Group Announcement Logic

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joint work with:

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Group Announcement Logic: background

- Two current trends in logics for multi-agent systems:

    1. Logics of coalitional ability (Coalition Logic, ATL, Stit logic, ...)

        - Recent interest: incomplete information

    2. Dynamic epistemic logic

        - Epistemic pre- and post- conditions of actions

        - Recent interest: quantification over formulae (APAL, ...)

- We combine ideas from both in order to analyse the logic of *group announcements*

Elevator pitch

Group Announcement Logic extends public announcement logic with:

$$\langle G\rangle\phi : \text{"Group } G \text{ can make an announcement after which } \phi \text{ is true"}$$

Public Announcement Logic (Plaza, 1989)

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2$$

ϕ_1 is true, and ϕ_2 is true after ϕ_1 is announced

Formally:

$M = (S, \sim_1, \ldots, \sim_n, V)$ $\quad \sim_i$ equivalence rel. over S

$$M, s \models K_i\phi \quad \Leftrightarrow \quad \forall t \sim_i s \; M, t \models \phi$$
$$M, s \models \langle\phi_1\rangle\phi_2 \quad \Leftrightarrow \quad M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2$$

The model resulting from removing states where ϕ_1 is false

Example

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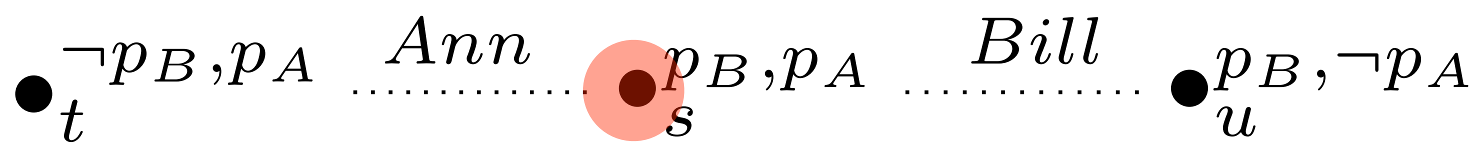

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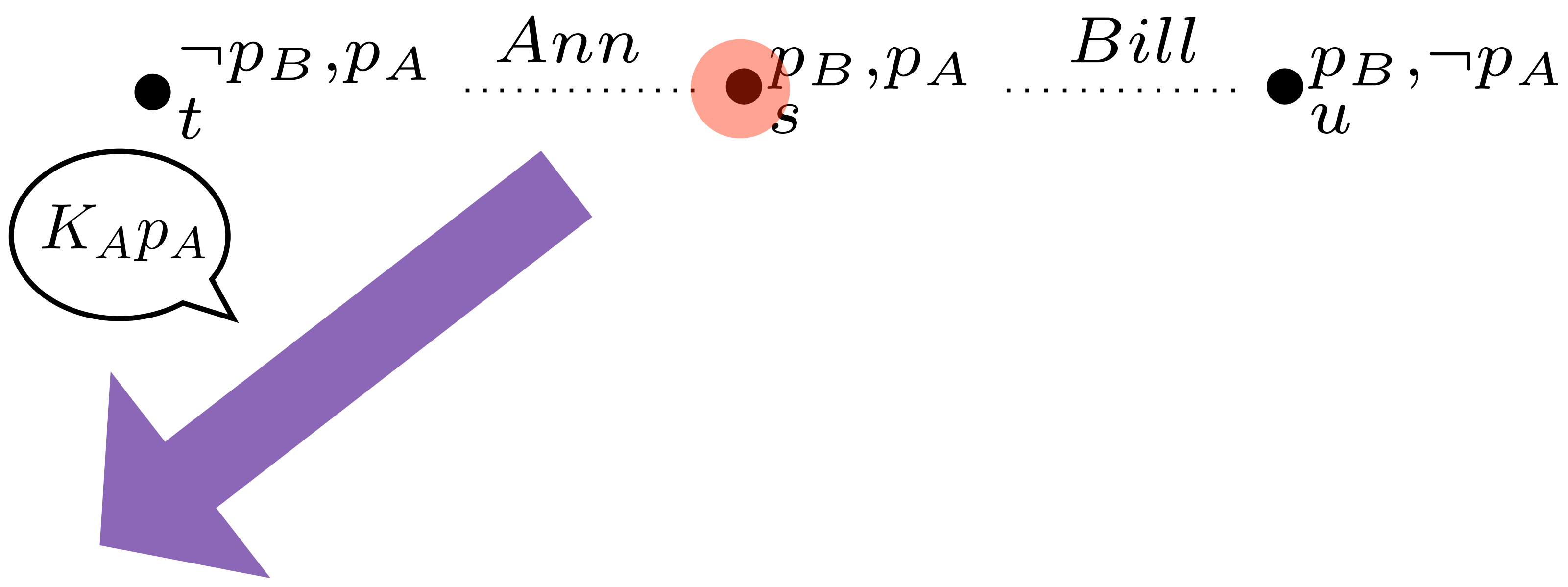

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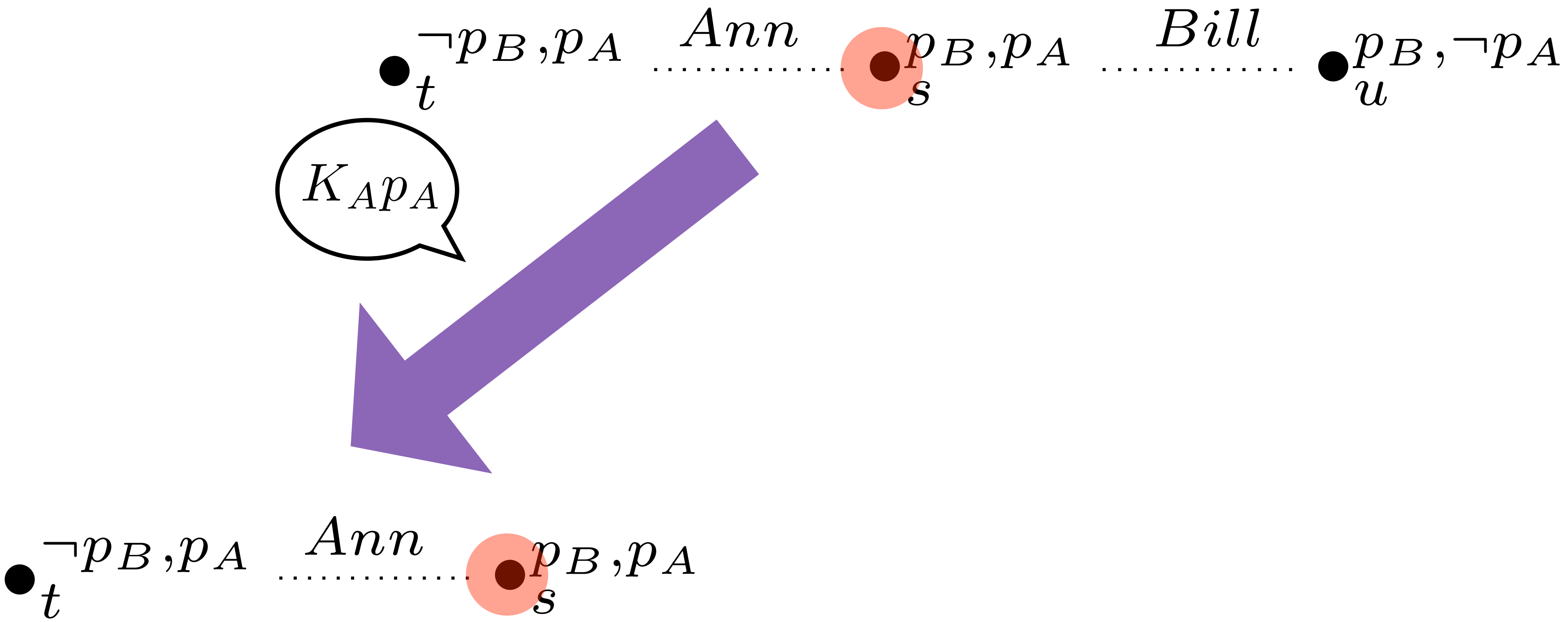

