

Quien soy

Que he hecho

Que estoy haciendo: Current Lines of Research

Que voy a hacer: Projects planned in a (near?) future

Complejidad en Cognición, Computación y Lógica

Joost J. Joosten

Viernes 25 de setiembre
Osuna

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda
- ▶ Formación: Matemáticas

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda
- ▶ Formación: Matemáticas
- ▶ Año de enseñanza: International Baccalaureate School

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda
- ▶ Formación: Matemáticas
- ▶ Año de enseñanza: International Baccalaureate School
- ▶ PhD: Filosofía

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda
- ▶ Formación: Matemáticas
- ▶ Año de enseñanza: International Baccalaureate School
- ▶ PhD: Filosofía
- ▶ Post-Doc

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda
- ▶ Formación: Matemáticas
- ▶ Año de enseñanza: International Baccalaureate School
- ▶ PhD: Filosofía
- ▶ Post-Doc
- ▶ Banco: Risk Management of;
 - Foreign Exchange Derivatives
 - Non-Program-Lending Credit Portfolio

- ▶ Joost J. Joosten: 10-10-1972 Diemen, Holanda
- ▶ Formación: Matemáticas
- ▶ Año de enseñanza: International Baccalaureate School
- ▶ PhD: Filosofía
- ▶ Post-Doc
- ▶ Banco: Risk Management of;
 - Foreign Exchange Derivatives
 - Non-Program-Lending Credit Portfolio
- ▶ Hijos: Paco, Anrés, Lucas

► Lógica de Demostrabilidad

- ▶ Lógica de Demostrabilidad
- ▶ Lógicas de Interpretabilidad (especialización)

- ▶ Lógica de Demostrabilidad
- ▶ Lógicas de Interpretabilidad (especialización)
- ▶ Lógica Modal

- ▶ Lógica de Demostrabilidad
- ▶ Lógicas de Interpretabilidad (especialización)
- ▶ Lógica Modal
- ▶ Proof and Model Theory of Arithmetic

- ▶ Lógica de Demostrabilidad
- ▶ Lógicas de Interpretabilidad (especialización)
- ▶ Lógica Modal
- ▶ Proof and Model Theory of Arithmetic
- ▶ Proof Complexity, Computational Complexity Theory

- ▶ Lógica de Demostrabilidad
- ▶ Lógicas de Interpretabilidad (especialización)
- ▶ Lógica Modal
- ▶ Proof and Model Theory of Arithmetic
- ▶ Proof Complexity, Computational Complexity Theory
- ▶ Intuitionistic Logic and semantics for Sub-Intuitionistic Logics

► Interpretability Logics and Arithmetic;

- ▶ Interpretability Logics and Arithmetic;
- ▶ Provability Algebras and Ordinal Notation Systems based thereon;

- ▶ Interpretability Logics and Arithmetic;
- ▶ Provability Algebras and Ordinal Notation Systems based thereon;
- ▶ Speed-up Phenomena in Small Turing Machines;

- ▶ Interpretability Logics and Arithmetic;
- ▶ Provability Algebras and Ordinal Notation Systems based thereon;
- ▶ Speed-up Phenomena in Small Turing Machines;
- ▶ At the Limits of the Central Limit Theorem.

- ▶ Interpretability logics consider formalized interpretability

- ▶ Interpretability logics consider formalized interpretability
- ▶ $A \triangleright_T B$ stands for $T + A^* \triangleright T + B^*$

- ▶ Interpretability logics consider formalized interpretability
- ▶ $A \triangleright_T B$ stands for $T + A^* \triangleright T + B^*$
- ▶ Different theories have different logics.

- ▶ Interpretability logics consider formalized interpretability
- ▶ $A \triangleright_T B$ stands for $T + A^* \triangleright T + B^*$
- ▶ Different theories have different logics.
- ▶ What logic is the "intersection" of all these logics, that is, holds in any theory?

- ▶ Interpretability logics consider formalized interpretability
- ▶ $A \triangleright_T B$ stands for $T + A^* \triangleright T + B^*$
- ▶ Different theories have different logics.
- ▶ What logic is the "intersection" of all these logics, that is, holds in any theory?
- ▶ This perverse question is the question of the

- ▶ Interpretability logics consider formalized interpretability
- ▶ $A \triangleright_T B$ stands for $T + A^* \triangleright T + B^*$
- ▶ Different theories have different logics.
- ▶ What logic is the "intersection" of all these logics, that is, holds in any theory?
- ▶ This perverse question is the question of the
Interpretability logic of all reasonable arithmetical theories.

