Connectivity constraints: the continuous version

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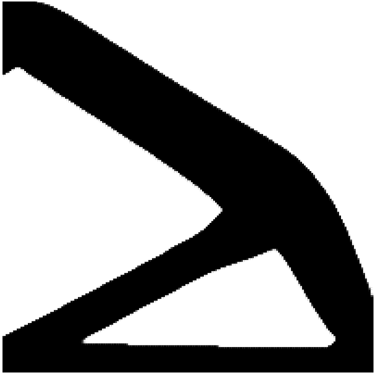
Departamento de Matemáticas (UCLM)

Avoiding enclosed voids



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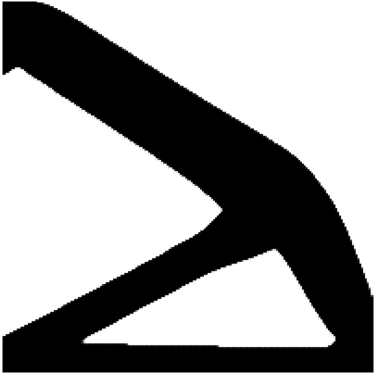

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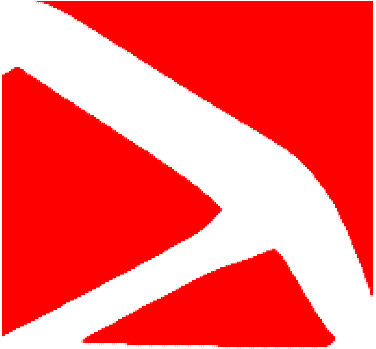

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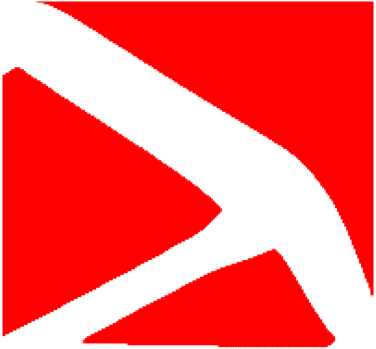
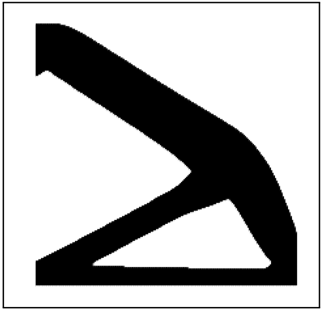


Characterization of enclosed voids



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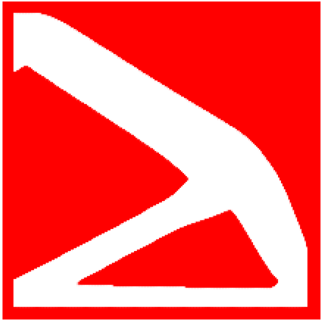

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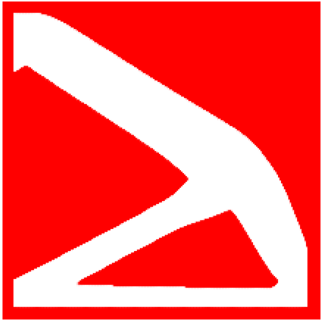


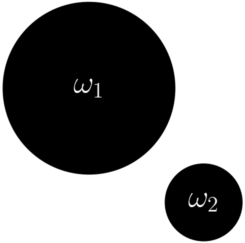
There is no enclosed voids $\Longleftrightarrow$ "Void set" of extended domain is connected

Connectivity characterization

Neumann-Laplacian eigenvalue problem:

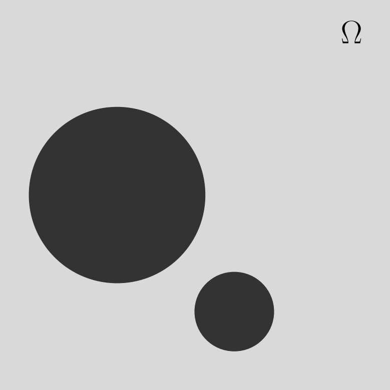
$$\begin{cases} \Delta\phi = \mu\phi & \text{in } \omega, \\ \frac{\partial\phi}{\partial n} = 0 & \text{on } \partial\omega. \end{cases}$$