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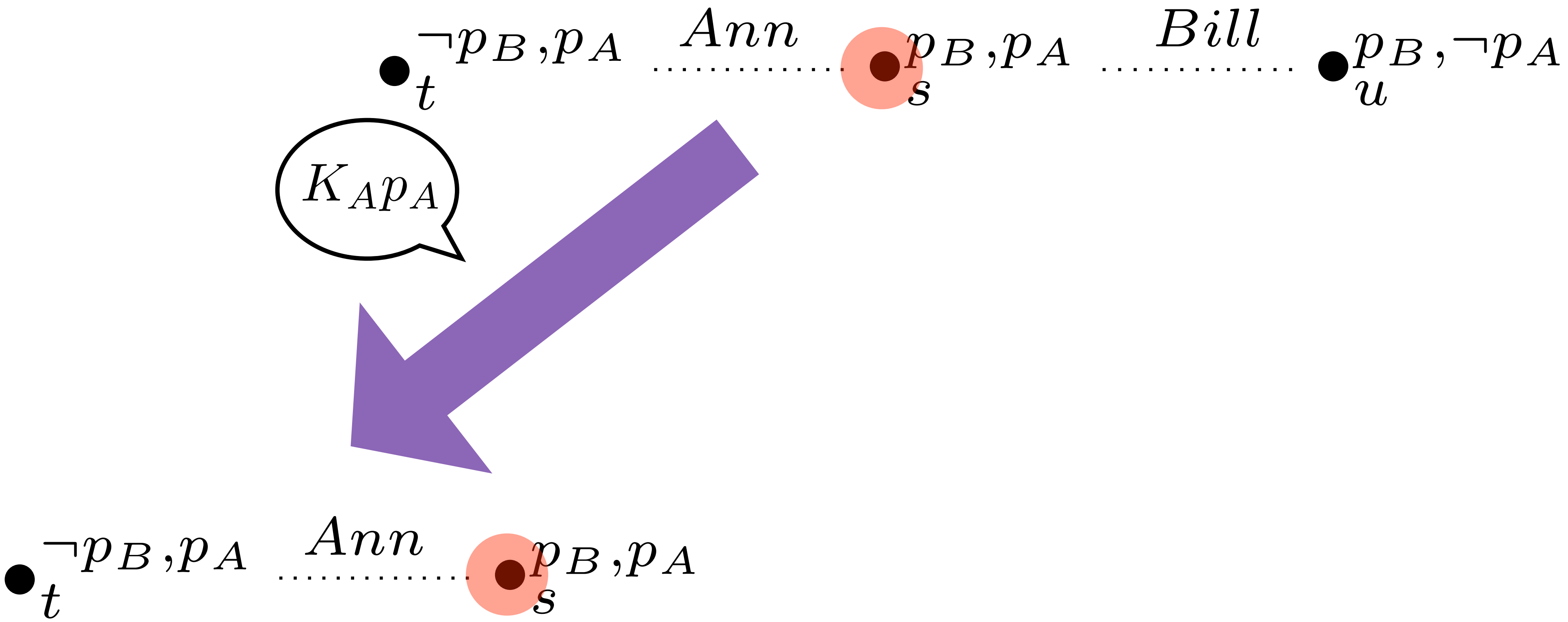
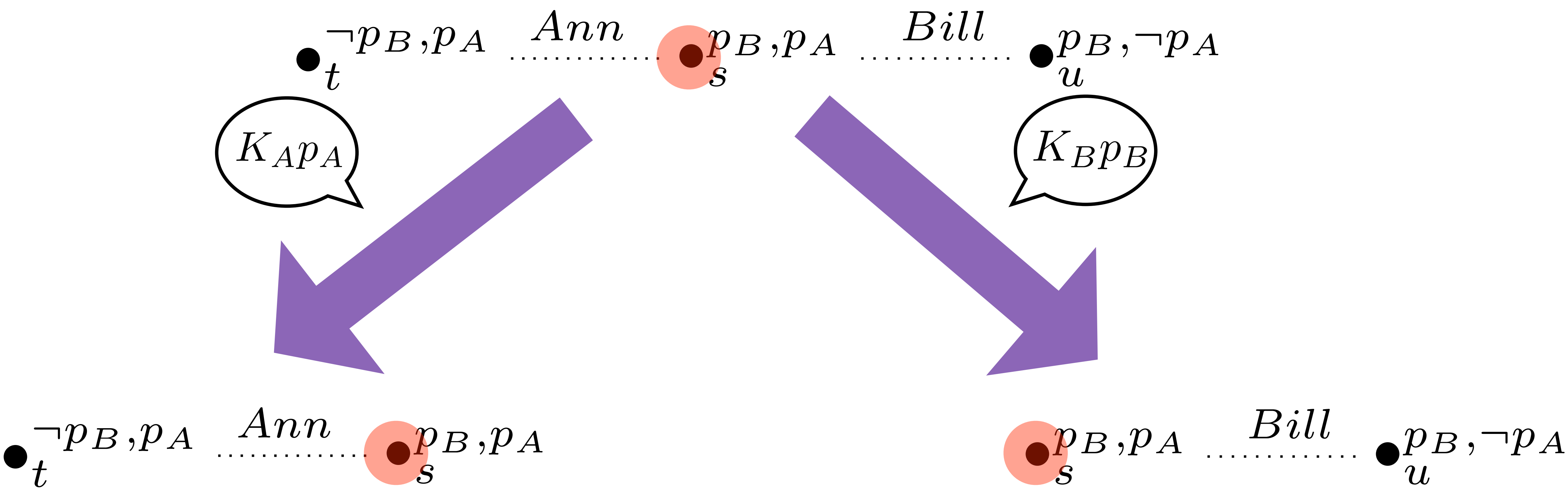


$$M, s \models \langle K_A p_A \rangle K_B p_A$$

Example



$$M, s \models \langle K_A p_A \rangle K_B p_A \qquad M, s \models \langle K_B p_B \rangle K_A p_B$$

Adding quantification: APAL

$$M, s \models \langle \phi_1 \rangle \phi_2 \Leftrightarrow M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2$$

Idea (van Benthem, Analysis, 2004): interpret the modal diamond as "there is an announcement such that.."

Arbitrary announcement logic (APAL) (Balbiani et al., TARK 2007):

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2 \mid \Diamond\phi$$

$$M, s \models \Diamond\phi \Leftrightarrow \exists\psi \; M, s \models \langle\psi\rangle\phi$$

Quantification in APAL

$$M, s \models \Diamond\phi \Leftrightarrow \exists\psi\ M, s \models \langle\psi\rangle\phi$$

Note: the quantification includes announcements that cannot be truthfully made in the system

Quantification: announcements by an agent

$$K_i\psi$$

Quantification: announcements by an agent

$$M, s \models \langle i \rangle \phi \Leftrightarrow \exists \psi \ M, s \models \langle K_i \psi \rangle \phi$$

Quantification: announcements by a group

$$M, s \models \langle G \rangle \phi \quad \Leftrightarrow \quad \exists \{\psi_i : i \in G\} \; M, s \models \langle \bigwedge_{i \in G} K_i \psi \rangle \phi$$

Group Announcement Logic (GAL):

$$\varphi ::= p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \mid \langle G \rangle \phi$$

Similar to *coalitional ability* operators of Coalition Logic (Pauly, 2002) and ATL (Alur, Henzinger, Kupferman, 1997), with actions = public announcements

But GAL is not a Coalition Logic

H. van Ditmarsch, *The Russian Cards Problem*, Studia Logica **75**, 2003

Example: **The Russian Cards Problem**

From a pack of seven known cards 0,1,2,3,4,5,6 Anne and Bill each draw three cards and Cath gets the remaining card. How can Anne and Bill openly inform each other about their cards, without Cath learning who holds any of their cards?

