

FIXED POINT THEOREMS UNDER THE INTERIOR CONDITION

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In 2006, A. Jiménez-Melado and Claudio H. Morales introduced the so-called Interior Condition (I-C), and showed that it could be a substitute of the Leray-Schauder boundary condition (L-S) in some well known fixed point theorems for set condensing operators, and also for nonexpansive mappings. To be more concrete, if G is a bounded, open subset of a Banach space X, with $0 \in G$, we say that a mapping $T : \overline{G} \to X$ satisfies the condition (I-C) if there exists $\delta > 0$ such that

 $T(x) \neq \lambda x$ for $x \in G^*, \lambda > 1$ and $T(x) \notin \overline{G}$,

where $G^{\star} = \{x \in G : \operatorname{dist}(x, \partial G) < \delta\}$. A. Jiménez-Melado and Claudio H. Morales proved that if G is strictly star-shaped and $T : \overline{G} \to X$ is a set-condensing mapping satisfying (I-C) then T has a fixed point. In the above context, the strictly star-shaped sets (with respect to the origin) are a class of sets which remain between the star-shaped sets and the convex sets, and are defined as those G for which any ray departing from the origin intersects its boundary at most at one point.

In this talk we present these basic facts about (L-S), (I-C) and its relations.