The logic IL

- ▶ Axioms

The logic IL

- ▶ Axioms

- ▶ All tautologies

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$
- ▶ $\Box(\Box A \rightarrow A) \rightarrow \Box A$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$
- ▶ $\Box(\Box A \rightarrow A) \rightarrow \Box A$
- ▶ $\Box(A \rightarrow B) \rightarrow (A \triangleright B)$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$
- ▶ $\Box(\Box A \rightarrow A) \rightarrow \Box A$
- ▶ $\Box(A \rightarrow B) \rightarrow (A \triangleright B)$
- ▶ $(A \triangleright B) \wedge (B \triangleright C) \rightarrow (A \triangleright C)$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$
- ▶ $\Box(\Box A \rightarrow A) \rightarrow \Box A$
- ▶ $\Box(A \rightarrow B) \rightarrow (A \triangleright B)$
- ▶ $(A \triangleright B) \wedge (B \triangleright C) \rightarrow (A \triangleright C)$
- ▶ $(A \triangleright C) \wedge (B \triangleright C) \rightarrow ((A \vee B) \triangleright C)$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$
- ▶ $\Box(\Box A \rightarrow A) \rightarrow \Box A$
- ▶ $\Box(A \rightarrow B) \rightarrow (A \triangleright B)$
- ▶ $(A \triangleright B) \wedge (B \triangleright C) \rightarrow (A \triangleright C)$
- ▶ $(A \triangleright C) \wedge (B \triangleright C) \rightarrow ((A \vee B) \triangleright C)$
- ▶ $(A \triangleright B) \rightarrow (\Diamond A \rightarrow \Diamond B)$

The logic IL

▶ Axioms

- ▶ All tautologies
- ▶ $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- ▶ $\Box A \rightarrow \Box \Box A$
- ▶ $\Box(\Box A \rightarrow A) \rightarrow \Box A$
- ▶ $\Box(A \rightarrow B) \rightarrow (A \triangleright B)$
- ▶ $(A \triangleright B) \wedge (B \triangleright C) \rightarrow (A \triangleright C)$
- ▶ $(A \triangleright C) \wedge (B \triangleright C) \rightarrow ((A \vee B) \triangleright C)$
- ▶ $(A \triangleright B) \rightarrow (\Diamond A \rightarrow \Diamond B)$
- ▶ $\Diamond A \triangleright A$

The logic IL

► Axioms

- All tautologies
- $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
- $\Box A \rightarrow \Box \Box A$
- $\Box(\Box A \rightarrow A) \rightarrow \Box A$
- $\Box(A \rightarrow B) \rightarrow (A \triangleright B)$
- $(A \triangleright B) \wedge (B \triangleright C) \rightarrow (A \triangleright C)$
- $(A \triangleright C) \wedge (B \triangleright C) \rightarrow ((A \vee B) \triangleright C)$
- $(A \triangleright B) \rightarrow (\Diamond A \rightarrow \Diamond B)$
- $\Diamond A \triangleright A$

► Rules: MP + Nec

- ▶ For example, with full induction, we can prove Montagna's principle:

$$A \triangleright B \rightarrow (A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ For example, with full induction, we can prove Montagna's principle:

$$A \triangleright B \rightarrow (A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Without induction, we only have isomorphism on a definable cut

- ▶ For example, with full induction, we can prove Montagna's principle:

$$A \triangleright B \rightarrow (A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Without induction, we only have isomorphism on a definable cut
- ▶ Which is sufficient to prove the principle M_0 :

$$A \triangleright B \rightarrow (\Diamond A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ For example, with full induction, we can prove Montagna's principle:

$$A \triangleright B \rightarrow (A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Without induction, we only have isomorphism on a definable cut
- ▶ Which is sufficient to prove the principle M_0 :

$$A \triangleright B \rightarrow (\Diamond A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Using the principle of "Outside big, inside small"

- ▶ For example, with full induction, we can prove Montagna's principle:

$$A \triangleright B \rightarrow (A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Without induction, we only have isomorphism on a definable cut
- ▶ Which is sufficient to prove the principle M_0 :

$$A \triangleright B \rightarrow (\Diamond A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Using the principle of "Outside big, inside small"
- ▶ Project: classify this logic of all reasonable arithmetical theories by stretching the principle of Obis to the extreme