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Formalisation: 012_a : "Ann has cards 0,1 and 2"

$$(one)\ \bigwedge_{ijk}(ijk_b \rightarrow K_a ijk_b) \quad (two)\ \bigwedge_{ijk}(ijk_a \rightarrow K_b ijk_a)$$

$$(three)\ \bigwedge_{q=0}^{6}((q_a \rightarrow \neg K_c q_a) \wedge (q_b \rightarrow \neg K_c q_b))$$

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Known solution:

$$\textit{anne} \equiv 012_a \vee 034_a \vee 056_a \vee 135_a \vee 246_a$$

$$\textit{bill} \equiv 345_b \vee 125_b \vee 024_b$$

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PAL: $\langle K_a anne \rangle \langle K_b bill \rangle (one \wedge two \wedge three)$

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GAL: $\langle a\rangle\langle b\rangle(one \wedge two \wedge three)$

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Quantification: sequences of announcements

$$\langle G\rangle\langle G\rangle\phi \rightarrow \langle G\rangle\phi?$$

Quantification: sequences of announcements

$$\langle G \rangle \langle G \rangle \phi \to \langle G \rangle \phi?$$

Answer: Yes.

Quantification: sequences of announcements

$$\langle G\rangle\langle G\rangle\phi \rightarrow \langle G\rangle\phi$$

$$M, s \models \langle G\rangle\phi \Leftrightarrow \text{ there is an announcement by } G\text{, after which } \phi$$

Quantification: sequences of announcements

$$\langle G\rangle\langle G\rangle\phi \to \langle G\rangle\phi$$

$$M, s \models \langle G\rangle\phi \Leftrightarrow \text{ there is an announcement by } G\text{, after which } \phi$$

$$\Leftrightarrow \text{ there is a sequence of announcements by } G\text{, after which } \phi$$

Example: **Russian Cards** (ctnd.)

$$\langle K_a anne \rangle \langle K_b bill \rangle (one \wedge two \wedge three)$$

$$\langle a \rangle \langle b \rangle (one \wedge two \wedge three)$$

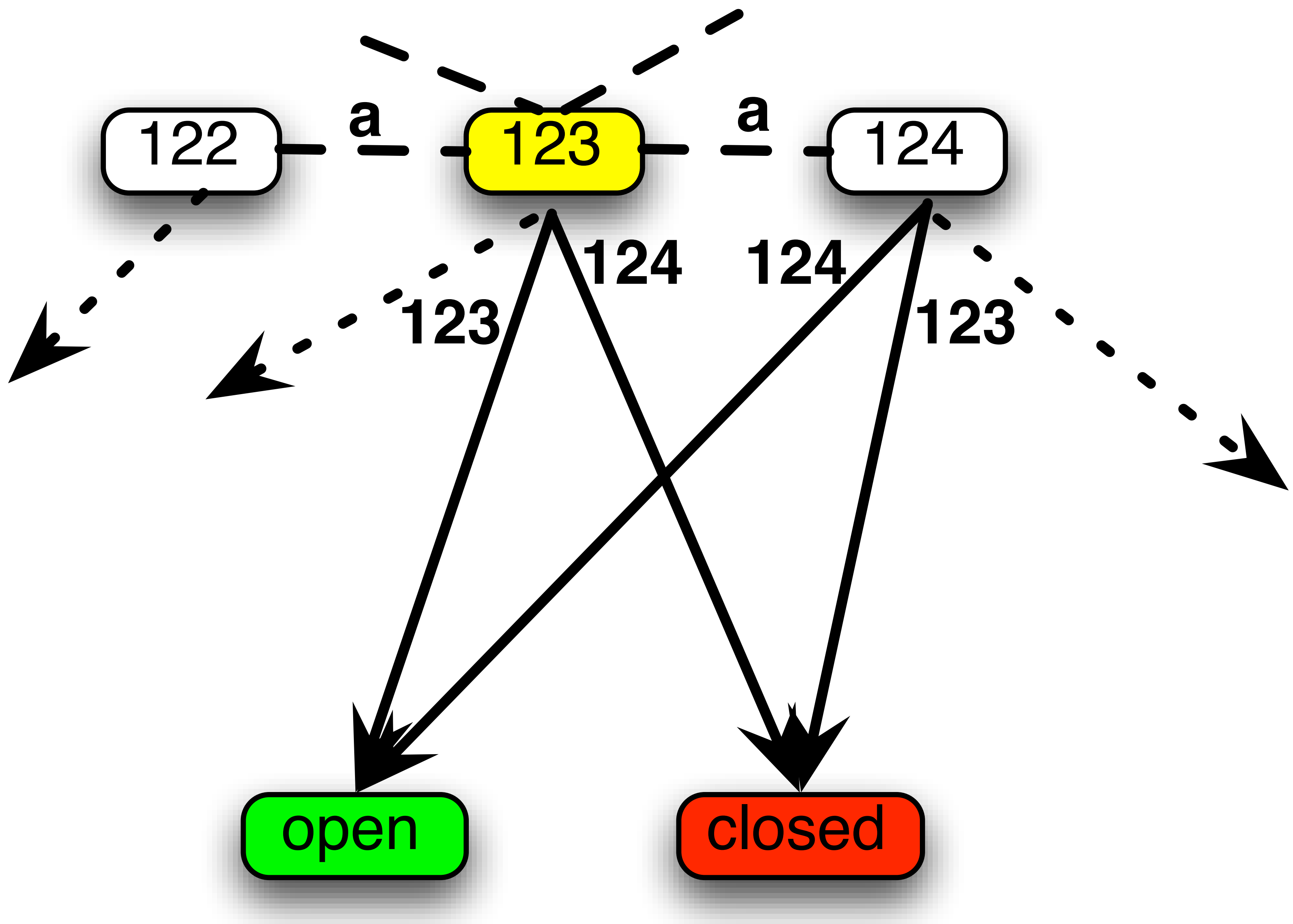
$$\langle ab \rangle (one \wedge two \wedge three)$$

Knowledge and Ability: general actions

- Consider the general case that agents have arbitrary joint actions (and not only group announcements) available, that will take the system to a new state
- Two variants of ability under incomplete information:
  - Knowing *de dicto* that you can achive something: in all the states you consider possible, you can achive the goal (by performing some action)
  - Knowing *de re* that you can achieve something: there is some action which will achieve the goal in all the states you consider possible

Knowledge and Ability: general actions

- Example: agent in front of a combination-lock safe; does not know the combination; correct combination is 123



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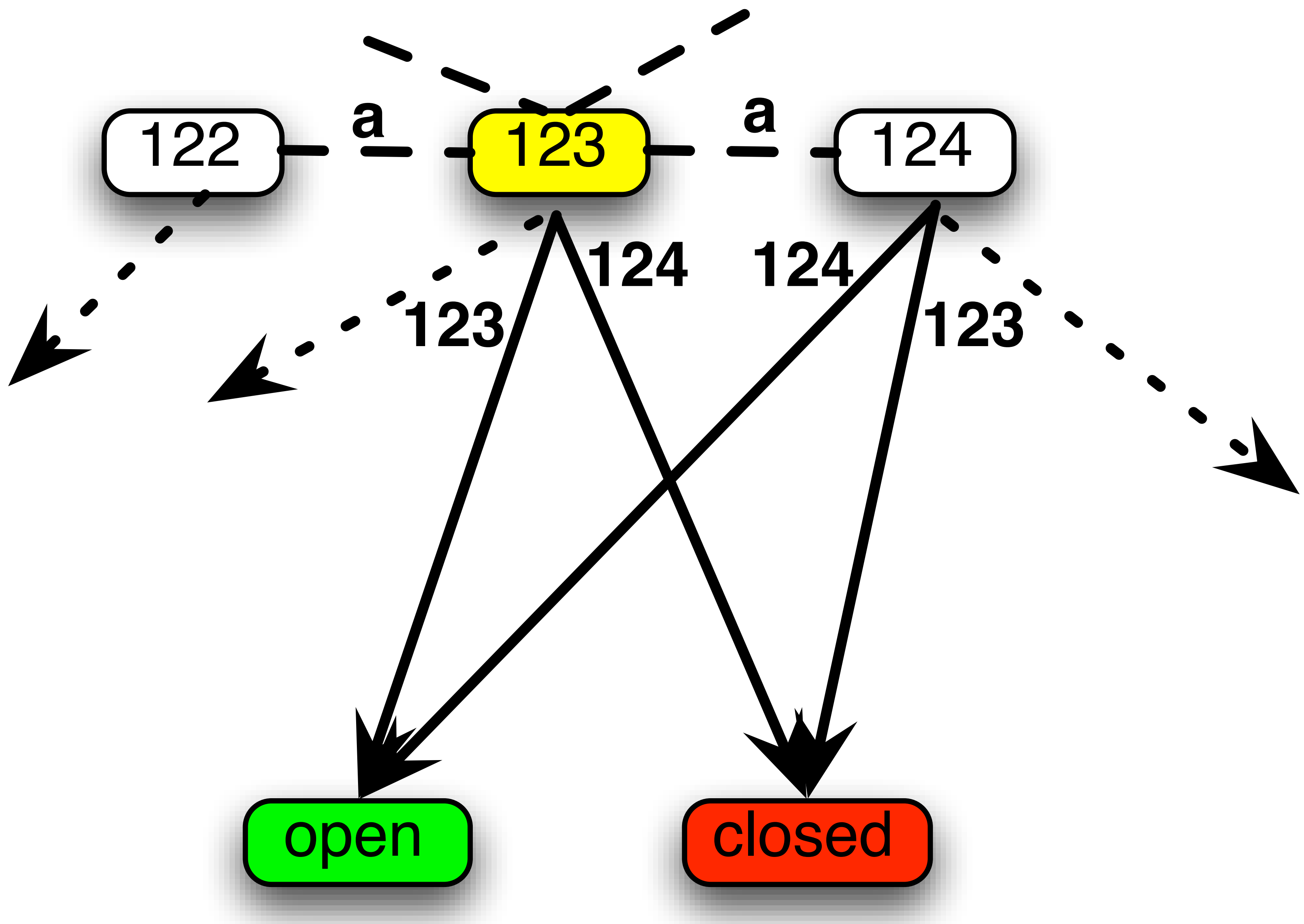

