On two optimization methods for optimal control problems

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Among the different optimization strategies that are used in practice to solve control-constrained optimal control problems governed by partial differential equations, there are two that stand out due to the high orders of convergence achieved both from a theoretical and a practical point of view. We are referring to the Sequential Quadratic Programming (SQP) technique and the SemiSmooth Newton (SSN) method.

While in finite dimensional problems it is known that no-gap second order sufficient conditions and a strict complementarity condition are sufficient to obtain quadratic order of convergence for the SQP and superlinear convergence of the SSN, the analysis of the application of these methods to optimal control problems governed by nonlinear partial differential equations that can be found in the literature requires stronger second order optimality conditions, and even in some references, local strict convexity.

Some of the last works of our group have been devoted to prove that we can achieve the same kind of results for infinite dimensional optimal control problems as the existing ones for finite dimensional constrained optimization problems. We have developed results in an abstract framework that can be applied to a wide variety of optimal control problems including problems governed by semilinear elliptic and parabolic equations, and Navier-Stokes equations, with boundary or distributed controls acting either in an additive or multiplicative (bilinear) way.

We will report on those results. Furthermore, we will expose the algorithms in detail for a prototypical example problem and will explain how to solve the subproblems arising at each of the methods. Finally, we will dig into some of the computational issues that have to be taken into account to solve the problem in practice.

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