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# FIXED POINT THEOREMS UNDER THE INTERIOR CONDITION

**Antonio Jiménez Melado**

***University of Málaga, Spain***

**melado@uma.es**

In 2006, A. Jiménez-Melado and Claudio H. Morales introduced the so-called Interior Condition (I-C), and showed that it could be a substitute of the Leray-Schauder boundary condition (L-S) in some well known fixed point theorems for set condensing operators, and also for nonexpansive mappings. To be more concrete, if $G$ is a bounded, open subset of a Banach space $X$, with $0 \in G$, we say that a mapping $T : \overline{G} \to X$ satisfies the condition (I-C) if there exists $\delta > 0$ such that

$$T(x) \neq \lambda\, x \quad \text{for } x \in G^{\star}, \lambda > 1 \text{ and } T(x) \notin \overline{G},$$

where $G^{\star} = \{x \in G : \operatorname{dist}(x, \partial G) < \delta\}$. A. Jiménez-Melado and Claudio H. Morales proved that if $G$ is strictly star-shaped and $T : \overline{G} \to X$ is a set-condensing mapping satisfying (I-C) then $T$ has a fixed point. In the above context, the strictly star-shaped sets (with respect to the origin) are a class of sets which remain between the star-shaped sets and the convex sets, and are defined as those $G$ for which any ray departing from the origin intersects its boundary at most at one point.

In this talk we present these basic facts about (L-S), (I-C) and its relations.