- ▶ For example, with full induction, we can prove Montagna's principle:

$$A \triangleright B \rightarrow (A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Without induction, we only have isomorphism on a definable cut
- ▶ Which is sufficient to prove the principle M_0 :

$$A \triangleright B \rightarrow (\Diamond A \wedge \Box C) \triangleright (B \wedge \Box C)$$

- ▶ Using the principle of "Outside big, inside small"
- ▶ Project: classify this logic of all reasonable arithmetical theories by stretching the principle of Obis to the extreme
- ▶ This is not in particular a research line I would like to pursue a lot further.

- ▶ Let $\langle n \rangle \varphi$ denote that φ is consistent in $EA + Tr_{\Pi_n}$

- ▶ Let $\langle n \rangle \varphi$ denote that φ is consistent in $EA + Tr_{\Pi_n}$
- ▶ One can consider a calculus of *worms* based on a poly-modal version of **GL**

- ▶ Let $\langle n \rangle \varphi$ denote that φ is consistent in $EA + Tr_{\Pi_n}$
- ▶ One can consider a calculus of *worms* based on a poly-modal version of **GL**
- ▶ Natural theories are represented in a natural way by worms:

$$I\Sigma_n \equiv \langle n + 1 \rangle T$$

- ▶ Let $\langle n \rangle \varphi$ denote that φ is consistent in $EA + Tr_{\Pi_n}$
- ▶ One can consider a calculus of *worms* based on a poly-modal version of **GL**
- ▶ Natural theories are represented in a natural way by worms:

$$I\Sigma_n \equiv \langle n + 1 \rangle T$$

- ▶ Various classical conservation results just follow from this calculus together with some minimal additional machinery (Reduction Property, etc)

- ▶ Let $\langle n \rangle \varphi$ denote that φ is consistent in $EA + Tr_{\Pi_n}$
- ▶ One can consider a calculus of *worms* based on a poly-modal version of **GL**
- ▶ Natural theories are represented in a natural way by worms:

$$I\Sigma_n \equiv \langle n + 1 \rangle T$$

- ▶ Various classical conservation results just follow from this calculus together with some minimal additional machinery (Reduction Property, etc)
- ▶ However, many natural theories do not live in this space:

$$PRA \equiv (EA)_{\omega}^1 \quad \omega \text{ times iterated } \langle 1 \rangle$$

- ▶ Project: develop calculus for extended language

- ▶ Project: develop calculus for extended language
- ▶ Plus corresponding machinery (Reduction Property, etc)

- ▶ Project: develop calculus for extended language
- ▶ Plus corresponding machinery (Reduction Property, etc)
- ▶ Obtain alternative and new conservation results

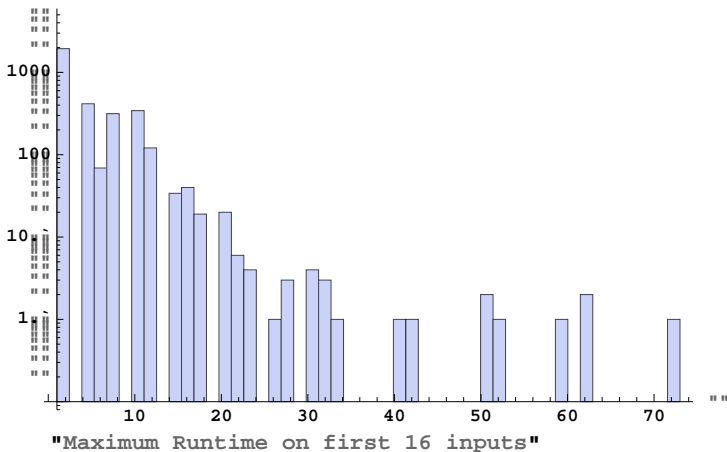
- ▶ Project: develop calculus for extended language
- ▶ Plus corresponding machinery (Reduction Property, etc)
- ▶ Obtain alternative and new conservation results
- ▶ Alternative proof of Solovay's Theorem?

- ▶ Main question: can we witness the phenomenon of intractability already on a very low level?