$$\langle a \rangle open$$

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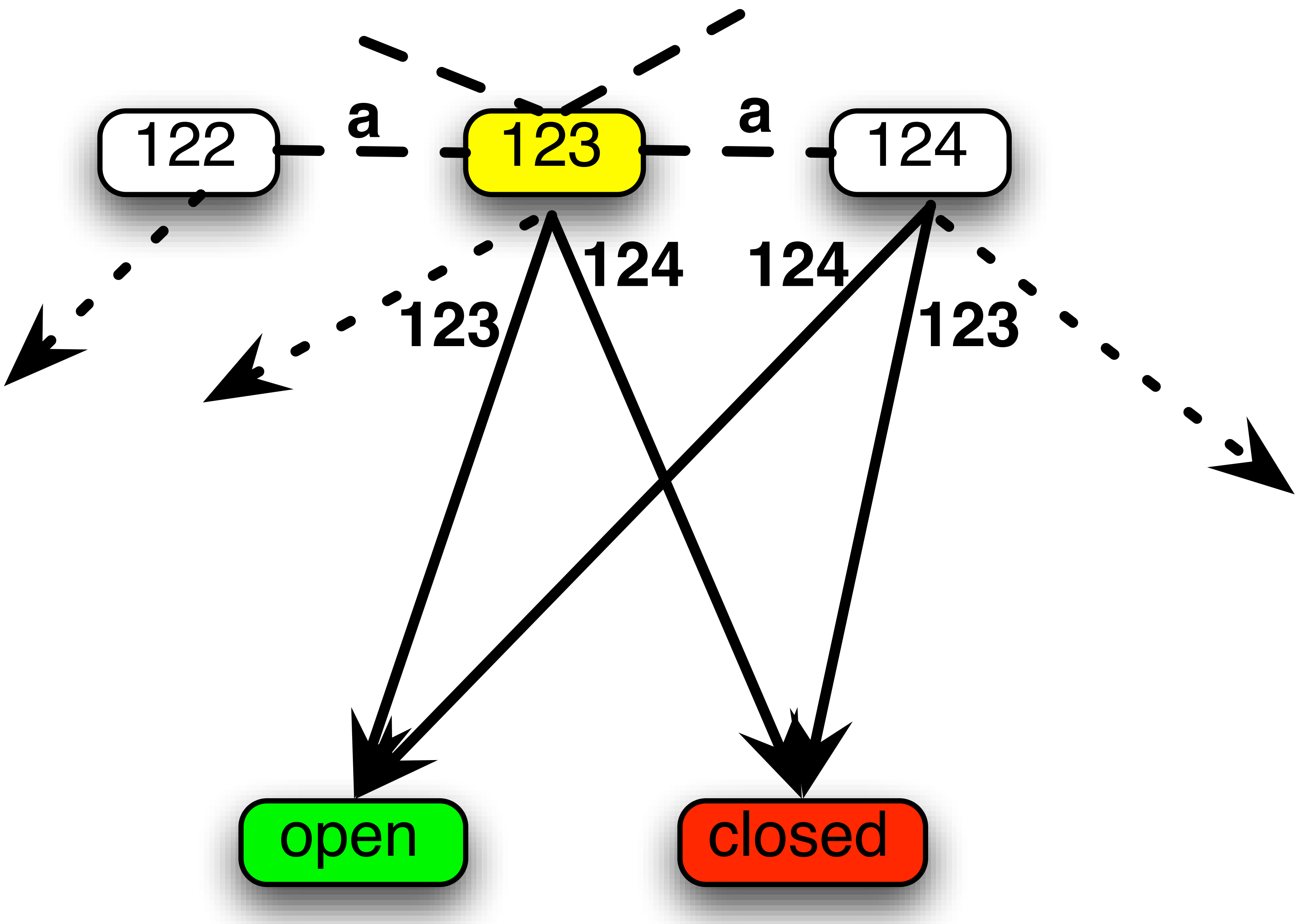

$$\langle a \rangle open$$

$$K_a \langle a \rangle open$$

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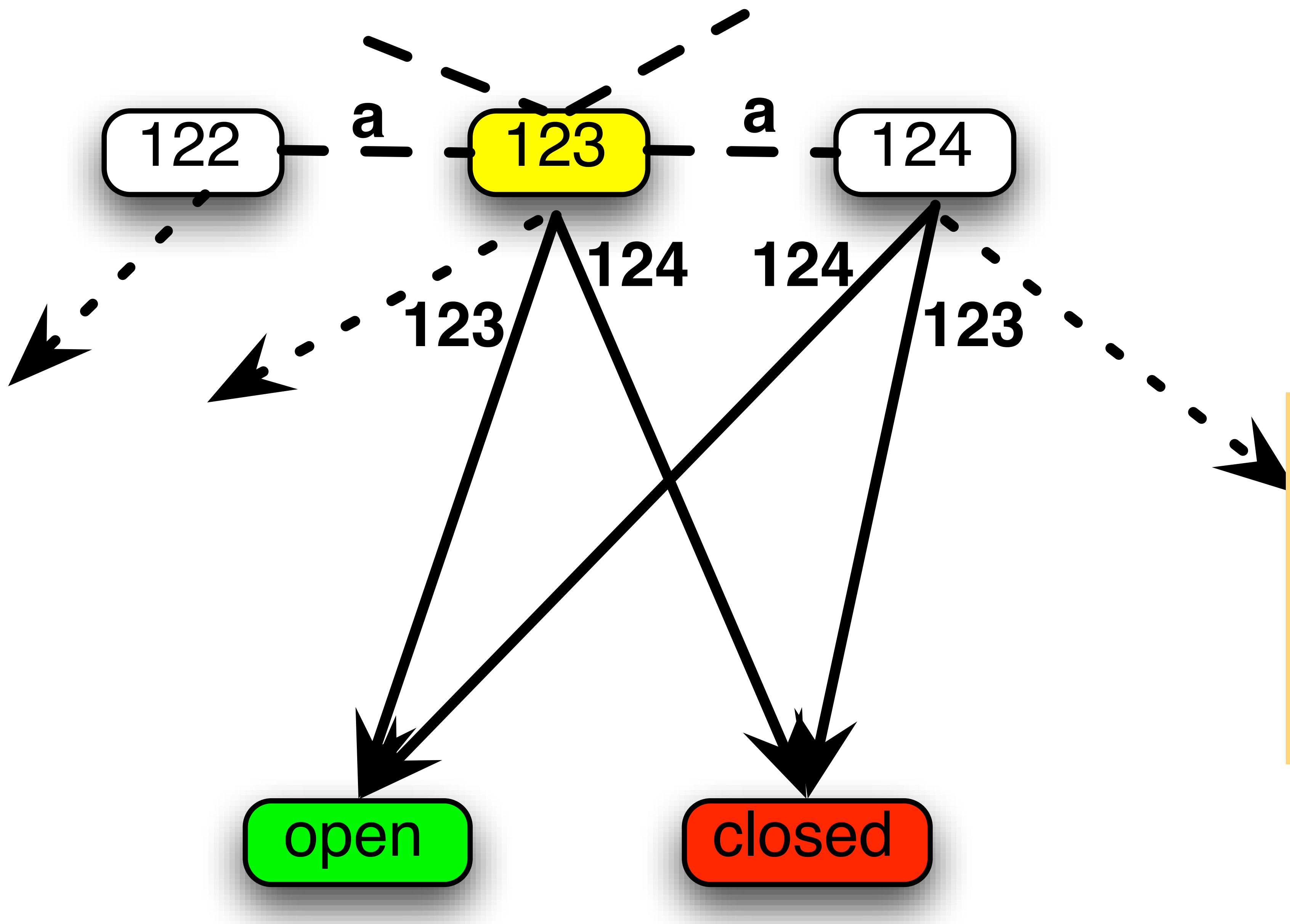