$\omega = \omega_1 \cup \omega_2$



Multiplicity of zero eigenvalue = Number of connected components

Densities in a reference domain Ω



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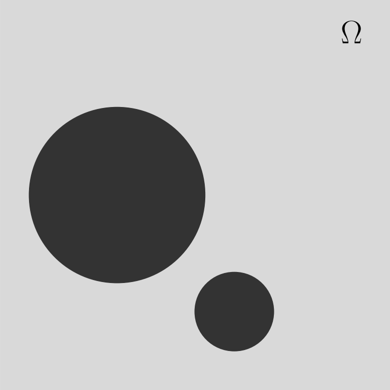

$$\rho = \varepsilon + (1-\varepsilon)\chi_\omega$$

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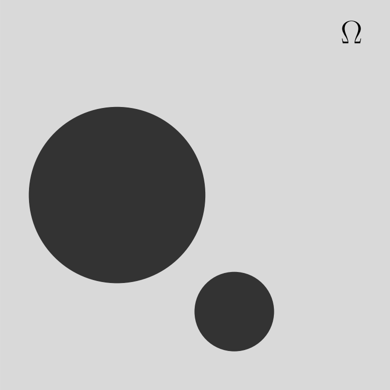

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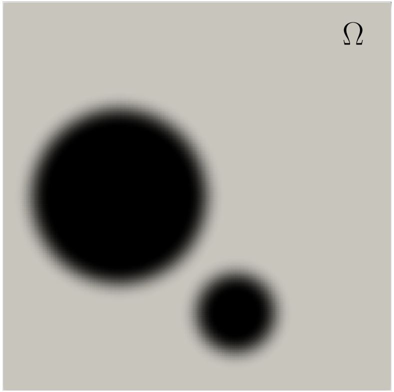
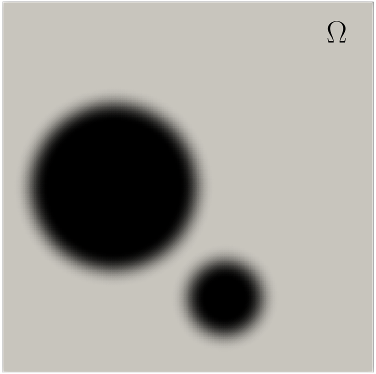


$\rho_n \longrightarrow \varepsilon + (1-\varepsilon)\chi_\omega$

The weighted Laplacian



$$(P_n) \quad \begin{cases} -\operatorname{div}(\rho_n \nabla \phi) = \lambda_n \rho_n \phi & \text{in } \Omega, \\ \dfrac{\partial \phi}{\partial n} = 0 & \text{on } \partial\Omega. \end{cases}$$

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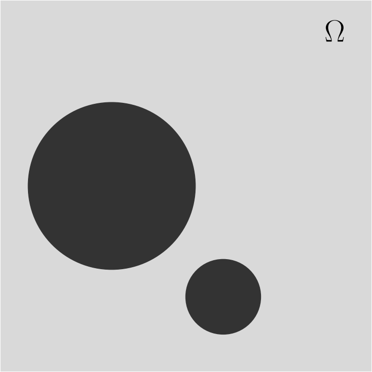

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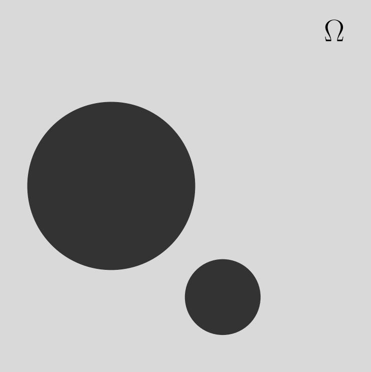

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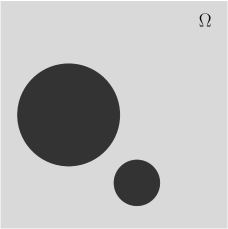
$$\rho_n \to \rho \Longrightarrow \lambda_n \longrightarrow \lambda$$