- ▶ Main question: can we witness the phenomenon of intractability already on a very low level?
- ▶ Approach: look at very small one-sided Turing machines

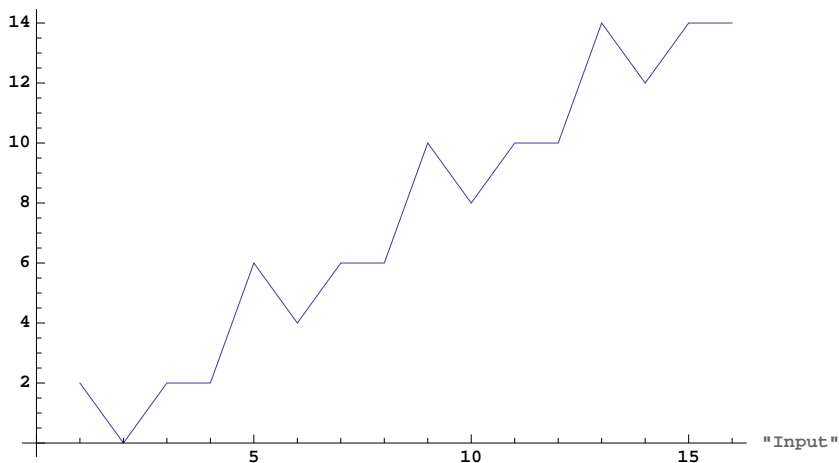
- ▶ Main question: can we witness the phenomenon of intractability already on a very low level?
- ▶ Approach: look at very small one-sided Turing machines
- ▶ See what functions they compute and if there are functions that are not sped-up when computed on larger TM's

"Number of Halting Turing Machines"

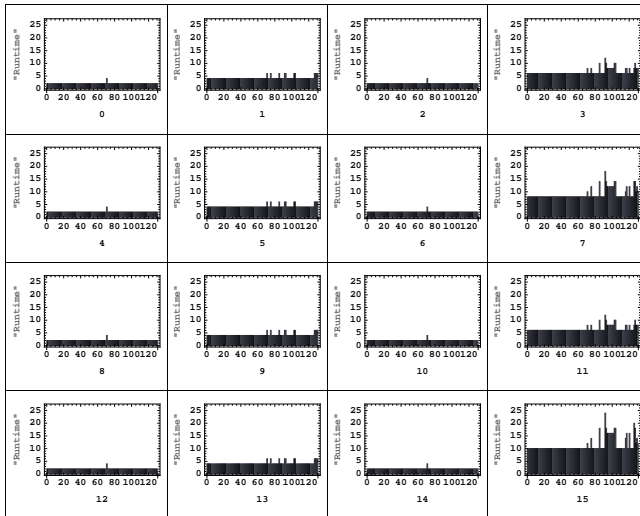


Do these blocks correspond to classes? (2,2 TMs)

"Output"



This function has 130 2,2 TMs that calculate it.

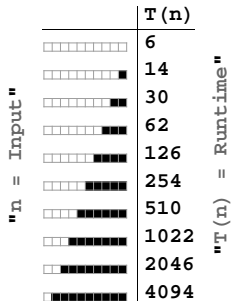


Not all these TMs have the same runtime.

But looking at minima, can we witness (non) speed-up?

"Rulenumber" "2 750 078"							
"f[1]=1 T=6"	"f[2]=2 T=2"	"f[3]=3 T=2"	"f[4]=4 T=14"	"f[5]=5 T=2"	"f[6]=6 T=2"	"f[7]=7 T=6"	"f[8]=8 T=2"
"f[9]=9 T=2"	"f[10]=10 T=6"	"f[11]=11 T=2"	"f[12]=12 T=2"	"f[13]=13 T=30"	"f[14]=14 T=2"	"f[15]=15 T=2"	"f[16]=16 T=6"

We witnessed slow-down!



- ▶ Findings so far:

- ▶ Findings so far:
- ▶ It seems that "slow-down" is more likely than "speed-up"

- ▶ Findings so far:
- ▶ It seems that "slow-down" is more likely than "speed-up"
- ▶ Can we quantify this by means of distributions and theory?

- ▶ Findings so far:
- ▶ It seems that "slow-down" is more likely than "speed-up"
- ▶ Can we quantify this by means of distributions and theory?
- ▶ Is this at the core of some evolutionary principle?