$\langle a \rangle open$

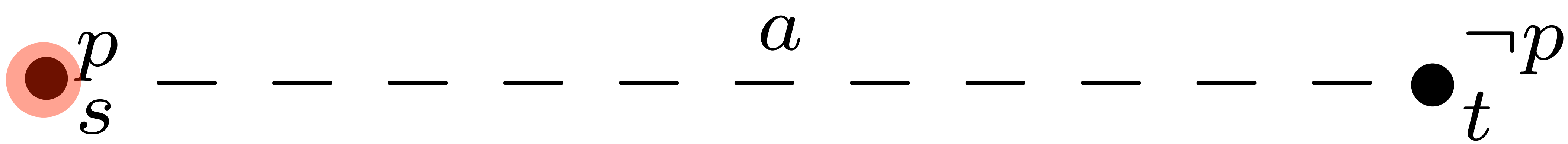
$K_a \langle a \rangle open$

But *a* does not know *de re* that she can open the safe

Knowledge and Ability: general actions

- It turns out that knowledge of ability *de re* is not expressible in standard logics combining knowledge and ability
  - Alternating-time Temporal Epistemic Logic (ATEL) (van der Hoek & Wooldridge)
- Several recent works, e.g. (Jamroga and van der Hoek, 2004), (Jamroga and Ågotnes, 2007), have focused on extending ATEL to be able to express knowledge *de re*
- In GAL, knowledge and action are intimately connected
  - How do the previous observations apply to GAL?

Being able to without knowing it



$$s \models \langle a \rangle p \wedge \neg K_a \langle a \rangle p$$

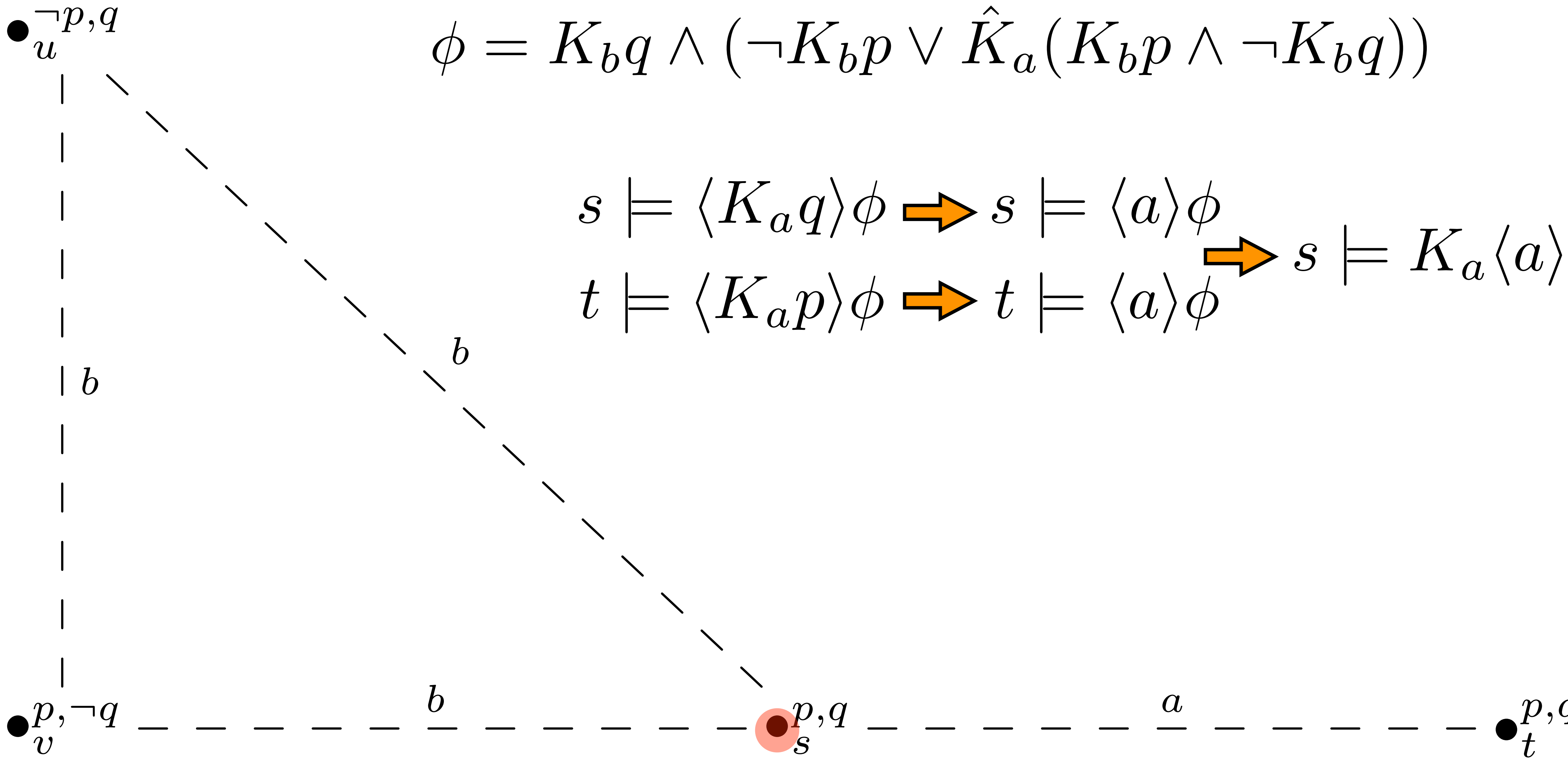
Being able to, knowing that, but not knowing how

$$\phi = K_b q \wedge (\neg K_b p \vee \hat{K}_a(K_b p \wedge \neg K_b q))$$

$$s \models \langle K_a q\rangle\phi \Rightarrow s \models \langle a\rangle\phi$$

$$t \models \langle K_a p\rangle\phi \Rightarrow t \models \langle a\rangle\phi$$

$$\Rightarrow s \models K_a\langle a\rangle\phi$$



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$$\phi = \boxed{K_b q} \wedge (\boxed{\neg K_b p} \vee \hat{K}_a(K_b p \wedge \neg K_b q))$$

$$\boxed{s \models \langle K_a q \rangle \phi} \Rightarrow s \models \langle a \rangle \phi$$

