A new formulation to the discrete ordered median problem

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Abstract

A new formulation for the ordered median problem is proposed in this work. This formulation decreases considerably the number of constraints to define the problem with respect to others previous formulations. Furthermore, the bound which is obtained in the relaxed problem is better than the ones obtained with the previous formulations in the literature even when strengthened is not applied.

1 Introduction

In [3], it was proposed a formulation which could be used to model different locations problems, as the p-median or the p-center. It is a flexible formulation based in ordering the costs as appear in the solution and accounting them with a suitable n-vector λ. This first formulation produces a nonlinear program.

For the sake of comprehension, let us give a brief explanation of the Discrete Optimization Median Problem, DOMP. In DOMP, we have several facilities, which need to satisfy a demand. Let C be the cost matrix, C_{ij} is the cost to serve completely the demand point in position i with a facility located at j.

We have n demand points, which at the same time are the candidate sites to be facilities. We have a set I, with n elements and finally we will obtain a set J, with p elements, such that $J \subseteq I$. Hence, $|I| = n$ and $|J| = p$

The DOMP is characterized by a set of constraints: the problem must have exactly p facilities, where $1 \leq p \leq n$ and we consider the uncapacitated problem.

Let I be a set with n locations, J will be a feasible solution if $|J| = p$.

In order to define the objective function in DOMP, we need to calculate the vector c. This vector has n components, which are the sorted cost for each location

$$c_i(J) = \min_{j \in J} C_{ij}$$

By means of a permutation we obtains the vector $c_{\leq}$, such that it satisfies

$$c^1_{\leq}(J) \leq \cdots \leq c^n_{\leq}(J)$$

Then, we have to solve this problem

$$\min_J \sum_{k=1}^{n} \lambda_k c^k_{\leq}(J)$$

where λ is a n-vector with $\lambda_i \geq 0 \, \forall i = 1, \ldots, n$.

Later this formulation was linearizated in different ways [1] and improved taking into account the possible ties in the cost matrix and branch&cut techniques [2, 4].

In this work, we define a new formulation related to the set packing polytope. This formulation can be applied to all the cases (even without free self-service). We provide new families of inequalities which inprove the performance of the formulation. In addition, we analyze the relationship between the polytopes of the different known formulations of these problems. Finally, some computational experiments are reported.

References

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