- ▶ Next step: move on to only exp-time bounded functions

- ▶ Next step: move on to only exp-time bounded functions
- ▶ Find examples which do not seem to allow speed-up

- ▶ Next step: move on to only exp-time bounded functions
- ▶ Find examples which do not seem to allow speed-up
- ▶ Try to prove necessary properties of those functions (e.g. NP completeness)

- ▶ Next step: move on to only exp-time bounded functions
- ▶ Find examples which do not seem to allow speed-up
- ▶ Try to prove necessary properties of those functions (e.g. NP completeness)
- ▶ Work from here on. . .

► Bold statement of the CLT

- ▶ Bold statement of the CLT
- ▶ Sums of arbitrary equal probability distributions tend to a normal distribution

- ▶ Bold statement of the CLT
- ▶ Sums of arbitrary equal probability distributions tend to a normal distribution
- ▶ Rephrased:

- ▶ Bold statement of the CLT
- ▶ Sums of arbitrary equal probability distributions tend to a normal distribution
- ▶ Rephrased:

Averages of samples of whatever distribution will be normally distributed around the expected value

- ▶ Bold statement of the CLT
- ▶ Sums of arbitrary equal probability distributions tend to a normal distribution
- ▶ Rephrased:

Averages of samples of whatever distribution will be normally distributed around the expected value

- ▶ More formally:

$$\sum_{i=1}^n X_i \longrightarrow N(\mu n, \sigma \sqrt{n})$$

- ▶ Bold statement of the CLT
- ▶ Sums of arbitrary equal probability distributions tend to a normal distribution
- ▶ Rephrased:

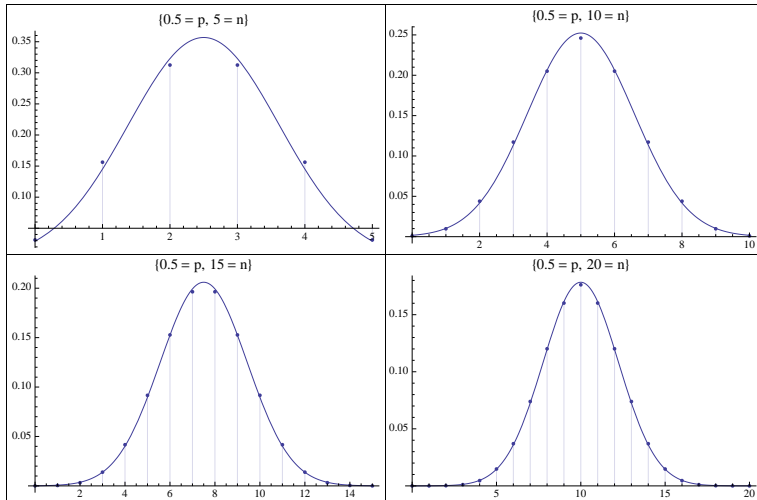
Averages of samples of whatever distribution will be normally distributed around the expected value

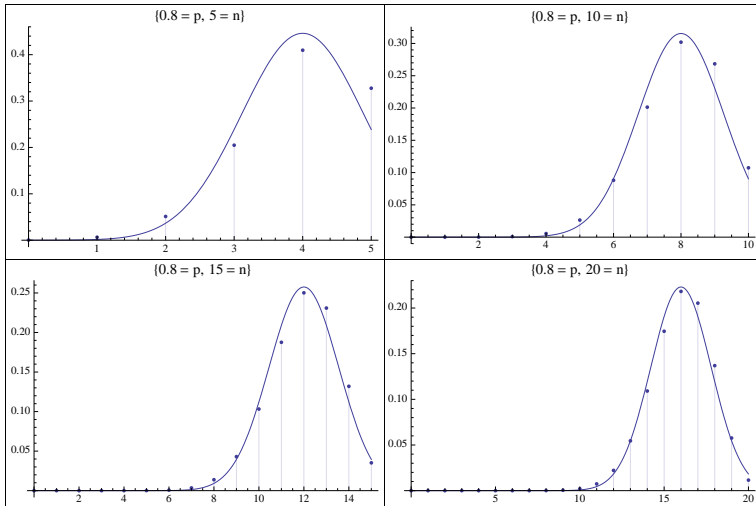
- ▶ More formally:

