Fuzzy Answer Set semantics for Residuated Logic programs

Nicolás Madrid & Manuel Ojeda-Aciego
Universidad de Málaga

September 23, 2009

Aims of this paper

We are studying the introduction of two kinds of negations into residuated logic programs:

- Default negation: This negation enables non-monotonic reasoning and is introduced into residuated logic programs by a generalization of the Gelfond-Lifschitz reduct.
- Strong negation: Monotonicity of positive programs is not affected by including this kind of negation. In this sense, we define the coherence restriction to generalize the concept of consistency.

Structure of this paper

This talk is structured as follows:

- We start recalling the basic definitions needed to describe our semantics.
- We focus mainly on strong negation:
  - Recalling the notion of coherent interpretation.
  - Providing properties of the notion of coherence.
  - Comparing the notion of coherence with α-consistency.
- Finally we present the definition of Fuzzy Answer Set in General Residuated Logic Programs

Preliminaries

Definition

A *residuated lattice* is a tuple $(L, \leq, *, \leftarrow)$ such that:

1. $(L, \leq)$ is a complete bounded lattice, with top and bottom elements 1 and 0.
2. $(L, *, 1)$ is a commutative monoid with unit element 1.
3. $(*, \leftarrow)$ forms an adjoint pair, i.e. $z \leq (x \leftarrow y)$ iff $y * z \leq x \quad \forall x, y, z \in L$.

Definition

A negation operator, over $(L, \leq, *, \leftarrow)$, is any decreasing mapping $n : L \to L$ satisfying $n(0) = 1$ and $n(1) = 0$.

Notation and terminology

We denote by $\neg$ the default negation and by $\sim$ the strong negation. The difference between them is uniquely semantic, and relates to the method used to infer the truth-value of one negated propositional symbol

Definition

- Π denotes the set of propositional symbols.
- If $p \in \Pi$ then both p and $\sim p$ are called literals.

General logic programs

Syntax

Definition

Given a residuated lattice with negations $(L, *, \leftarrow, \sim, \neg)$, a *general residuated logic program* $\mathbb{P}$ is a set of weighted rules of the form

$$\langle \ell \leftarrow \ell_1 * \cdots * \ell_m * \neg \ell_{m+1} * \cdots * \neg \ell_n; \quad \vartheta \rangle$$

where ϑ is an element of L and $\ell, \ell_1, \ldots, \ell_n$ are literals.

A general residuated logic program $\mathbb{P}$ is said to be:

- *positive* if it does not contain negation operators.
- *normal* if it does not contain strong negation.
- *extended* if it does not contain default negation.

General logic programs
Semantics

Rules will be frequently denoted as $\langle \ell \leftarrow \mathcal{B}; \vartheta \rangle$. As usual, the formula $\mathcal{B}$ is called the *body* of the rule whereas ℓ is called its *head*.

Definition

A fuzzy *L-interpretation* is a mapping $I\colon \mathit{Lit} \to L$.

I *satisfies* a rule $\langle \ell \leftarrow \mathcal{B}; \quad \vartheta \rangle$ if and only if $I(\mathcal{B}) * \vartheta \leq I(\ell)$ or, equivalently, $\vartheta \leq I(\ell \leftarrow \mathcal{B})$.

I is a *model* of $\mathbb{P}$ if it satisfies all rules in $\mathbb{P}$.

Extended Logic Programs

The immediate consequence operator defined on positive residuated logic programs is generalized to the extended ones.

Definition

Let $\mathbb{P}$ be an extended residuated logic program and let I be an interpretation. The immediate consequence operator of I wrt $\mathbb{P}$ is the interpretation defined as follows:

$$T_{\mathbb{P}}(I)(\ell) = lub\{I(\mathcal{B}) * \vartheta : \langle \ell \leftarrow \mathcal{B}; \vartheta \rangle \in \mathbb{P}\}$$

The minimal model

- The immediate consequence operator $T_{\mathbb{P}}$ is monotonic.
- By the Knaster-Tarski fix-point theorem, $T_{\mathbb{P}}$ has a least fix-point; $lfp(\mathbb{P})$.
- $lfp(\mathbb{P})$ is a model of $\mathbb{P}$
- The semantics of an extended residuated logic program $\mathbb{P}$ is given by the $lfp(\mathbb{P})$.