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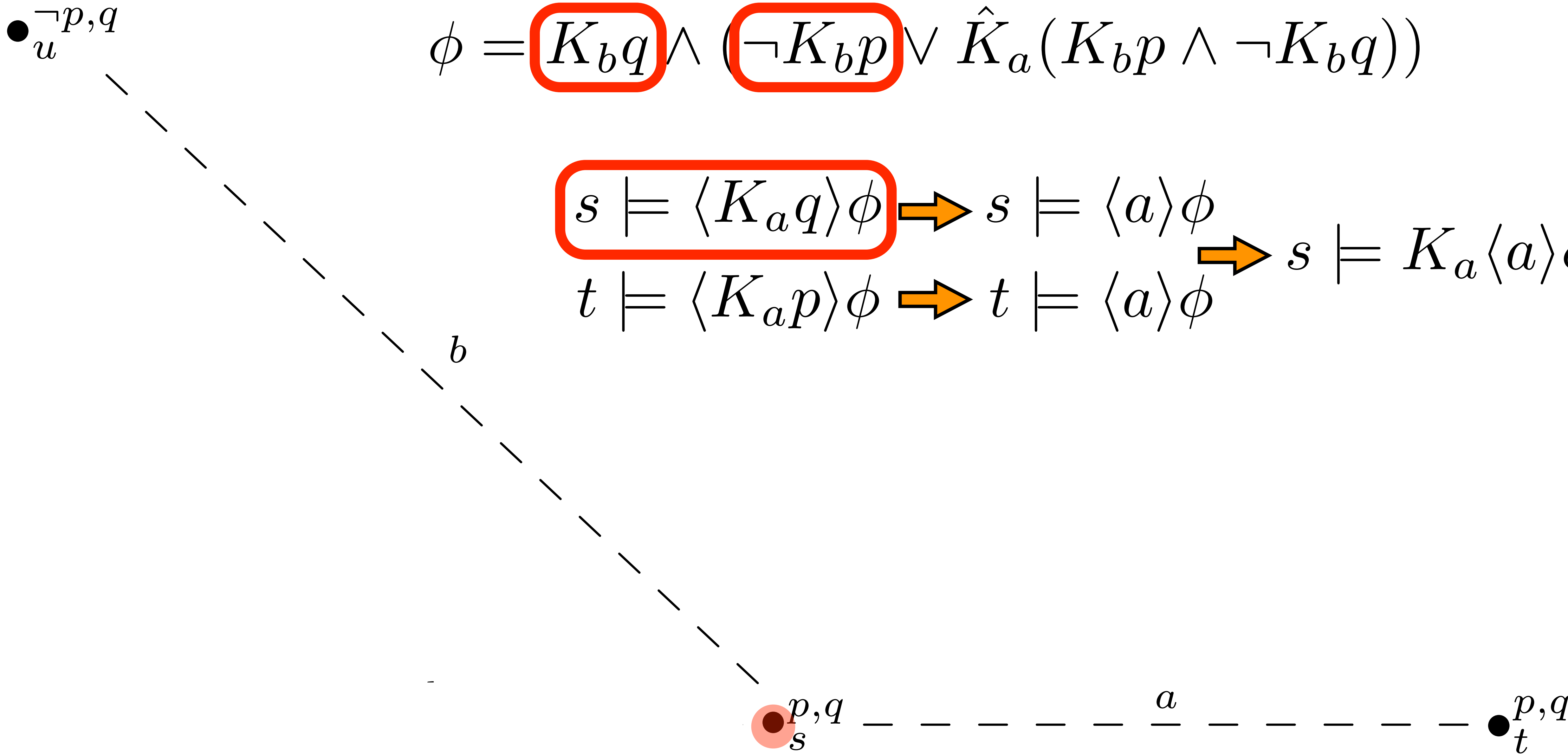

$$s \models \phi$$

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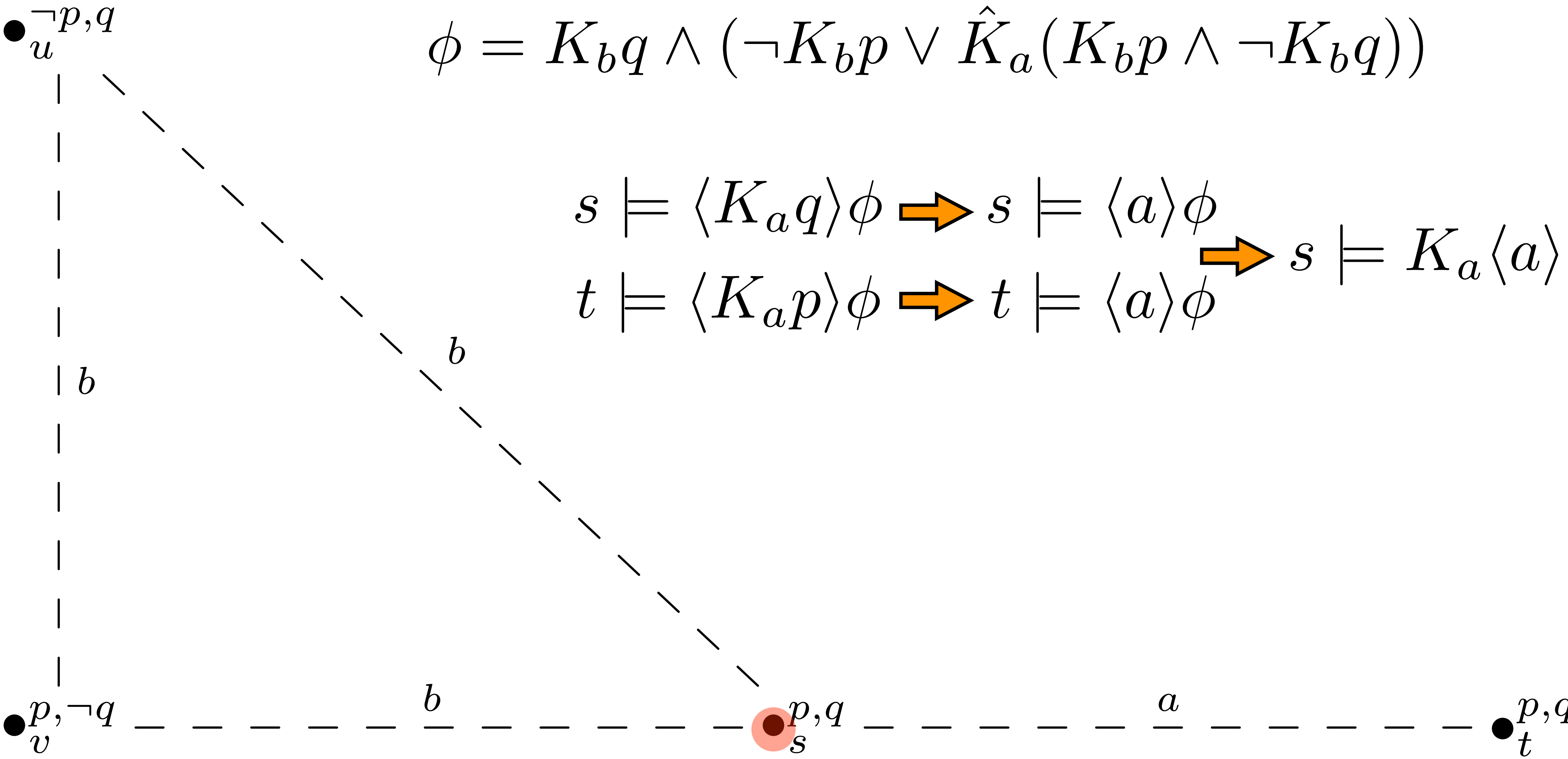