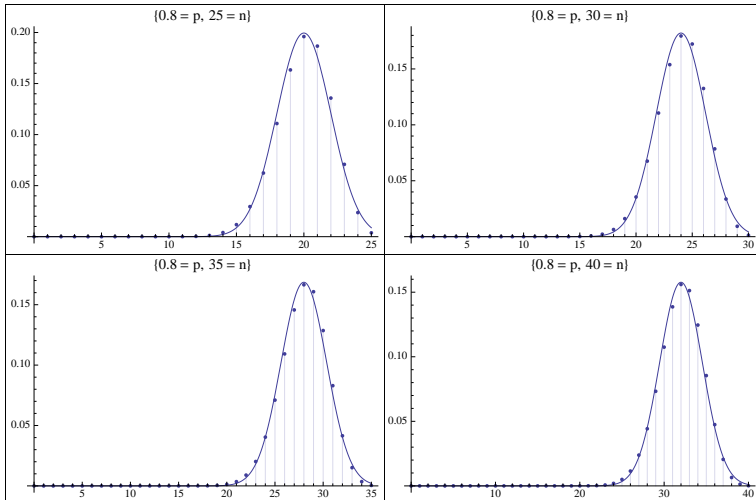
$$\sum_{i=1}^n X_i \longrightarrow N(\mu n, \sigma \sqrt{n})$$

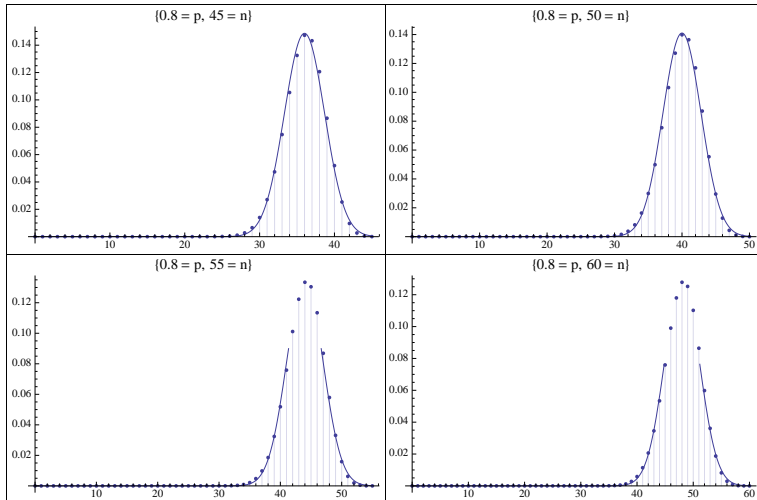
- ▶ Equivalently:

$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i - \mu n}{\sigma \sqrt{n}} = N(0, 1)$$











$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i - \mu n}{\sigma \sqrt{n}} = N(0, 1)$$



$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i - \mu n}{\sigma \sqrt{n}} = N(0, 1)$$

- ▶ Where X_i are all equal and independent but arbitrarily distributed with finite mean μ and finite standard deviation σ



$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i - \mu n}{\sigma \sqrt{n}} = N(0, 1)$$

- ▶ Where X_i are all equal and independent but arbitrarily distributed with finite mean μ and finite standard deviation σ
- ▶ There are tons of variations and generalizations of this theorem



$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i - \mu n}{\sigma \sqrt{n}} = N(0, 1)$$

- ▶ Where X_i are all equal and independent but arbitrarily distributed with finite mean μ and finite standard deviation σ
- ▶ There are tons of variations and generalizations of this theorem
- ▶ One can drop the demand for identicality of the X_i by adding some other requirements (Lyapunov, Lindeberg)



$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i - \mu n}{\sigma \sqrt{n}} = N(0, 1)$$

- ▶ Where X_i are all equal and independent but arbitrarily distributed with finite mean μ and finite standard deviation σ
- ▶ There are tons of variations and generalizations of this theorem
- ▶ One can drop the demand for identicality of the X_i by adding some other requirements (Lyapunov, Lindeberg)
- ▶ One can weaken the independence requirement

► Our interest in the Theorem

- ▶ Our interest in the Theorem
- ▶ Understand the ins and outs of the CLT

- ▶ Our interest in the Theorem
- ▶ Understand the ins and outs of the CLT
- ▶ Why is it so stable?

