Frequencies of invariant tori in degenerate or quasi-periodically non-autonomous Hamiltonian systems

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Abstract

We revisit two important achievements in the finite-dimensional Hamiltonian KAM theory of the last decade, namely, 1) the existence of invariant tori of dimensions from N to $n + N$ in Hamiltonian systems with n degrees of freedom and with a perturbation depending on time quasi-periodically with N basic frequencies [A.Jorba and J.Villanueva, J. Nonlinear Sci. 7 (1997), 427-473] and 2) the partial preservation of the frequencies of invariant m-tori in moderately degenerate Hamiltonian systems with n degrees of freedom under small perturbations [S.-N.Chow, Y.Li, and Y.Yi, J. Nonlinear Sci. 12 (2002), 585-617 for the case $m = n$; Y.Li and Y.Yi, J. Differential Equat. 208 (2005), 344-387 for the case $m \leq n$]. The original proofs of these results are very complicated.

In the talk, new proofs of the results by A.Jorba, J.Villanueva, S.-N.Chow, Y.Li, and Y.Yi will be presented. These new proofs are exceptionally simple and are based on the so called 'decoupling' technique invented by M.R.Herman in 1990 and then developed in the mid nineties in the works by H.W.Broer, G.B.Huitema, and the speaker. The 'decoupling' approach enables one to reduce the degenerate problem at hand to a nondegenerate set-up by introducing external parameters. In this way, one succeeds in avoiding a lengthy and tiring KAM machinery (especially involved in degenerate settings), while all the degeneracies are translated into suitable number-theoretical prepositions: the KAM iteration procedure and the Diophantine approximations on submanifolds are 'decoupled'. Some new results are also expected to be expounded.