However, one has to take into account the interaction between opposite literals.

For this purpose we define the concept of coherence.

Coherence

Definition

A fuzzy L-interpretation I over Lit is coherent if the inequality $I(\sim p) \leq \dot{\sim} I(p)$ holds for every propositional symbol p.

- The notion of coherence coincides with consistency in the classical framework.
- It only depends on the negation operator.
- It allows to handle missing information (i.e I such that $I(\ell) = 0$ for all $\ell \in Lit$ is always coherent).

Coherent Programs

Definition

Let $\mathbb{P}$ be an extended residuated logic program, we say that $\mathbb{P}$ is coherent if its least model is coherent.

Proposition

Let I and J be two interpretations satisfying $I \leq J$. If J is coherent, then I is coherent as well.

Corollary

An extended residuated logic program is coherent if and only if it has at least one coherent model.

More properties of the notion of coherence

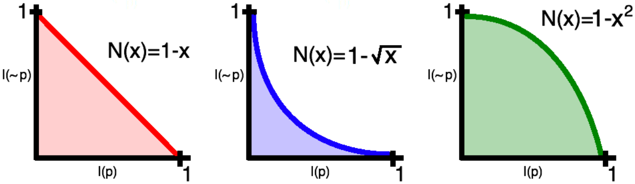
An interpretation assigns to any negative literal a truth degree independently from the negation operator. This way, if we have two different negation operators we can talk about the coherence wrt any of these operators.

Proposition

Let $\sim_1$ and $\sim_2$ be two negation operators such that $\sim_1 \leq \sim_2$, then any interpretation I that is coherent wrt $\sim_1$ is coherent wrt $\sim_2$ as well.

Example

The previous proposition provides us a way to filter admissible (coherent) interpretations by changing the strong negation operator.



Comparison with the notion of α-consistency

Definition

Let $*$ be a t-norm and let $\sim$ be a negation operator. We say that an interpretation $I : Lit \to L$ on the set of literals is α-consistent if for all propositional symbol p we have that $I(p) * I(\sim p) \leq \alpha$.

Remark

By the adjoint condition, we have that

$$I(p) * I(\sim p) \leq \alpha \quad \textit{iff} \quad I(\sim p) \leq \alpha \leftarrow I(p)$$

General Logic Programs: The reduct of $\mathbb{P}$

Let $\mathbb{P}$ and I be a general residuated logic program and a L-interpretation respectively, we will construct a new normal program $\mathbb{P}_I$ by substituting each rule in $\mathbb{P}$ of the form:

$$\langle \ell \leftarrow \ell_1 * \cdots * \ell_m * \neg \ell_{m+1} * \cdots * \neg \ell_n; \quad \vartheta \rangle$$

by the rule

$$\langle \ell \leftarrow \ell_1 * \cdots * \ell_m; \quad \neg I(\ell_{m+1}) * \cdots * \neg I(\ell_n) * \vartheta \rangle$$

Definition

The program $\mathbb{P}_I$ is called the reduct of $\mathbb{P}$ wrt the interpretation I.

Fuzzy Answer Sets

Definition

- Let $\mathbb{P}$ be a coherent extended residuated logic program; the unique fuzzy answer set of $\mathbb{P}$ is its least coherent model $\mathbb{P}$.
- Let $\mathbb{P}$ be a general residuated logic program, a coherent L-interpretation I is said to be a fuzzy answer set of $\mathbb{P}$ iff I is the minimal of $\mathbb{P}_I$.

Theorem

Any fuzzy answer set of $\mathbb{P}$ is a minimal model of $\mathbb{P}$.

Conclusions

- We have recalled the definition of the syntax and semantics of extended (general) residuated logic programs
- We have concentrated on the handling of strong negation and, therefore, on extended residuated logic programs.
- We have provided the notion of coherence and we have showed several properties of coherent interpretations.
- We have presented the definition of Fuzzy Answer Set for General Residuated Logic Programs.

Future work

- Relate our approach of fuzzy answer set semantics with other existing approaches and study their possible interactions.
- Obtain further the properties of coherence and establish a more extensive comparison between the notion of coherence and the notion of α-consistency.

Fuzzy Answer Set semantics for Residuated Logic programs

Nicolás Madrid & Manuel Ojeda-Aciego
Universidad de Málaga

September 23, 2009