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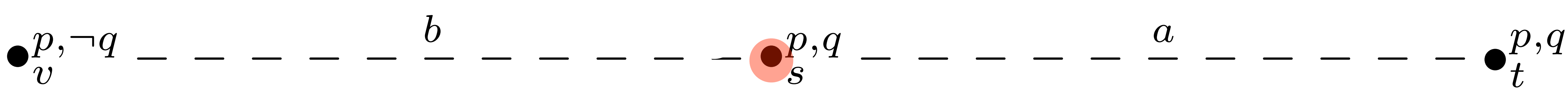

$$t \models \phi$$

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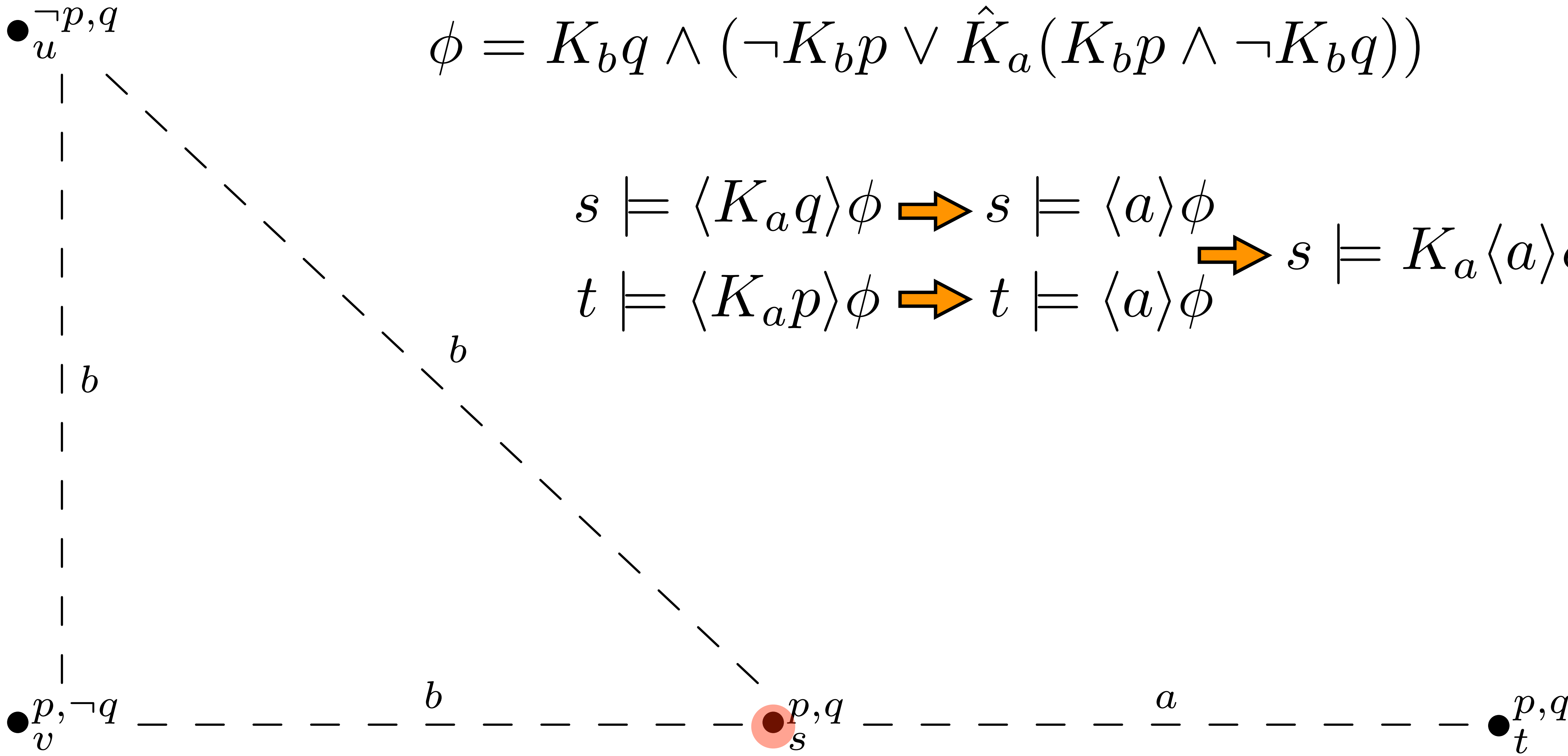

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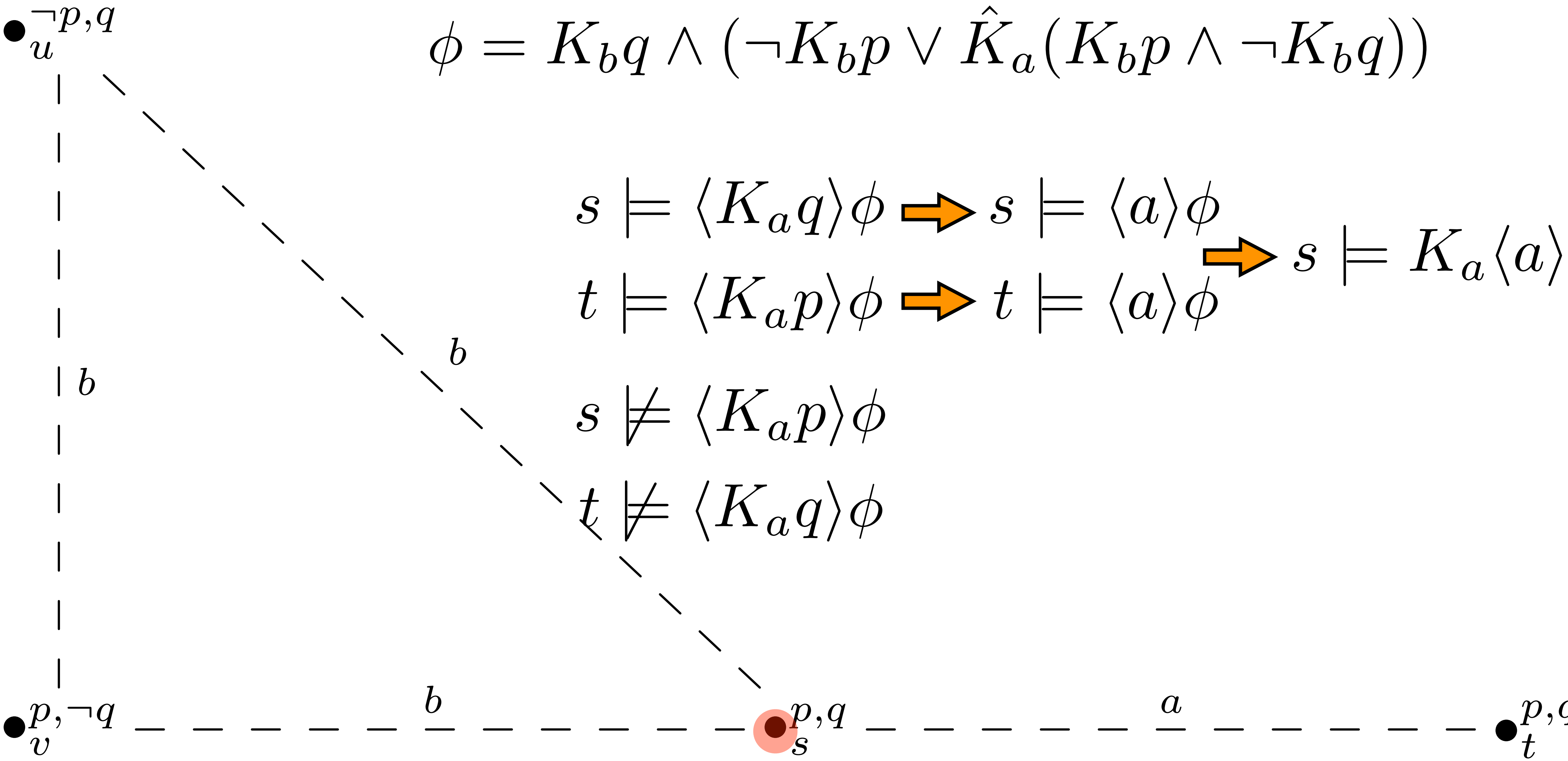

$s \models \langle K_a q \rangle \phi \Rightarrow s \models \langle a \rangle \phi$

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$\Rightarrow s \models K_a \langle a \rangle \phi$