- ▶ Our interest in the Theorem
- ▶ Understand the ins and outs of the CLT
- ▶ Why is it so stable?
- ▶ Run simulations at the border of convergence to see if other forms than the Gaussian are possible
(Within convergence there are other forms possible (stable distributions))

- ▶ Our interest in the Theorem
- ▶ Understand the ins and outs of the CLT
- ▶ Why is it so stable?
- ▶ Run simulations at the border of convergence to see if other forms than the Gaussian are possible
(Within convergence there are other forms possible (stable distributions))
- ▶ What are epistemic consequences of the CLT?

- ▶ Our interest in the Theorem
- ▶ Understand the ins and outs of the CLT
- ▶ Why is it so stable?
- ▶ Run simulations at the border of convergence to see if other forms than the Gaussian are possible
(Within convergence there are other forms possible (stable distributions))
- ▶ What are epistemic consequences of the CLT?
- ▶ To what extent are the existing proofs *explanatory* or just *merely justificatory*?

- ▶ Our interest in the Theorem
- ▶ Understand the ins and outs of the CLT
- ▶ Why is it so stable?
- ▶ Run simulations at the border of convergence to see if other forms than the Gaussian are possible
(Within convergence there are other forms possible (stable distributions))
- ▶ What are epistemic consequences of the CLT?
- ▶ To what extent are the existing proofs *explanatory* or just *merely justificatory*?
- ▶ What was the original proof of de Moivre (without Taylor expansion?)?

► Complexity in Cognition

- ▶ Complexity in Cognition
- ▶ Complexity in Reductionism

- ▶ Complexity in Cognition
- ▶ Complexity in Reductionism
- ▶ Complexity measures in dynamical systems and links to complexity measures in other fields

- ▶ Complexity in Cognition
- ▶ Complexity in Reductionism
- ▶ Complexity measures in dynamical systems and links to complexity measures in other fields
- ▶ Overview paper on form-generating mechanisms

Questions:

- ▶ "Do we prefer a certain coarseness in our scientific analyses?"

Questions:

- ▶ "Do we prefer a certain coarseness in our scientific analyses?"
- ▶ Where is our pattern recognition optimal and is this related to the first question?

- ▶ At first: take a cognitive psychological approach

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?
- ▶ Kolmogorov complexity says that few distributions are compressible

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?
- ▶ Kolmogorov complexity says that few distributions are compressible
- ▶ However, we are not always compressing

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?
- ▶ Kolmogorov complexity says that few distributions are compressible
- ▶ However, we are not always compressing
- ▶ My phone number has a long but not so easy "compression"

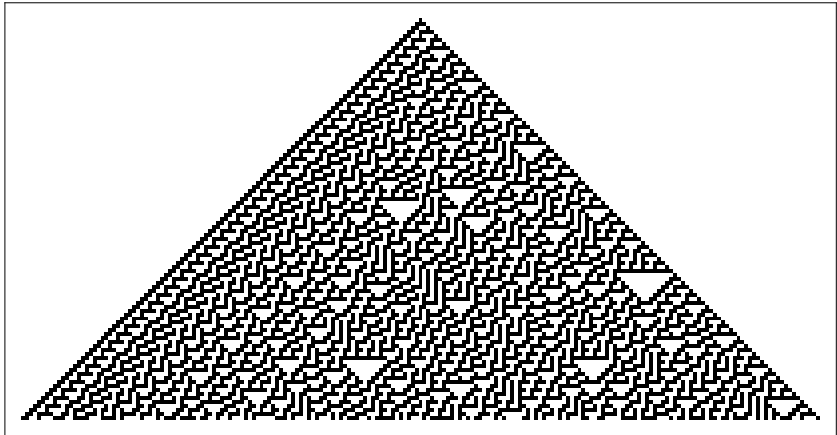
- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?
- ▶ Kolmogorov complexity says that few distributions are compressible
- ▶ However, we are not always compressing
- ▶ My phone number has a long but not so easy "compression"
- ▶ Might it be that in certain areas there are just a lot of "compressions" around that we accept as "compressions"?

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?
- ▶ Kolmogorov complexity says that few distributions are compressible
- ▶ However, we are not always compressing
- ▶ My phone number has a long but not so easy "compression"
- ▶ Might it be that in certain areas there are just a lot of "compressions" around that we accept as "compressions"?
- ▶ Can this account for the phrase

- ▶ At first: take a cognitive psychological approach
- ▶ For example, draw squares with groups of points and ask if there is a pattern?
- ▶ For which ratio $\frac{\text{size of square}}{\text{amount of dots}}$ is the pattern density "optimal"?
- ▶ Kolmogorov complexity says that few distributions are compressible
- ▶ However, we are not always compressing
- ▶ My phone number has a long but not so easy "compression"
- ▶ Might it be that in certain areas there are just a lot of "compressions" around that we accept as "compressions"?
- ▶ Can this account for the phrase

Ain't that a coincidence?