With of without weights

$$\begin{cases} -\operatorname{div}(\rho\nabla\phi) = \lambda\rho\phi & \text{in } \Omega, \\ \dfrac{\partial\phi}{\partial n} = 0 & \text{on } \partial\Omega. \end{cases}$$

$$\rho = \varepsilon + (1-\varepsilon)\chi_\omega$$



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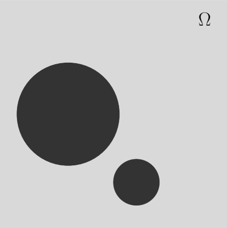
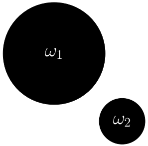


$\omega = \omega_1 \cup \omega_2$



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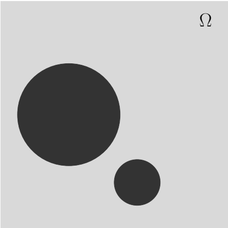

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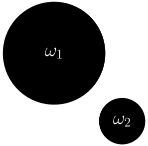


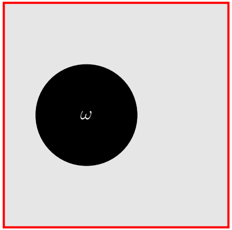
$$\begin{cases} -\Delta\phi = \mu\phi & \text{in } \omega, \\ \dfrac{\partial\phi}{\partial n} = 0 & \text{on } \partial\omega. \end{cases}$$

Theorem: $\exists C > 0$ such that $\lambda_2 - \mu_2 < C\varepsilon$

Dirichlet boundary values without extended domain

$$\begin{cases} -\operatorname{div}(\rho \nabla \phi) = \lambda \rho \phi & \text{in } \Omega, \\ \phi = 0 & \text{on } \partial\Omega. \end{cases}$$

$$\rho = \varepsilon + (1 - \varepsilon)\chi_\omega$$



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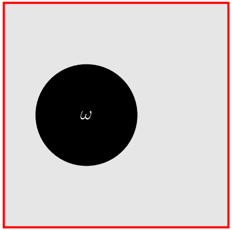
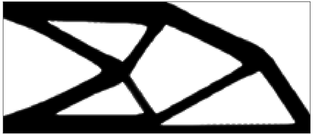


Theorem: $\exists C > 0$ such that $\lambda_1 < C\varepsilon$

Numerical results with different boundary conditions



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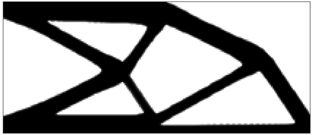

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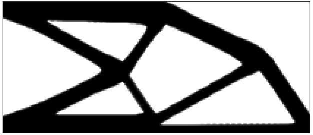  

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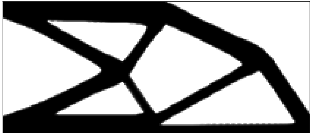

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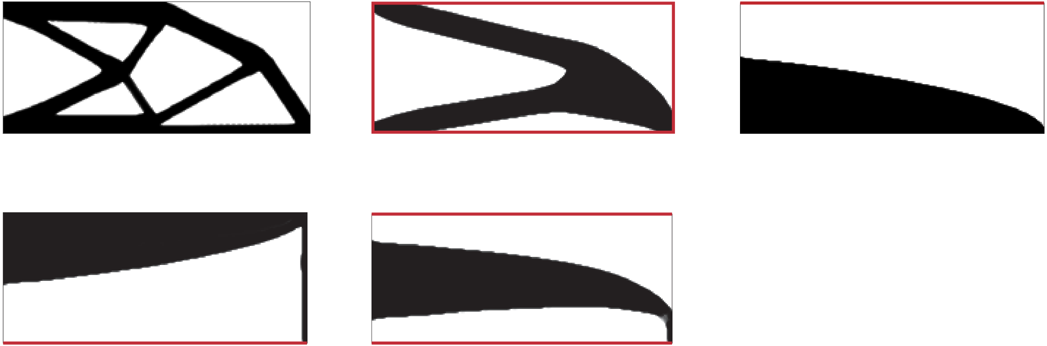

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