$s \not\models \langle K_a p \rangle \phi$

$t \not\models \langle K_a q \rangle \phi$

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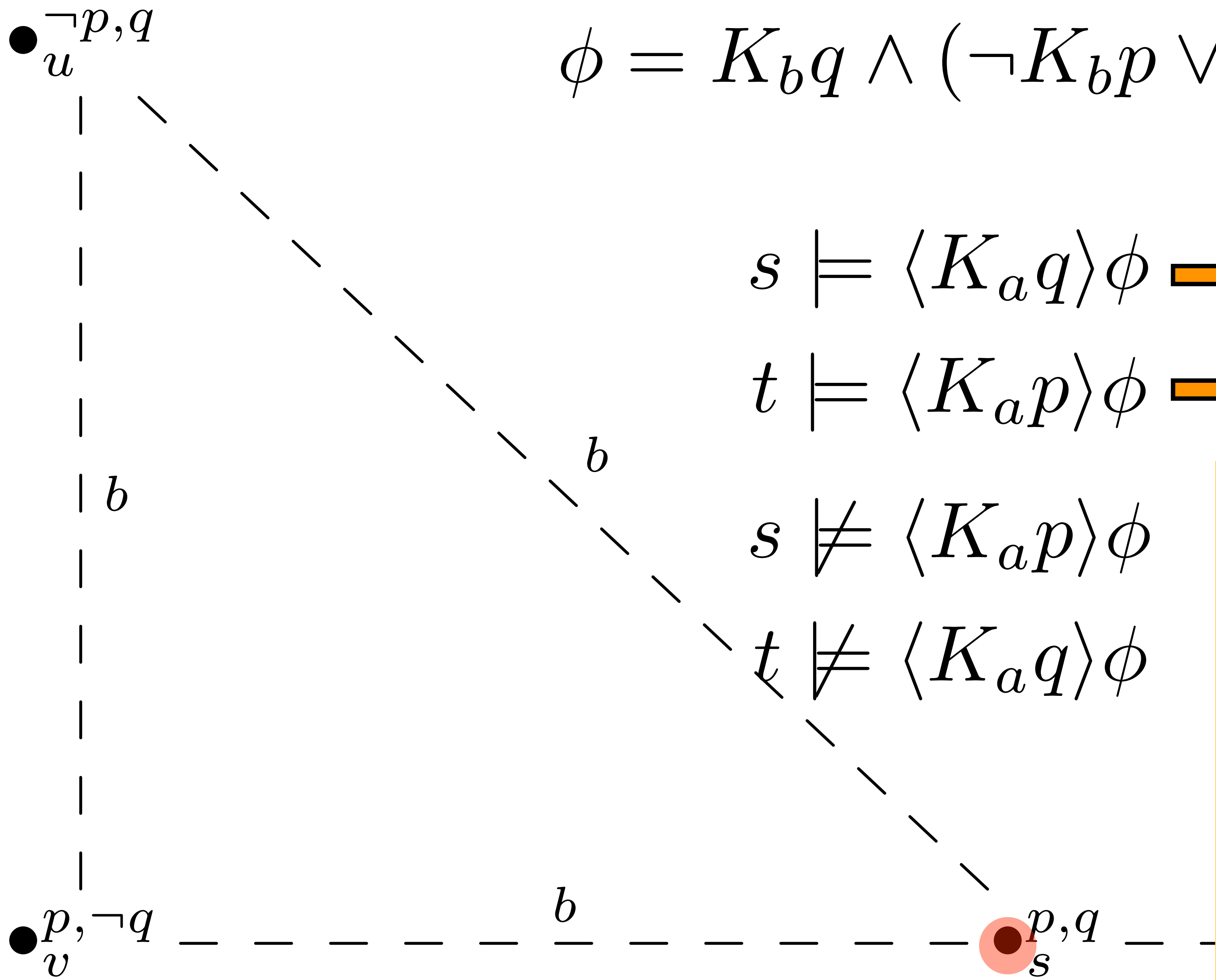

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$$\Rightarrow s \models K_a \langle a \rangle \phi$$

$$s \not\models \langle K_a p \rangle \phi$$

$$t \not\models \langle K_a q \rangle \phi$$

The same announcement will not achieve the goal in both s and t - *a* does not know *how* to achieve it

Expressing knowledge *de dicto*/*de re*

Ability	Knowledge of ability, *de dicto*	Knowledge of ability, *de re*
$\exists\psi\ s \models \langle K_a\psi\rangle\phi$	$\forall s \sim_a t\ \exists\psi\ t \models \langle K_a\psi\rangle\phi$	$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi$
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
$s \models \langle a\rangle\phi$	$s \models K_a\langle a\rangle\phi$	??

Expressing knowledge *de dicto*/*de re*

Ability

$$\exists\psi\ s \models \langle K_a\psi\rangle\phi$$

$$s \models \langle a\rangle\phi$$

Knowledge of ability, *de dicto*

$$\forall s \sim_a t\ \exists\psi\ t \models \langle K_a\psi\rangle\phi$$

$$s \models K_a\langle a\rangle\phi$$

Knowledge of ability, *de re*

$$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi$$

??

$$s \models \langle a\rangle K_a\phi$$

$$\exists\psi\ s \models \langle K_a\psi\rangle K_a\phi$$

Expressing knowledge *de dicto*/*de re*

Ability	Knowledge of ability, *de dicto*	Knowledge of ability, *de re*
$\exists\psi\ s \models \langle K_a\psi\rangle\phi$	$\forall s \sim_a t\ \exists\psi\ t \models \langle K_a\psi\rangle\phi$	$\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi$
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
$s \models \langle a\rangle\phi$	$s \models K_a\langle a\rangle\phi$	$s \models \langle a\rangle K_a\phi$
		$\Updownarrow$
		$\exists\psi\ s \models \langle K_a\psi\rangle K_a\phi$

Expressing knowledge *de dicto/de re*

Ability

$$\exists\psi \;\; s \models \langle K_a\psi\rangle\phi$$

$$\Updownarrow$$

$$s \models \langle a\rangle\phi$$

Knowledge of ability, *de dicto*

$$\forall s \sim_a t \;\exists\psi \;\; t \models \langle K_a\psi\rangle\phi$$

$$\Updownarrow$$

$$s \models K_a\langle a\rangle\phi$$

Knowledge of ability, *de re*

$$\exists\psi \;\forall s \sim_a t \;\; t \models \langle K_a\psi\rangle\phi$$

$$\Updownarrow$$

$$s \models \langle a\rangle K_a\phi$$

$$\Updownarrow$$

$$\exists\psi \;\; s \models \langle K_a\psi\rangle K_a\phi$$

Depends on
(1) the fact that actions are *announcements*
(2) the S5 properties

Example: **Russian Cards** (ctnd.)

Ann and Bill *knows how* to exectute a successful protocol:

$$\langle a \rangle K_a(two \wedge three \wedge \langle b \rangle K_b(one \wedge two \wedge three))$$

Some logical properties

$$[G \cup H]\phi \to [G][H]\phi$$

$$\langle G \rangle [G]\phi \to [G]\langle G \rangle \phi$$ (Church-Rosser)

$$\langle G \rangle [H]\phi \to [H]\langle G \rangle \phi$$ (..generalised)

Axiomatisation

$S5_n$ axioms and rules

PAL axioms and rules

$[G]\phi \rightarrow [\bigwedge_{i \in G} K_i \psi_i]\phi \qquad$ where $\psi_i \in \mathcal{L}_{el}$

From ϕ, infer $[G]\phi$

From $\phi \rightarrow [\theta][\bigwedge_{i \in G} K_i p_i]\psi$, infer $\phi \rightarrow [\theta][G]\psi$
where $p_i \notin \Theta_\phi \cup \Theta_\theta \cup \Theta_\psi$

Theorem:
Sound & complete.

Model Checking

The model checking problem:

$$\text{Given } M, s \text{ and } \phi, \text{ does } M, s \models \phi \text{ hold?}$$

Theorem:
The model checking problem is PSPACE-complete

(also extends to APAL)

Directions

- More general actions/events
- Coalition Announcement Logic
  - a coalition logic
  - there are announcements by G such that for all announcements by the other agents, ...
- Public Announcement Games
  - To appear in *Synthese/KRA*
  - Question-Answer Games (LOFT 2010)

For more details:

T. Ågotnes and H. van Ditmarsch, *Coalitions and Announcements*, Proc. AAMAS 2008

T. Ågotnes, P. Balbiani, H. van Ditmarsch and P. Seban, *Group Announcement Logic*, Journal of Applied Logic **8**(1), 2010