Interesting and simple dynamical systems are cellular automata



- ▶ For example, certain very simple cellular automata are known to be Turing Complete

- ▶ For example, certain very simple cellular automata are known to be Turing Complete
- ▶ The pictures they generate can be given fractal dimensions

- ▶ For example, certain very simple cellular automata are known to be Turing Complete
- ▶ The pictures they generate can be given fractal dimensions
- ▶ Investigate the link between the dimensions and the computational complexity of the automata

- ▶ For example, certain very simple cellular automata are known to be Turing Complete
- ▶ The pictures they generate can be given fractal dimensions
- ▶ Investigate the link between the dimensions and the computational complexity of the automata
- ▶ More in general: can complexity notions in one framework be formally linked to complexity notions in another framework?

- ▶ Times is nothing but iterated addition

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition
- ▶ However, it is a lot more

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition
- ▶ However, it is a lot more
- ▶ Can we somehow use this as an analogy to incorporate the role of complexity in reductionism in theories of the philosophy of science

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition
- ▶ However, it is a lot more
- ▶ Can we somehow use this as an analogy to incorporate the role of complexity in reductionism in theories of the philosophy of science
- ▶ Or any other form of reductionism

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition
- ▶ However, it is a lot more
- ▶ Can we somehow use this as an analogy to incorporate the role of complexity in reductionism in theories of the philosophy of science
- ▶ Or any other form of reductionism
- ▶ Our behavior is optimizing our utility function

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition
- ▶ However, it is a lot more
- ▶ Can we somehow use this as an analogy to incorporate the role of complexity in reductionism in theories of the philosophy of science
- ▶ Or any other form of reductionism
- ▶ Our behavior is optimizing our utility function
- ▶ All matter exists of atoms

- ▶ Times is nothing but iterated addition
- ▶ The Ackermann function is nothing but iterated addition
- ▶ However, it is a lot more
- ▶ Can we somehow use this as an analogy to incorporate the role of complexity in reductionism in theories of the philosophy of science
- ▶ Or any other form of reductionism
- ▶ Our behavior is optimizing our utility function
- ▶ All matter exists of atoms
- ▶ Number Theory can be reduced to Logic

- ▶ I find the interplay between mechanisms that generate complexity (Rule 30) versus those that generate simplicity (Central Limit Theorem) fascinating

- ▶ I find the interplay between mechanisms that generate complexity (Rule 30) versus those that generate simplicity (Central Limit Theorem) fascinating
- ▶ It would be an interesting enterprise to make an overview of the main mechanisms that shape our universe

- ▶ I find the interplay between mechanisms that generate complexity (Rule 30) versus those that generate simplicity (Central Limit Theorem) fascinating
- ▶ It would be an interesting enterprise to make an overview of the main mechanisms that shape our universe
 - ▶ Symmetries and broken symmetries

- ▶ I find the interplay between mechanisms that generate complexity (Rule 30) versus those that generate simplicity (Central Limit Theorem) fascinating
- ▶ It would be an interesting enterprise to make an overview of the main mechanisms that shape our universe
 - ▶ Symmetries and broken symmetries
 - ▶ Catastrophe sets

- ▶ I find the interplay between mechanisms that generate complexity (Rule 30) versus those that generate simplicity (Central Limit Theorem) fascinating
- ▶ It would be an interesting enterprise to make an overview of the main mechanisms that shape our universe
 - ▶ Symmetries and broken symmetries
 - ▶ Catastrophe sets
 - ▶ Iterations (Cellular Automata, Dynamical Systems)

- ▶ I find the interplay between mechanisms that generate complexity (Rule 30) versus those that generate simplicity (Central Limit Theorem) fascinating
- ▶ It would be an interesting enterprise to make an overview of the main mechanisms that shape our universe
 - ▶ Symmetries and broken symmetries
 - ▶ Catastrophe sets
 - ▶ Iterations (Cellular Automata, Dynamical Systems)
 - ▶ Statistical convergence like the CLT