles is $\mathrm{PP}(\lambda(t))$
- A remarkable property of records :

  The points of a Poisson process with rate $\lambda(t)$, $t \geq 0$, are the records from the parent distribution $F(x) = 1 - \exp \Lambda(x)$, where $\Lambda(x) = \int_0^x \lambda(t)dt$

Formally, if $N \sim \mathrm{PP}(\lambda(t))$ then

$$N(B) \stackrel{d}{=} \sum_{k=1}^{\infty} \mathbf{1}_{\{R_k \in B\}}, \quad B \in \mathcal{B}(\mathbb{R}) \tag{5}$$

where $\{R_k\}_{k \geq 1}$ is distributed as the sequence of ordinary records from the parent distribution $F(x) = 1 - \exp \Lambda(x)$

Gouet et al. (2012) have shown that:

If the arrival process of particles is PP($\lambda(t)$) then the points of the process of counted particles by a type II counter are distributed as the δ-records from $F(x) = 1 - \exp \Lambda(x)$

We (ACM, FLB and BSM) (2013) have shown that:

If the arrival process of particles is PP($\lambda(t)$) then the points of the process of counted particles by a type I counter are distributed as the δ-excededances from $F(x) = 1 - \exp \Lambda(x)$

.... but the distribution theory of δ-exceedances, δ-record, geometric records is not well known!!

For that reason we have started a systematic study of these record-like objects:

- δ-records
  - $\delta \leq 0$ Distribution theory of δ-records. Case $\delta \leq 0$. To appear in Test
  - $\delta > 0$ (in progress...)
- ψ-exceedances (in progress...)
- Geometric records (in progress...)

Acknowledgements

This research has been supported by the Research Projects MTM2010-16949 of Ministerio de Educación y Ciencia, España and FQM331 Junta de Andalucía.