The Evans function and Fredholm determinants

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Abstract

This talk is based on a joint paper with Fritz Gesztesy and Konstantin A. Makarov. The Evans function is a widely known Wronskian-type object, which is being used to detect isolated eigenvalues of differential operators obtained as linearizations of nonlinear partial differential equations along such special solutions as traveling waves. The Fredholm determinants considered in the talk are the modified Fredholm determinants of certain integral operators with semi-separable kernels.

Using the theory of Lyapunov exponents, we give a definition of the Evans function for quite general first order nonatonomous matrix differential equations on the line in the context of perturbation theory. The central result of the talk is a formula relating the Evans function and the modified Fredholm determinant of the so-called "sandwiched resolvent". This result is obtained under the assumption that the unperturbed equation has exponential dichotomy on the line, and that the perturbation has certain exponential decay at infinities controlled by the general Lyapunov exponents of the unperturbed system.